ENGINEERING TRIPOS PART IA

Thursday 8 June 2006 9 to 12

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** marks allocated to each part of a question are indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachment:
Table for Question 3

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version 6

SECTION A

1 **(short)** Three identical circular cylinders are stacked as a triangular pile on a rough horizontal floor, as shown in Figure 1. The lower cylinders may roll but not slip at their points of contact on the floor, A and C. Find the minimum coefficient of friction required at points of contact B and D, between pairs of cylinders, if the pile is to be stable under its own weight. [10]

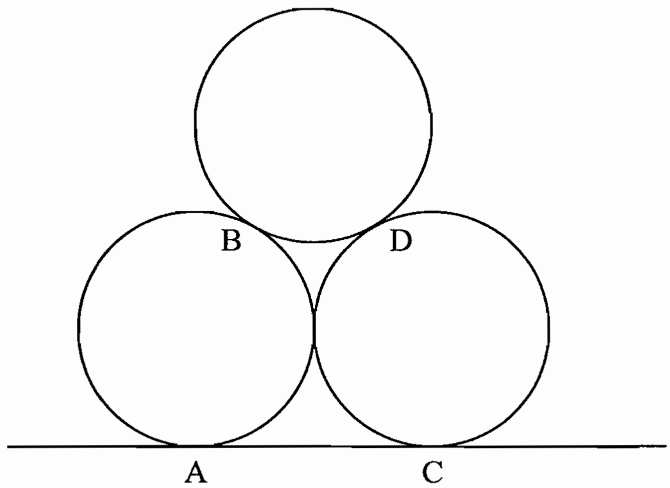

Fig. 1

2 **(short)** A thin, flexible cable of weight w per unit length is draped over two small, rough circular pegs spaced L apart so that it has a sag of D in the middle and drops H at each end: see Figure 2. The sag ratio D/L is sufficiently small that the weight of the cable between the pegs can be regarded as being uniformly distributed over the horizontal.

(a) Obtain an expression for the maximum tension in that part of the cable lying between the pegs. [7]

(b) What other calculation should be performed to determine whether the configuration is in static equilibrium? [3]

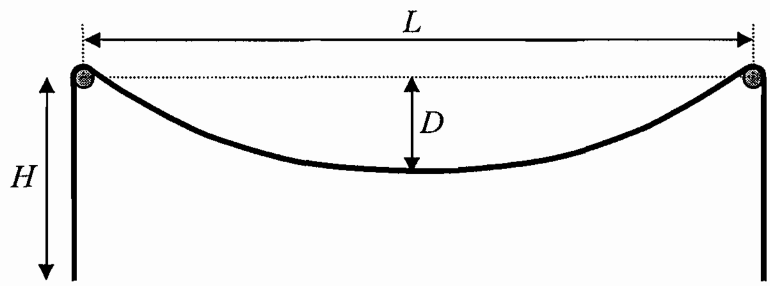

Fig. 2 (NOT drawn to scale)

3 (long) The plane truss shown in Fig. 3 forms part of a gantry, and is to be designed for the single loading condition shown – a vertical force W applied at K. Members are all constructed in steel of Young's Modulus E, and their lengths are L, $L\sqrt{2}$ or $2L$ as shown.

(a) Calculate all the bar forces T due to the imposition of load W. List all the results in the table given on the attached sheet marked *Table for Question 3*. A number of bars are found to carry zero force. Explain why they may nevertheless serve a valuable function. [10]

(b) Sectional areas are to be selected so that the magnitude of the axial stress in the members shown as carrying load, whether tensile or compressive, is a fixed proportion α (< 1) of the yield stress σ_y of the steel. Deduce all bar extensions e and list all the results in the Table provided. [5]

(c) Use the Virtual Work Principle to find expressions for both the vertical and horizontal components of the displacement of point K due to the application of the load W. [15]

