Crib for Section A of 1A Paper 3 2006
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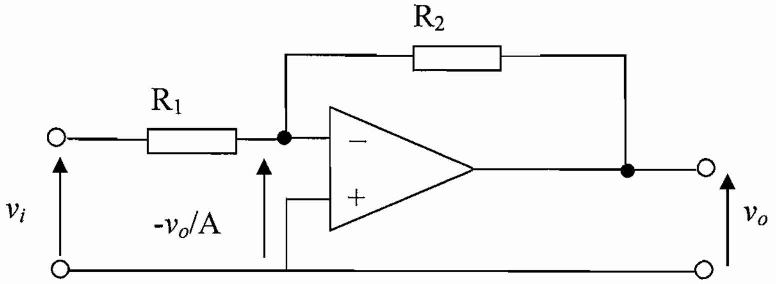
1. i) Open loop gain A is infinite.
Input resistance is infinite.
Output resistance is zero.

ii) Ideal op-amp so $v_- = v_+ = 0$ by virtual earth principle. Also no current flows into the inverting input since input resistance is infinite. Therefore:

$v_i/600 + v_o/12000 = 0$ and so Gain $= v_o/v_i = -12000/600 = -20$

Input current $i_{in} = v_i/600$ Input resistance $= v_i/i_{in} = 600\ \Omega$

iii)



Summing currents at the inverting input:

$v_i-(-v_o/A)/R_1 + v_o-(-v_o/A)/R_2 = 0$

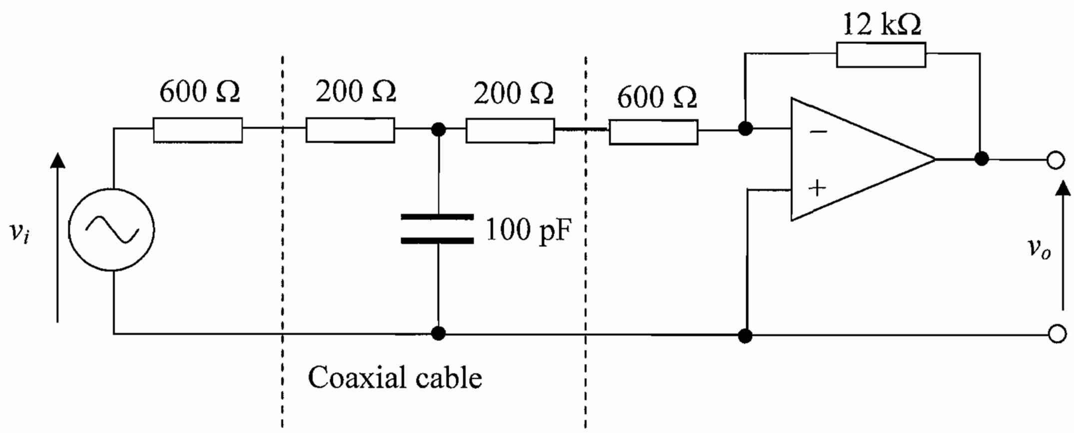
$v_o(R_2+R_1+AR_1) = -R_2Av_i$

$v_o/v_i = -AR_2/(R_2+R_1+AR_1)$ Check: As $A \to \infty$ $v_o/v_i \to -R_2/R_1$ as required.

Putting in numbers gives $v_o/v_i = -19.96$

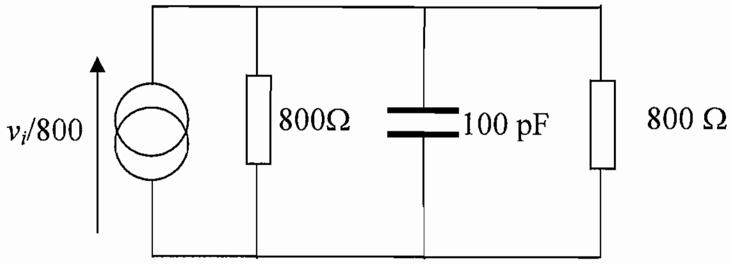
$i_{in} = (v_i-(-v_o/A))/R_1 = (v_i-19.96v_i/10^4)/600$ $R_{in} = v_i/i_{in} = 600/(1-19.96\times10\text{-}4) = 601.2\ \Omega$

iv)



