ENGINEERING TRIPOS PART IA

Wednesday 7 June 2006 9 to 12

Paper 1

MECHANICAL ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 **(short)** Water flows through a rectangular channel of uniform width with an upstream depth and speed of h and V respectively. A hump of height δ is placed on the channel bed over its entire width. The free surface then has a dip of depth d centred over the hump (Fig. 1). The velocity may be taken as uniform at each cross-section of the channel.

(a) By applying Bernoulli's equation to the free-surface streamline show that the fluid speed above the bump, V_*, is given by

$$V_*^2 = V^2 + 2gd.$$

State any assumptions that you make. [5]

(b) Find an independent expression relating V_* to V and hence show that, if $\delta = d$,

$$\frac{V^2}{2gd} = \left(\frac{V^2}{2gd}+1\right)\left(1-\frac{2d}{h}\right)^2$$

[5]

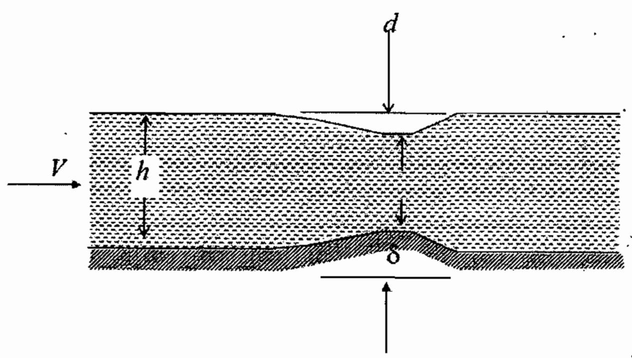

Fig. 1

2 (**short**) A jet of water is deflected by a stationary bucket as shown in Fig. 2. The speed of the fluid leaving the nozzle, as well as that leaving the bucket, is V and the flow may be treated as inviscid. You may neglect changes in potential energy.

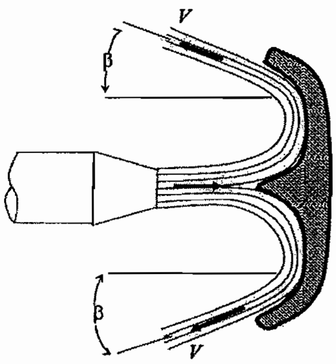

Fig. 2

(a) Why is the fluid speed leaving the nozzle equal to that leaving the bucket? [3]

(b) Show that the force exerted on the bucket by the jet is

$$F = \dot{m}V\,[1+\cos\beta]$$

[7]

where $\dot{m}$ is the mass flow rate and β is the angle shown in Fig. 2.

(TURN OVER

3 **(short)** A perfect gas enters a compressor at a pressure of $p_1 = 10^5$ N m^{-2}, a temperature of $T_1 = 290$ K, and a velocity of 18 m s^{-1} through an inlet area of 0.1 m^2. At the exit the temperature is 370 K and the velocity is 6 m s^{-1}. Heat transfer to the surroundings, as well as changes in potential energy, may be neglected. The gas has a specific heat and gas constant of $c_p = 10^3$ J kg^{-1}K^{-1} and $R = 287$ J kg^{-1} K^{-1} respectively.

(a) Calculate the density at the inlet and the mass flow rate through the compressor. [4]

(b) Find the compressor power. [6]

4 **(short)** The rate of heat transfer through a furnace wall of area A is determined in part by convection at the inner and outer surfaces and in part by conduction through the wall. The surface heat transfer coefficient is $h = 20$ Wm^{-2} K^{-1}, the wall thickness is $t = 0.1$m, and the thermal conductivity of the wall material $\lambda = 1.0$ Wm^{-1} K^{-1}.

Find an expression for the overall thermal resistance and hence show that the heat transfer rate *per unit area* is given by

$$\dot{q} = \frac{T_f - T_a}{\left(\frac{2}{h} + \frac{t}{\lambda}\right)}$$

where T_f is the temperature inside the furnace and T_a the ambient temperature outside. [10]

5 **(long)**

(a) A perfect gas expands at constant temperature in a cylinder from state 'a' to state 'b'. Starting from the expression

$$W = \int F dx$$

for the work done by a force F moving in the direction x, show that the work done by the gas on the piston is

$$W = p_a V_a \ln \frac{V_b}{V_a}.$$

How much heat is transferred to or from the gas during the expansion? [9]

(b) A perfect gas of mass m undergoes a closed cycle consisting of:

process 1 - 2: pressure rise at constant volume from p_1 to p_2
process 2 - 3: constant temperature expansion from p_2 to $p_3 = p_1$
process 3 - 1: constant pressure compression back to state 1.