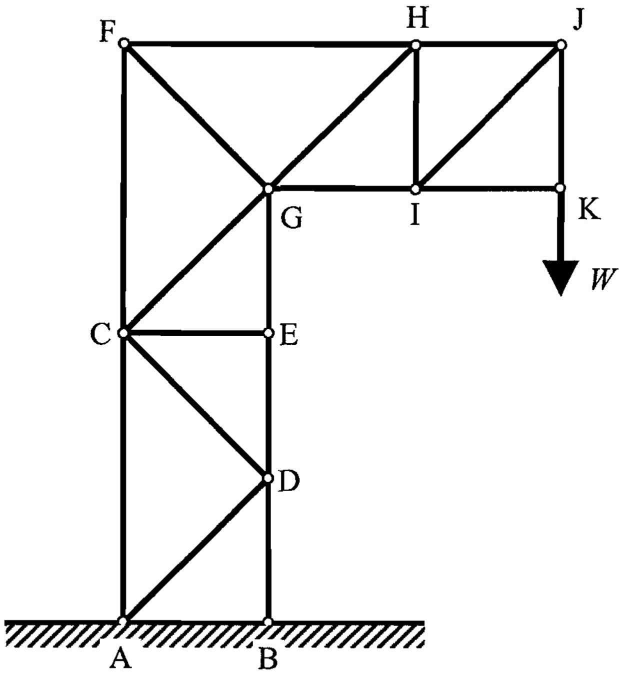

Fig. 3

4 **(short)** Figure 4 shows a beam AE simply supported over length $4L$. Its loading can be considered in four subdivisions of length L, with a downward force $2W$ distributed over AB, an upward force W at C, and a downward force W at D. Self-weight can be neglected.

Show that the reactions at A and E are $3W/2$ and $W/2$ respectively. Sketch both shear force and bending moment diagrams for the beam and calculate all key values and critical locations. [10]

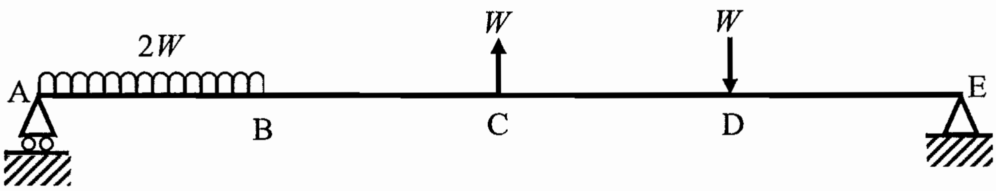

Fig. 4

5 **(short)** Figure 5 shows a uniform elastic beam AC of length $4L$ and bending stiffness EI, built-in at A to form a cantilever, and then provided with an additional simple support at B so that zero force is applied to the unloaded beam. A load W is then applied to the tip C.

Find the reaction at B. [10]

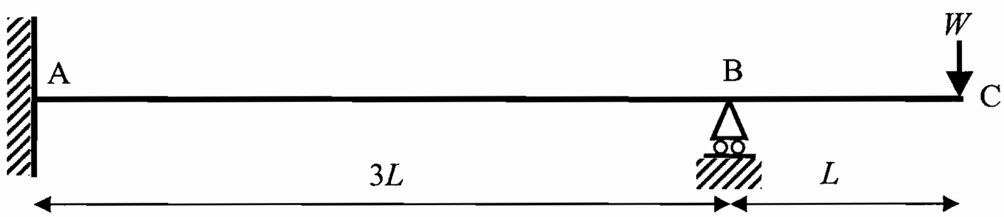

Fig. 5

6 **(long)** Old brick chimneys, some as high as 100 m, must occasionally be preserved for posterity. You are required to establish some elementary relationships that could serve as a guide in considering their safety. For this purpose consider a cylindrical chimney, with constant inner and outer radii a and b respectively, and height $h >> b$. The weight per unit volume of a representative element of bricks and mortar can be taken to be γ. It is required to evaluate the safety of such a chimney under a combination of gravity loading and wind pressure p that can be considered to act uniformly over the area of elevation of the chimney. The chimney may be regarded as a cantilever carrying both axial compression and bending. Safety should be evaluated against two limit criteria:

(i) for protection against crushing, the first achievement of the maximum permissible compressive stress σ_c at any point;

(ii) for protection against the onset of tensile cracking, the first elimination of compressive stress at any point.

Between these limits, the material can be considered to be linear elastic.

(a) Explain carefully why each of these conditions will be achieved first at the base of a chimney. [6]

(b) Develop inequalities from which the maximum safe wind pressure can be estimated. [24]

SECTION B

7 **(short)** (a) Explain briefly why ceramics are usually much stronger in compression than in tension. [5]

(b) Long beams of a ceramic, of square cross-section, are subjected to a uniform bending moment. This generates a bending stress field, of maximum value $+|\sigma_b|$ (in the upper face of the beam) and minimum value $-|\sigma_b|$ (in the lower face of the beam). It is found that 50% of the beams break when or before the tensile stress in the upper face of the beam reaches 500 MPa. Identical beams are loaded in tension along their length, and it is found that 50% of these beams break when or before the tensile stress reaches 367 MPa. Give a brief qualitative explanation for this difference in strength. [5]

8 **(short)** Briefly explain the following.