The mid-band voltage gain assumes that ω is low enough so that the 100 pF capacitor is an open-circuit. In this case all the series resistors in the circuit above may be lumped together to give 1600 Ω. Using earlier results:
Mid-band voltage gain v_o/v_i = -12000/1600 = -7.5

For –3 dB frequency the voltage across the 100 pF capacitor needs to fall to 1/√2 of its mid-band value (since the output voltage is proportional to the capacitor voltage). To obtain this voltage, convert left-hand side of circuit to Norton equivalent and use the fact that the inverting input of the op-amp is at 0 V by virtual earth principle:



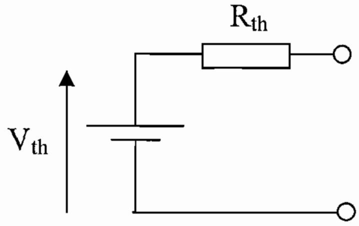
The two 800 Ω resistors may be combined in parallel to give 400 Ω. Denoting R = 400Ω and C = 100 pF the capacitor voltage, v_c, is:

v_c = (R× 1/jωC)/(R+1/jωC) × v_i/2R = R/(1+jωCR)× v_i/2R = v_i /(2(1+jωCR))

-3dB frequency is found by equating the magnitudes of the real and imaginary parts of the denominator:

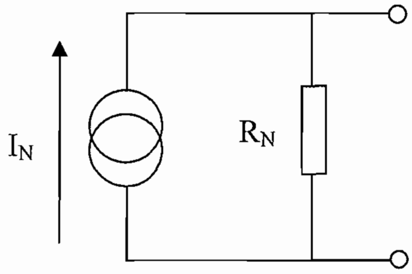
ωCR = 1 so f_{3dB} = 1/(2πCR) = 3.98 MHz

2. a) Thevenin's theorem: Any linear circuit, insofar as the load connected to it is concerned, may be represented as a voltage, V_{th}, in series with a resistance, R_{th}:



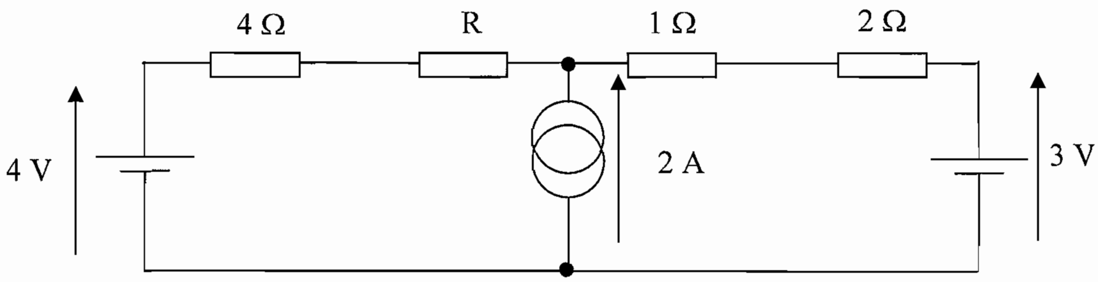
where V_{th} is the open-circuit voltage of the linear circuit, V_{OC}, and $R_{th} = V_{OC}/I_{SC}$ where I_{SC} is the short-circuit current of the linear circuit.