(i) Sketch the process on a pV diagram and show that $T_2 = rT_1$ and $V_3 = rV_1$ where r is the pressure ratio, $r = p_2/p_1$. [6]

(ii) Calculate the heat transfer during each part of the cycle expressing the results in terms of r, m, R, T_1 and c_v where R is the gas constant and c_v the specific heat at constant volume. You may use the result of part (a). [8]

(iii) Show that the cycle efficiency is [7]

$$\eta = \frac{r \ln r - (r-1)}{r \ln r + (r-1)c_v/R}$$

6 **(long)**

(a) In a steady, two-dimensional flow the components of acceleration of a fluid particle parallel and perpendicular to a streamline are

$$a_{//} = V\frac{\partial V}{\partial s} \quad , \quad a_{\perp} = -\frac{V^2}{R}$$

where V is the fluid speed, R is the radius of curvature of the streamline, s is a curvilinear coordinate measured along the streamline, and $a_{\perp}$ is measured in a direction away from the centre of curvature of the streamline.

(i) Using Fig. 3, show that the net pressure force acting on a small rectangular element of fluid, $\delta s \delta n \delta w$, has components

$$F_{//} = -\frac{\partial p}{\partial s}\delta s \delta n \delta w \quad , \quad F_{\perp} = -\frac{\partial p}{\partial n}\delta s \delta n \delta w$$

where n is a coordinate normal to the streamline, pointing away from the centre of curvature. [6]

(ii) By applying Newton's second law to the small rectangular fluid element shown in Fig. 3, show that the pressure gradients parallel and perpendicular to the streamline are

$$\frac{\partial p}{\partial s} = -\rho V\frac{\partial V}{\partial s}, \qquad \frac{\partial p}{\partial n} = \rho\frac{V^2}{R},$$

where ρ is the density. You may neglect gravitational and viscous forces. [6]

(iii) Confirm that, by integrating the first of the expressions in (ii) above, we obtain Bernoulli's equation in the absence of gravity. [4]

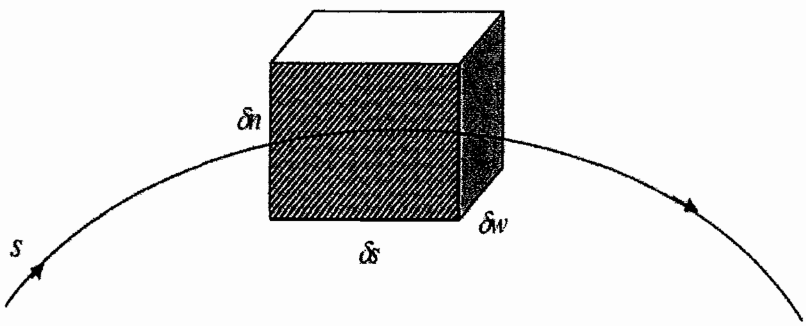

Fig. 3

(cont.

(b) A cylindrical container rotates with constant angular velocity, ω, and is partially filled with a liquid. The fluid rotates as a rigid body at the same rate as the container and it is observed that its free surface is curved as shown in Fig. 4. The depth, h, of the liquid is therefore a function of radius r. We wish to determine the shape of the free surface, $h(r)$.

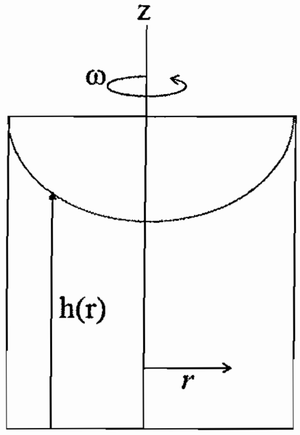

Fig. 4

(i) The surface shape is found to be independent of ρ, and so we have

$$h - h_0 = F(g, \omega, r)$$

where h_0 is the depth of fluid at $r = 0$, g is the acceleration due to gravity and F is some unknown function. Use dimensional analysis to show that

$$\frac{h - h_0}{r} = G\left(\frac{\omega^2 r}{g}\right)$$

for some function G. [6]

(ii) Verify that the radial variation in pressure is governed by

$$\frac{\partial p}{\partial r} = \rho\omega^2 r$$

and hence show that the pressure distribution *on the base of the container* takes the form

$$p = p_0 + \frac{1}{2}\rho\omega^2 r^2$$

where p_0 is the pressure at the centre. [4]

(iii) Noting that there is no vertical acceleration in the fluid, confirm that

$$h(r) - h_0 = \frac{\omega^2 r^2}{2g}.$$

[4]