(a) When exposed to air at high temperature, stainless steel oxidises much less rapidly than mild steel. [3]

(b) Steel nails used to fasten copper roofing sheet fail rapidly by wet corrosion. [3]

(c) The coefficient of static friction between two pieces of the same metal is always found to be approximately 1/6. [4]

9 **(short)** (a) Explain briefly what is meant by an edge dislocation. Show with sketches how the motion of an edge dislocation under an applied shear stress leads to the plastic deformation of a crystal. [4]

(b) State briefly how dislocations can account for the following.

(i) Cold working makes aluminium harder. [2]

(ii) An alloy of 80% copper plus 20% zinc is harder than pure copper. [2]

(iii) The hardness of nickel is increased by adding hard particles of thorium oxide. [2]

10 **(short)** Referring to the graphs of uniaxial tensile response (Figures 2.1 and 2.2) on p15 of the Materials Data book, consider the graphs for the following materials.

(i) Medium-carbon steel.

(ii) PMMA.

(iii) Nylon 66.

(a) In each case, estimate the yield stress (where appropriate), the tensile strength (maximum nominal stress) and the percentage elongation to cause fracture. [3]

(b) Account for the main differences between the shapes of the three graphs. [7]

11 **(long)** A cylindrical pressure vessel made from steel has an internal diameter of 1000 mm and a wall thickness of 20 mm. The working pressure (WP) is 10 MPa (gauge pressure), and the test pressure (TP) is 15 MPa (gauge pressure). The vessel has a longitudinal welded seam containing a number of cracks, each of which passes through the full thickness of the weld and extends along the line of the weld by distance $2a$. In order to prevent corrosion, the vessel is lined with a skin of polyethylene. Because this prevents the contents leaking out through the cracks, they cannot be detected by leak testing.

(a) Describe briefly the methods you would use to detect the cracks, and explain why the methods you describe are appropriate. [8]

(b) Assuming that K_{IC} = 3125 MPa $\sqrt{\text{mm}}$ and $Y = 1$, where K_{IC} and Y are defined on p6 of the Materials Data Book, calculate the value of a for fast fracture at:

(i) the working pressure (a_{WP});

(ii) the test pressure (a_{TP}). [6]

(c) In normal service, the vessel is subjected to repeated cycles of pressure of (0 to 10 to 0) MPa. As a result, the cracks grow by fatigue according to the equation

$$\frac{da}{dN} = A(\Delta K)^3$$

where N and ΔK are defined on p7 of the Materials Data Book, and $A = 2 \times 10^{-13}$ for a in units of mm and ΔK in units of MPa $\sqrt{\text{mm}}$.

(i) Calculate the number of pressure cycles required to grow the cracks from a_{TP} to a_{WP}; [10]

(ii) Discuss the use of regular pressure testing in establishing the continuing safety of pressure vessels. [6]

12 **(long)** You have been asked to design the pressure hull for a deep-sea submersible vehicle capable of descending to the bottom of the Pacific Ocean. The hull is to be a thin-walled sphere with a specified radius r (equal to 1 m) and a uniform wall thickness t (which must be selected). The design pressure is 200 MPa. The sphere can fail by one of the following two mechanisms:

(i) external-pressure buckling at a pressure given by $p_b = 0.3E\left(\frac{t}{r}\right)^2$,

(ii) compressive failure at a pressure given by $p_f = 2\sigma_f\left(\frac{t}{r}\right)$.

E is Young's modulus, and σ_f is the compressive failure stress.

(a) Derive a materials performance index which must be maximised in order to achieve the minimum mass:

(i) in the case of external-pressure buckling; [8]

(ii) in the case of compressive failure. [8]

(b) For *each* material listed in the Table below, calculate the *mass* and the *wall thickness* of the hull for *each* failure mechanism at the design pressure. [8]

(c) Hence determine the limiting failure mechanism for each material. [4]

(d) What is the optimum material for the hull? [2]

Material	E (GPa)	σ_f (MPa)	ρ (kg m^{-3})
Alumina	390	5000	3900
Glass	70	2000	2600
Alloy steel	210	2000	7800
Titanium alloy	120	1200	4700
Aluminium alloy	70	500	2700

END OF PAPER

Table for Question 3

bar	length	$\frac{T}{W}$	$\frac{eE}{L\alpha\sigma_y}$			
AC						
AD						
BD						
CD						
CE						
CF						
CG						
DE						
EG						
FG						
FH						
GH						
GI						
HI						
HJ						
IJ						
IK						
JK						