Norton's theorem: Any linear circuit, insofar as the load connected to it is concerned, may be represented as a current source, I_N, in parallel with a resistance R_N:

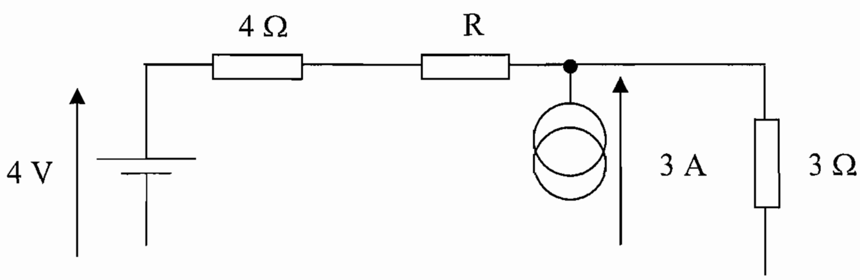


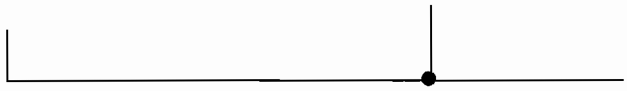
where $I_N = I_{SC}$ and $R_N = V_{OC}/I_{SC}$

b) Convert left-hand side of the circuit to Thevenin equivalent, and do the same to the right-hand side:

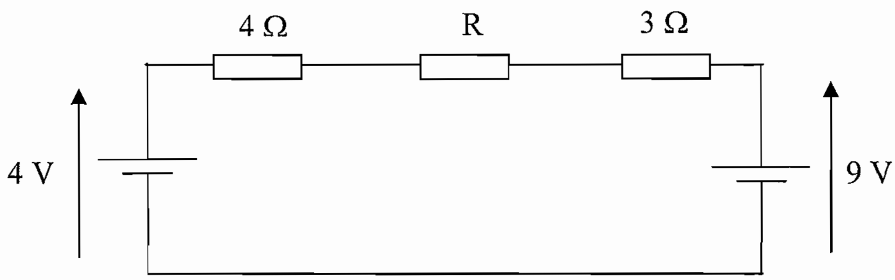


Now convert right-hand side to Norton to give a 1 A current source in parallel with a 3 Ω resistor, and combine this 1 A current source with the 2 A current source by adding (since they are in parallel):





Finally, convert the right-hand side of the circuit back to Thevenin to give:



i) $R = 13\ \Omega$ $I = (9-4)/20 = 0.25$ A

ii) $R = 3\ \Omega$ $I = (9-4)/10 = 0.5$ A

c) i) Find the total impedance as seen by the 240 V voltage source:

Impedance of the series combination of the capacitor, inductor and 1 Ω resistor is

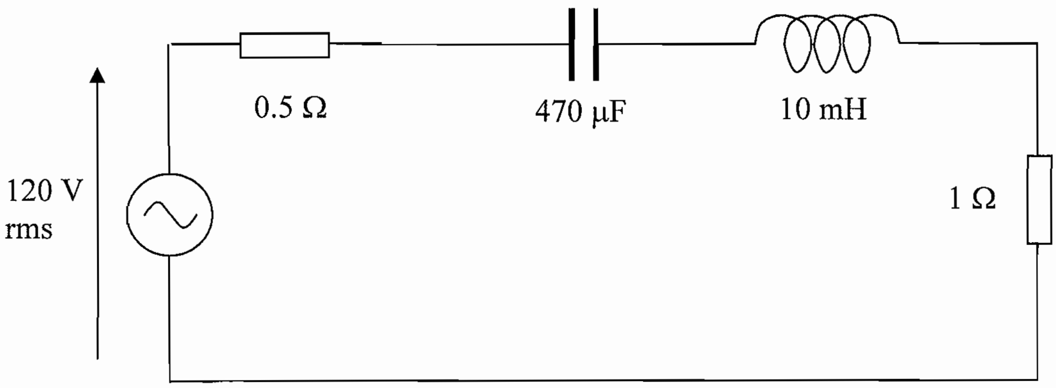
$1 + jwL + 1/jwC = 1 + j(3.14 - 6.77) = 1 - j3.63$

Combine this in parallel with the 1 Ω resistor and add the series 1 Ω resistor:

