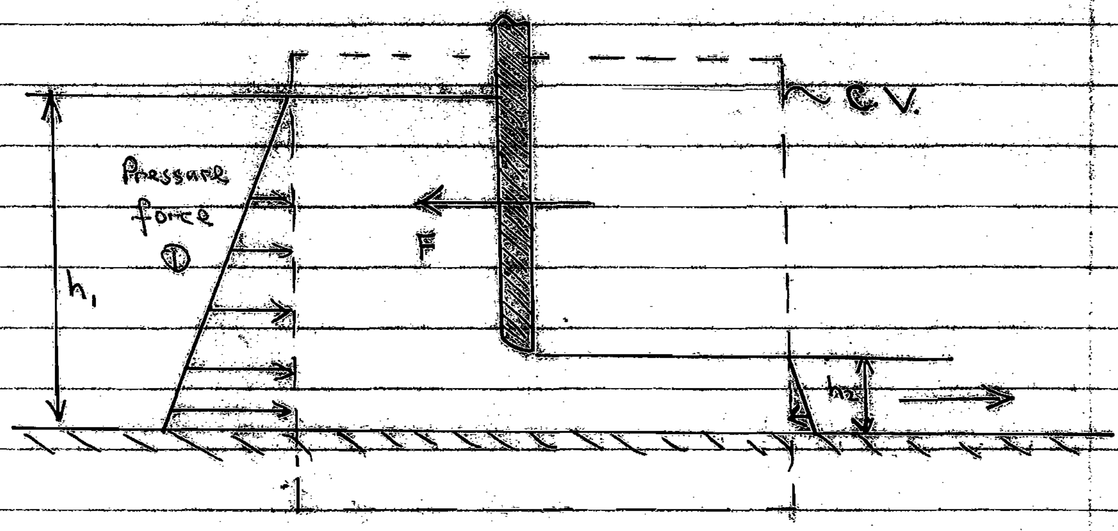
[IA] PAPER I CRIB SECTION A

1 (a) No vertical acceleration of fluid ⇒ static balance
⇒ $\frac{\partial p}{\partial z}$ balances ρg [2

(b)



$$\text{Net Force on CV} = \sum \dot{m}\,\underline{u}_{OUT} - \sum \dot{m}\,\underline{u}_{IN}$$

Take component in → direction

$$-F + \tfrac{1}{2}\rho g h_1 \times h_1 - \tfrac{1}{2}\rho g h_2 \times h_2 = (\rho h_2 V_2)V_2 - (\rho h_1 V_1)V_1$$

$$\Rightarrow \quad F = \tfrac{1}{2}\rho g h_1^2 - \tfrac{1}{2}\rho g h_2^2 + \rho h_1 V_1^2 - \rho h_2 V_2^2 \qquad [6]$$

(c) $$F/\rho = \frac{g}{2}(h_1^2 - h_2^2) + \frac{\dot{m}}{\rho}(V_1 - V_2)$$

Continuity: $h_1 V_1 = h_2 V_2 \Rightarrow \underline{\underline{V_2 = 6\ m/s}}$

$$F/\rho = \frac{9.81}{2}\left(3^2 - \left(\tfrac{1}{2}\right)^2\right) + 3(1-6) = 27.9\ m^3 s^{-2}$$

$$\Rightarrow \quad F = 27.9 \times 10^3\ N. \qquad [2]$$

2 (a) From piston to nozzle; Bernoulli gives:

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \rho g h = \cancelto{0}{p_2} + \tfrac{1}{2}\rho V_2^2$$

But $p_1 = F/A_1$

$$\Rightarrow \quad F/A_1 + \tfrac{1}{2}\rho V_1^2 + \rho g h = \tfrac{1}{2}\rho V_2^2$$

$$\Rightarrow \quad \underline{\underline{F/A_1 = \tfrac{1}{2}\rho\left(V_2^2 - V_1^2\right) - \rho g h}}$$ [7]

(Assumptions: inviscid and quasi-steady. Cannot be exactly steady.)