7 . **(short)**

(a) Find from first principles the mass moment of inertia of a disc with uniformly distributed mass m and radius a about an axis that passes through the centre of mass and is perpendicular to the plane of the disc. [5]

(b) The disc pivots on a horizontal axis A that passes through the circumference of the disc and is perpendicular to the plane of the disc, as shown in Fig. 5. The disc is initially held so that the centre of mass G is at the same height as the pivot axis. The disc is then released. Calculate the initial angular acceleration of the disc. [5]

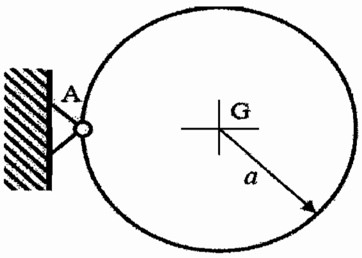

Fig. 5

8 **(short)** Figure 6 shows a planar mechanism consisting of a crank OA of length L, a connecting rod AB of length $\sqrt{3}L$ and a sliding piston at B. The crank rotates at speed ω in an anti-clockwise direction. At the instant shown there is a right angle between the crank and the connecting rod and the distance OB is $2L$.

(a) For the instant shown calculate the angular velocity (magnitude and direction) of the connecting rod by instantaneous centres or otherwise. [5]

(b) If there is a friction torque Q resisting rotation of joint A calculate the torque required to rotate the crank. [5]

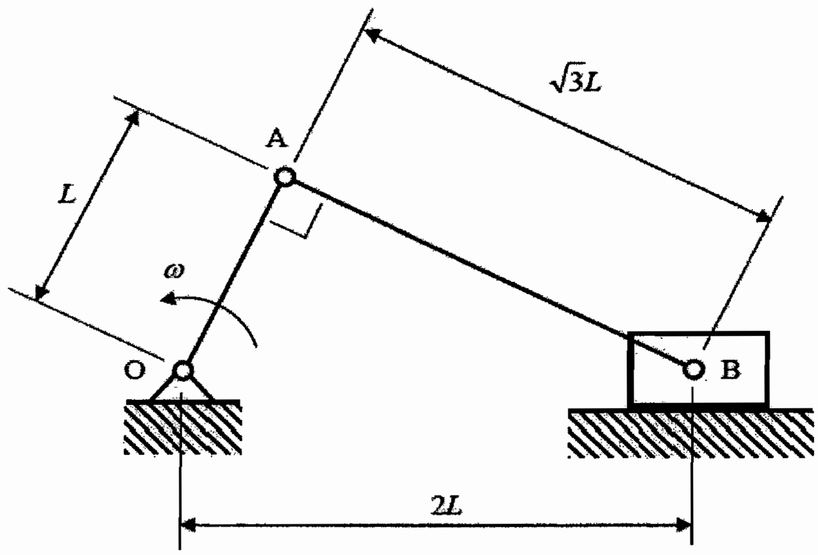

Fig. 6

9 **(long)** An aircraft flies along a path in a horizontal plane. At one instant the aircraft is 1 km east of a radar station, and travelling north-east at 300 m s^{-1} as shown in Fig. 7. The acceleration of the aircraft is 1 m s^{-2} in an easterly direction.

(a) Express the velocity and acceleration of the aircraft in intrinsic coordinates. [6]

(b) Calculate the magnitude of the instantaneous radius of curvature of the path. [5]

(c) Express the velocity and acceleration of the aircraft in polar coordinates, with the origin at the radar station. [7]

(d) The radar points continually at the aircraft and measures the horizontal distance to the aircraft. Calculate the acceleration rate at which this distance is changing and calculate the angular acceleration (magnitude and direction) of the radar. [12]

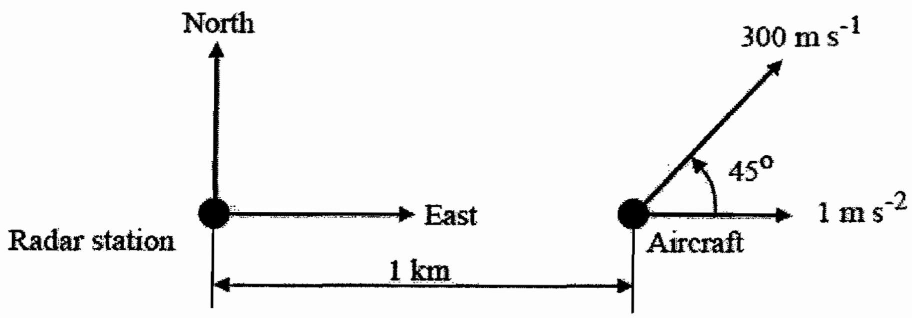

Fig. 7

10 **(short)** Figure 8 shows a damper (damping rate λ) and spring (stiffness k) connected in series. One end of the spring is connected to ground and a force $f(t)$ is applied to the free end of the damper. Displacement $x(t)$ is defined in the figure.