$Z = 1 + 1\times(1 - j3.63)/(1+1 - j3.63) = 1.884 - j\,0.211$
Finally $I = 240/Z = 126.6 \angle +6.4^0$

Examiner's comment: A common mistake here was to convert to Thevenin and find the current. This gives the current flowing in the inductor/capacitor ie the 'load' current, but not the supply current as asked for.

ii) Convert left-hand side of circuit to Thevenin:



ii) Resonant frequency $\omega_0 = 1/\sqrt{LC}$ and $f_0 = \omega_0/2\pi$ giving 73.4 Hz

At resonance the impedance of the capacitor and inductor cancel out so that the only impedance is the 1.5 Ω of resistance. Therefore:

$I = 120/1.5 = 80\ A \angle 0$

Capacitor voltage $V_c = X_C \times I$ where X_C is the reactance of the capacitor
$= 80/\omega C = 369\ V \angle -90$

$Q = \omega_0 L/R = 4.61/1.5 = 3.07$

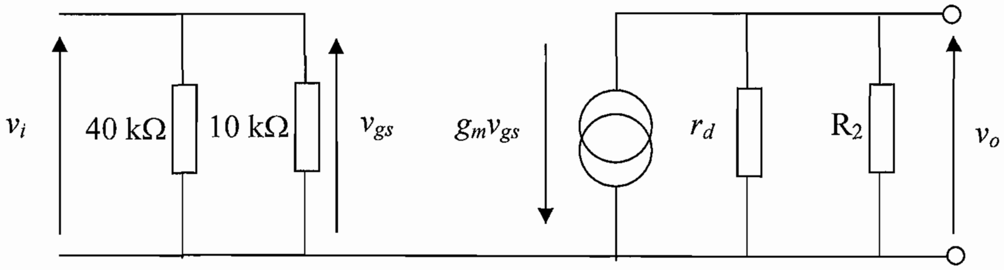
3. i) For $V_{GS} = 4$ V by potential divider:

$10000 \times 20/(10000 + R_1) = 4$ giving $R_1 = 40$ kΩ

For $V_{DS} = 10$ V, 10 V must be dropped across R_2 with 5 mA flowing through it:

$10/R_2 = 0.005$ giving $R_2 = 2$ kΩ

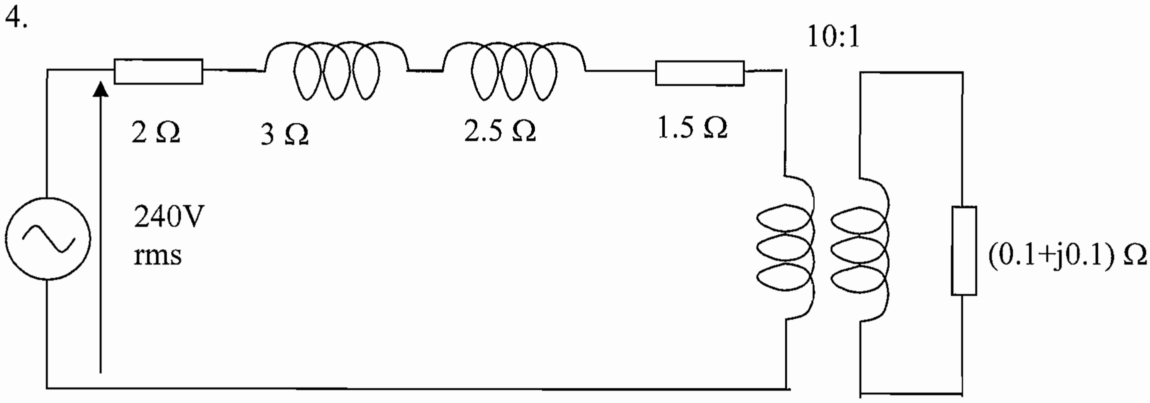
ii) To construct small-signal circuit valid at mid-band, treat all capacitors as short circuits and short-circuit all points at fixed voltage ie 20 V supply rail to ground:



$v_{gs} = v_i$

$v_o = g_m v_{gs} r_d R_2/(r_d+R_2)$ giving $v_o/v_i = -9.375$

4.



i) Load referred to primary = $(N_1/N_2)^2 Z_L$ = 100 (0.1+j0.1) = (10 + j 10) Ω

ii) I_L' = 240/(13.5 + j 15.5) = 11.7 A in magnitude.