(b) Continuity $\Rightarrow \quad A_1 V_1 = A_2 V_2$

$$\Rightarrow \quad V_2 = 10^3 V_1 = \underline{\underline{10\ \text{m/s}}}$$

$$F/A_1 = \tfrac{1}{2}\times 10^3\left(10^2 - 0.01^2\right) - 10^3 \times 9.81 \times 1.5 \ \text{N/m}^2$$

$$\Rightarrow \quad \underline{\underline{F = 35.3\ \text{kN}}}$$ [3]

3 (a) $dq = du + p\,dv$

$h = u + pv \Rightarrow dh = du + p\,dv + v\,dp$

$\Rightarrow dq = dh - v\,dp$

Adiabatic $\Rightarrow dq = 0 \Rightarrow \underline{dh = v\,dp}$.

Incompressible $\Rightarrow v = \frac{1}{\rho} =$ constant $\Rightarrow dh = \frac{dp}{\rho}$

Integrate: $\underline{\Delta h = \Delta p / \rho}$. [4]

(b) SFEE

$$\dot{Q} - \dot{W}_p = \dot{m}\left[\left(h + \tfrac{1}{2}v^2\right)_2 - \left(h + \tfrac{1}{2}v^2\right)_1\right]$$

(P.E. neglected)

$\dot{Q} = 0$, $\Delta h = \Delta p / \rho$

$$\Rightarrow \dot{W} = -\dot{m}\left[(p_2 - p_1)/\rho + \tfrac{1}{2}(v_2^2 - v_1^2)\right]$$

$$= -(\dot{m}/\rho)\left[(p_2 - p_1) + \tfrac{1}{2}\rho(v_2^2 - v_1^2)\right]$$

$$\dot{W}_p = -\dot{W} = (\dot{m}/\rho)\left[(p_2 - p_1) + \tfrac{1}{2}\rho(v_2^2 - v_1^2)\right] \quad [4]$$

(c) Turbulence will cause irreversible loss of mechanical energy due to viscous dissipation. [2]

4. (a) $PV = mRT \Rightarrow m = \dfrac{P_1 V_1}{RT_1} = \dfrac{0.25 \times 600}{0.520 \times 773} = \underline{\underline{0.373\ \text{kg}}}$

$V = \text{const.} \Rightarrow \dfrac{P_2}{P_1} = \dfrac{T_2}{T_1} \Rightarrow P_2 = \dfrac{193 \times 600}{773} = \underline{\underline{150\ \text{kPa}}}$

[6]

(b) $Q - \underbrace{W}_{0} = \Delta U = mc_v \Delta T = mc_v (T_2 - T_1)$

$Q = 0.373 \times 1.71 \times 580 = \underline{\underline{-370\ \text{kJ}.}}$ [4]

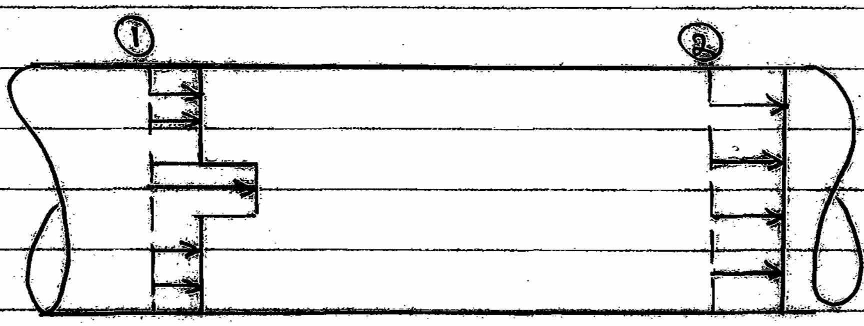
5. (a) The streamlines in the jet are straight and ∥.

$\partial p/\partial n = \rho V^2/R, \quad R \to \infty \Rightarrow \partial p/\partial n = 0$

No curvature ⇒ no cross-stream pressure gradient.