(a) Show that force $f(t)$ is related to displacement $x(t)$ by

$$\frac{\mathrm{d}f}{\mathrm{d}t} + f\frac{k}{\lambda} = \frac{\mathrm{d}x}{\mathrm{d}t}k$$

[5]

(b) Derive an expression for the force response to a unit step in dx/dt. Sketch the response. [5]

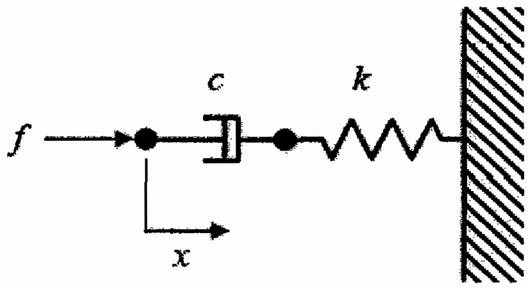

Fig. 8

11 **(short)** Figure 9 shows a mass isolated from a moving base by a parallel spring and damper. The absolute displacement of the base is x and the absolute displacement of the mass is y.

(a) Show that the equation of motion is:

$$m\frac{d^2y}{dt^2}+\lambda\frac{dy}{dt}+ky=\lambda\frac{dx}{dt}+kx$$

[3]

(b) The mass m is 10 kg and the stiffness k is 12 kN m^{-1}. The base displacement is sinusoidal with amplitude 1 mm, and can be at any frequency. Using the graphical data in the mechanics data book, or otherwise, estimate the smallest value of damping λ that ensures the displacement amplitude of the mass is no greater than 2 mm. [7]

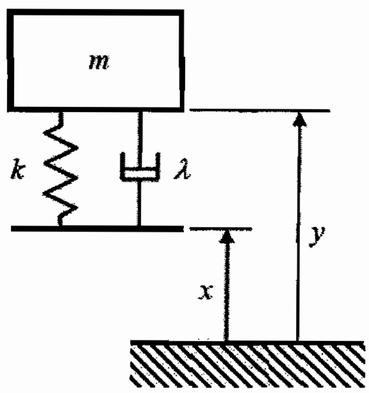

Fig. 9

12 **(long)** Figure 10 shows two masses, each of mass m, connected by a spring of stiffness k. The lower mass is connected to ground by a second spring of stiffness k. The displacements of the masses are y_1 and y_2.

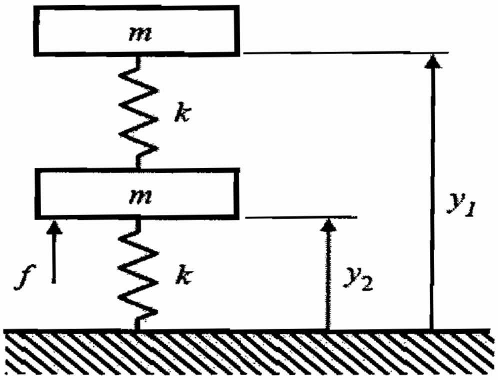

Fig. 10

(a) Show that the equations of motion can be written as:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{y}_1(t) \\ \ddot{y}_2(t) \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & 2k \end{bmatrix} \begin{Bmatrix} y_1(t) \\ y_2(t) \end{Bmatrix} = \begin{Bmatrix} 0 \\ f(t) \end{Bmatrix}$$

[5]

(b) By considering free vibration ($f(t) = 0$) and harmonic displacements ($y_1(t) = Y_1 e^{i\omega t}$, $y_2(t) = Y_2 e^{i\omega t}$), show that the natural frequencies ω_1 and ω_2 satisfy:

$$\left(k - \omega^2 m\right)\left(2k - \omega^2 m\right) - k^2 = 0$$

and find expressions for ω_1 and ω_2. [10]

(c) Show that the displacement response $y_2(t) = Y_2 e^{i\omega t}$ to a harmonic force excitation $f(t) = Fe^{i\omega t}$ on the lower mass is given by:

$$\frac{Y_2}{F} = \frac{k - \omega^2 m}{(k - \omega^2 m)(2k - \omega^2 m) - k^2}$$

[10]

(d) Sketch the magnitude of Y_2/F as a function of frequency ω. Also show on your sketch the effect of adding a viscous dashpot between the two masses. [5]

END OF PAPER