This is the load current referred to the primary.

$I_L = 10 I_L'$ = 117 A in magnitude.

Examiner's comment: A common mistake here was to forget to calculate the actual load current as opposed to the load current referred to the primary.

iii) $P_{LOAD} = 11.7^2 \times 10 = 1369$ W

$P_{LOSS} = 11.7^2 \times (2 + 1.5) = 479$ W

Therefore input power, $P_{IN} = P_{LOAD} + P_{LOSS} = 1848$ W

Efficiency = P_{LOAD}/P_{IN} = 1369/1848 = 74.1 %

5. i) Total impedance, $Z_T = 0.5 + j2 + 10 + j5 = (10.5 + j\,7)\ \Omega$

$I = 240/(10.5+j7) = 19$ A in magnitude

$P_{LOAD} = 19^2 \times 10 = 3.62$ kW $\quad Q_{LOAD} = 19^2 \times 5 = 1.81$ kVAr

ii) Need to find total reactive power supplied

$Q_{TOTAL} = 19^2 \times 7 = 2.53$ kVAr

Capacitor reactive power must equal total reactive power to correct power factor to unity:

$240^2/X_C = 2530$ where X_C is the capacitor reactance at the supply frequency of 50 Hz

$X_C = 22.75\ \Omega = 1/(100\pi C)$ giving $C = 140\ \mu F$

Engineering Tripos Part 1A 2006 - Paper 3 - Sections B and C answers

A R L Travis

q 6

C	A=00	01	11	10
B=00	00001	00001	00001	00001
01	00000	00001	00011	00010
11	00000	00001	11011	01000
10	00000	00001	01001	00100

C_4	A=00	01	11	10
B=00	0	0	0	0
01	0	0	0	0
11	0	0	1	0
10	0	0	0	0

C_3	A=00	01	11	10
B=00	0	0	0	0
01	0	0	0	0
11	0	0	1	1
10	0	0	1	0

C_2	A=00	01	11	10
B=00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	1

C_1	A=00	01	11	10
B=00	0	0	0	0
01	0	0	1	1
11	0	0	1	0
10	0	0	0	0

C_0	A=00	01	11	10
B=00	1	1	1	1
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

So:

$C_4 = A_1A_0B_1B_0,$

$C_3 = A_1A_0B_1 + A_1B_1B_0$

$C_2 = A_1\overline{A}_0B_1\overline{B}_0$

$C_1 = A_1A_0B_0 + A_1\overline{B}_1B_0$

$C_0 = A_0 + \overline{B}_1\overline{B}_0$

(b)