[2]

(b)



Momentum equation:

$$P_1 A_2 - P_2 A_2 = \rho A_2 V_2^2 - \rho (A_2 - A_J) V_1^2 - \rho A_J V_J^2$$

$$\Rightarrow \Delta P = P_2 - P_1 = \rho \left[\frac{A_J}{A_2} V_J^2 + \left(1 - \frac{A_J}{A_2}\right) V_1^2 - V_2^2 \right]$$

$$= \rho \left[\lambda V_J^2 + (1-\lambda) V_1^2 - V_2^2 \right]$$

$$\Rightarrow \underline{\Delta p = \rho \lambda V_J^2 + \rho (1-\lambda) V_1^2 - \rho V_2^2} \quad [8]$$

(c) Continuity:

$$A_3 V_3 + (A_2 - A_3) V_1 = A_2 V_2$$

$$\Rightarrow \quad V_2 = \lambda V_3 + (1-\lambda) V_1 \qquad \left(\lambda = \tfrac{1}{4}\right)$$

$$= 0.25 \times 4 + 0.75 \times 1 = \underline{\underline{1.75 \text{ m/s}}}$$

$$\Delta p = 10^3 \left[\tfrac{1}{4} \times 4^2 + \tfrac{3}{4} \times 1^2 - 1.75^2 \right] = \underline{\underline{1690 \text{ Pa}}}$$

$$C = \Delta p / \tfrac{1}{2} \rho V_3^2 = \underline{\underline{0.211}}$$ [4]

(d) $$H_1 = p_1 + \tfrac{1}{2} \rho V_3^2, \quad H_2 = p_2 + \tfrac{1}{2} \rho V_2^2$$

$$H_2 - H_1 = (p_2 - p_1) + \tfrac{1}{2} \rho (V_2^2 - V_3^2)$$ (Applied to mean or time-averaged streamline.)

$$= 1690 + \tfrac{1}{2} 10^3 (1.75^2 - 4^2)$$

$$= -4780 \text{ Pa}$$

$$\neq 0$$

This is due to mechanical energy loss caused by turbulence. Strong turbulence dissipates a lot of mechanical energy. [7]

(~~d~~)(e) $$\Delta p = f(\rho, V_3, V_1, A_3, A_2)$$

6 variables, 3 dimensions $\Rightarrow$ 3 dimensionless groups.

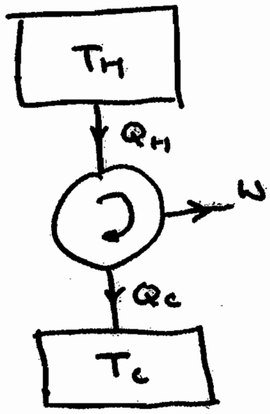
By inspection; $\Pi_1 = C = \dfrac{\Delta p}{\tfrac{1}{2} \rho V_3^2}$, $\Pi_2 = \dfrac{V_3}{V_1}$, $\Pi_3 = \dfrac{A_3}{A_2}$

$$\Pi_1 = f(\Pi_2, \Pi_3) \Rightarrow \underline{\underline{C = f\left(\frac{V_3}{V_1}, \frac{A_3}{A_1}\right)}}$$

If V_3 and V_1 both doubled, C is unchanged $\Rightarrow \Delta p$ [9]

$$\Rightarrow \Delta p = 4 \times (\Delta p)_{\text{OLD}} = \underline{6760 \text{ Pa}}$$

6 (a)



$$\eta = \frac{W}{Q_H}$$

$$= 1 - \frac{Q_C}{Q_H}$$

Clausius: $\frac{Q_H}{T_H} - \frac{Q_C}{T_C} \leq 0$

$$\therefore \quad \frac{Q_C}{Q_H} \geq \frac{T_C}{T_H}$$

$$\Rightarrow \quad \eta \leq 1 - \frac{T_C}{T_H} \quad \text{or} \quad \eta_{max} = 1 - \frac{T_C}{T_H}$$

[8]

(b) $\dot{Q}_H = 4000$ MW

$\dot{Q}_C = 2300$ MW

$\therefore \quad \dot{W} = \dot{Q}_H - \dot{Q}_C = \underline{\underline{1700 \text{ MW}}}$

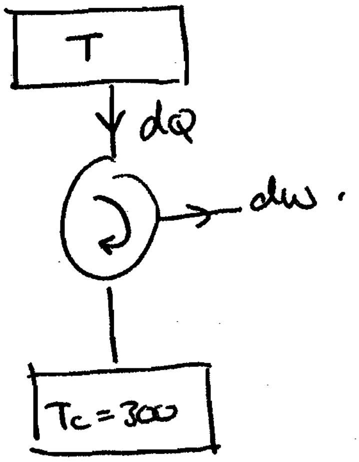
$$\eta = \frac{\dot{W}}{\dot{Q}_H} = \frac{1700}{4000} = \underline{\underline{42.5\%}}$$

$$\eta_{max} = 1 - \frac{T_C}{T_H} = 1 - \frac{303}{633} = \underline{\underline{52.1\%}}$$

The actual efficiency is lower due to irreversibilities within the cycle – eg. aerodynamic losses (frictional/viscous losses) etc.