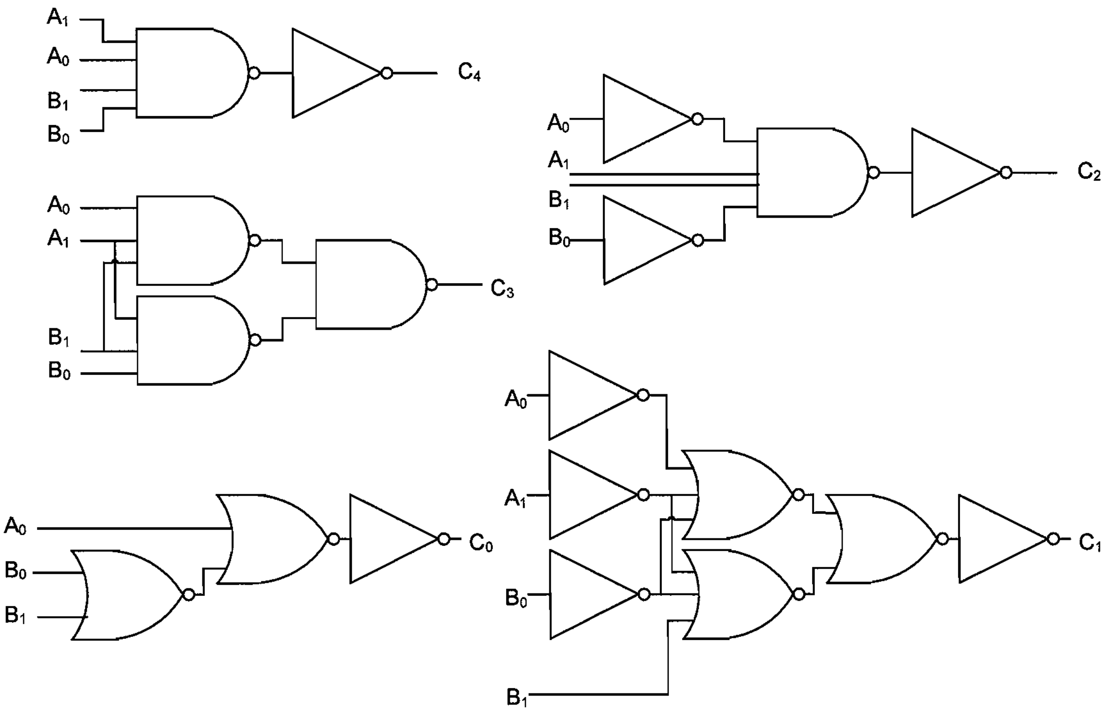
$C_4 = \overline{\overline{A_1A_0B_1B_0}}$

$C_3 = \overline{\overline{A_1A_0B_1}.\overline{A_1B_1B_0}}$

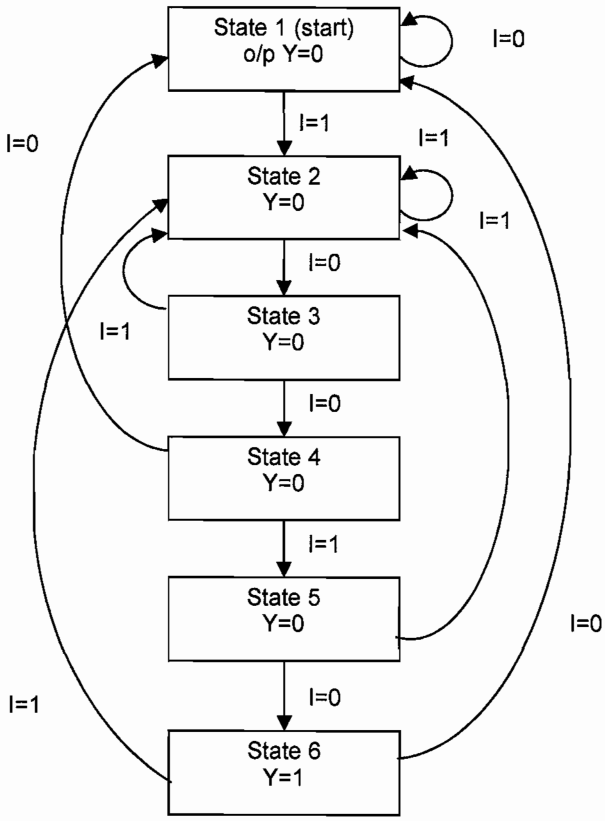
$C_2 = \overline{\overline{A_1\overline{A}_0B_1\overline{B}_0}}$

$C_1 = A_1A_0B_0 + A_1\overline{B}_1B_0 = \overline{\overline{\overline{\overline{A}_1 + \overline{A}_0 + \overline{B}_0} + \overline{\overline{A}_1 + B_1 + \overline{B}_0}}}$

$C_0 = A_0 + \overline{B}_1\overline{B}_0 = \overline{\overline{A_0 + \overline{B_1 + B_0}}}$



q 7



There are 6 states so 3 bistables are required to implement the system

q 8 (i) A←#FC = 11111100B
B←#3D = 00111101B
A←sum = 00111001B + carry

The carry into bit 7 equals the carry out of bit 7, so V is not set
There has been a carry so C is set
The result is not a negative number so N is not set
The result is not zero so Z is not set
The contents of B are 3DH
The contents of A are 00111001B

(ii) Sum of clock cycles per instruction = 2+2+2+2+2 = 10 so code takes 1 microsecond.

q 9

A	B	C	D	E	F	G	H	I	J	K	L	M	N
0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110
√				√	√		√	√		√	√	√	√

C	**X_1X_0=00**	**01**	**11**	**10**
X_3X_2=00	X	1	0	0
01	0	1	0	1
11	1	1	X	1
10	1	1	1	0

$$C = \overline{X}_1 X_0 + X_3 \overline{X}_1 + X_3 X_0 + X_2 X_1 \overline{X}_0$$

q 10 By symmetry, around any circle concentric with the annuliu, H is constant. Ampère's Law states that $NI = H2\pi r$

Now $\mathbf{B} = \mu_r \mu_o \mathbf{H}$ so:

$$\mathbf{B} = \mu_r \mu_o \frac{NI}{2\pi r} \mathbf{e}_\theta$$

where $\mathbf{e}_\theta$ is the tangential unit vector. So:

(a) $\mathbf{B} = 100 \times 4\pi \times 10^{-7} \dfrac{100 \times 1}{2\pi r} \mathbf{e}_\theta$

(b) $\mathbf{B} = 4\pi \times 10^{-7} \dfrac{100 \times 1}{2\pi r} \mathbf{e}_\theta = \dfrac{2 \times 10^{-5}}{r} \mathbf{e}_\theta$

$$\phi = \int_S \mathbf{B.dA} = \int_{r1}^{r2} \left(\frac{2 \times 10^{-5}}{r} \mathbf{e}_\theta \right) . (t dr \mathbf{e}_\theta) = 2 \times 10^{-5} t \int_{r1}^{r2} \frac{dr}{r} = 2 \times 10^{-5} t \ln \frac{r2}{r1} = 2 \times 10^{-8} \ln \frac{30}{20}$$

$= 8.1 \times 10^{-9}$ Wb

(c) $L = \phi / I = 8.1 \times 10^{-9}$H

(d) The relationship between B and H is non-linear in both Permalloy and Columax q 11 $Q = C_1 V_1 = C_2 V_2$ so:

$$V_2 = \frac{C_1}{C_2} V_1 = \frac{r_1}{r_2} V_1 = 200 \text{ kV}$$

$$E_2 - E_1 = ½(C_2V_2^2 - C_2V_1^2) = ½4\pi\varepsilon_0(r_2V_2^2 - r_1V_1^2) = 2\pi\varepsilon_0(½ \times 4 \times 10^9 - 10^9) = 2\pi\varepsilon_0 \times 10^9$$

q 12 By symmetry, the direction of the electric flux must be radial.

According to Gauss:

$$D4\pi r^2 = \int ar \times 4\pi r^2 dr$$

$$D4\pi r^2 = \pi a r^4$$

$$D = ar^2/4$$

Energy stored within a small volume, $V = \int_V \mathbf{B.dH} = \frac{1}{2}\mu_r\mu_0 H^2$

Energy stored within one annulus $= \frac{1}{2}\mu_r\mu_0 \times 2\pi \times t \times \int_{r1}^{r2} H^2 dr$

$$= \frac{\mu_r\mu_0 2\pi t}{2} \times \left(\frac{NI}{2\pi}\right)^2 \times \int_{r1}^{r2} \frac{1}{r^2} dr$$

$$= \frac{\mu_r\mu_0 t N^2 I^2}{4\pi} \times \left(\frac{1}{r_2^3} - \frac{1}{r_1^3}\right)$$