[7]

(c)



(c) $dw_{max} = \eta_{max}\, dQ$

$$= \left(1 - \frac{T_c}{T}\right)\left(-C\,dT\right)$$

$$\therefore\ w_{MAX} = \int_{T=T_1}^{T_c} dw_{MAX} = \int_{T_1}^{T_c} C\left(\frac{T_c}{T} - 1\right) dT$$

(i.e., maximum work is extracted when the heat engine is reversible, and the final state of the heat source is at 300 K).

$$\therefore\ w_{MAX} = C\left[T_c \ln\left(\frac{T_c}{T_1}\right) - T_c + T_1\right]$$

$$= 10^6 \times \left(300 \ln\left(\tfrac{1}{2}\right) + 600 - 300\right)$$

$$= 300\left(1 - \ln 2\right)\ \text{MJ}$$

[12]

$$w_{MAX} = 92.06\ \text{MJ}.$$

(d) The minimum work input for the heat pump will be when it is also reversible, and so must also be 92.06 MJ

[3]

7 a) $I_A = \underbrace{\frac{ma^2}{12}}_{I_G} + \underbrace{m\left(\frac{a}{2}\right)^2}_{\parallel \text{ axis}} = \frac{ma^2}{3}$

so $h_A = \frac{\omega m a^2}{3}$

b) $I_A' = \frac{ma^2}{3} + ma^2 = \frac{4ma^2}{3} = 4 I_A$

M of M conserved, so $\omega' = \omega/4$

c) KE before $= \frac{1}{2} I_A \omega^2$, KE after $= \frac{1}{2} I_A' \omega'^2$

$I'\omega'^2 = \frac{I\omega^2}{4}$ so 3/4 lost

8 a) $F = \frac{mV^2}{r} = mg$

so $V^2 = gr \longrightarrow V = \sqrt{gr}$

b) If pom drops by 0.5r, then $r' = r/2$

M of M conserved, so $V' = 2V$

Also, increase in KE = loss of PE

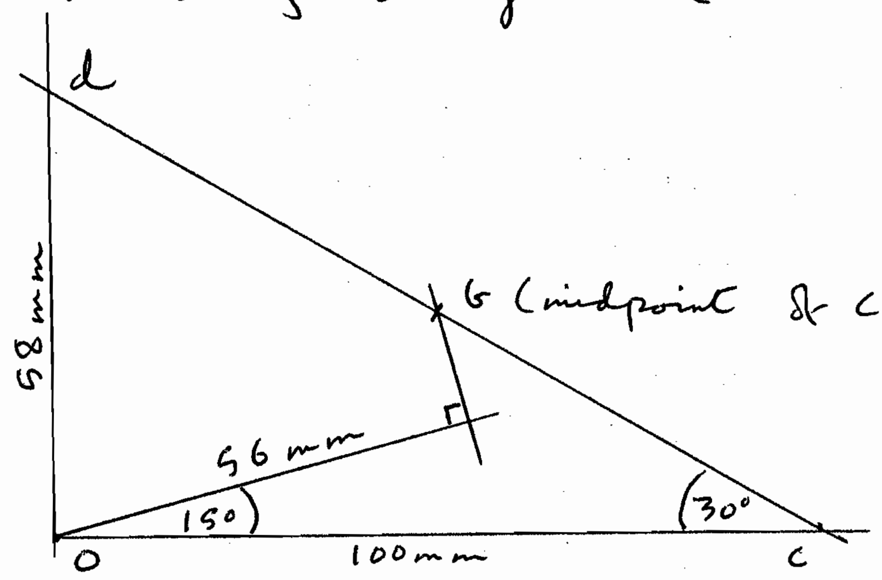
so $\frac{1}{2} m V^2 (4-1) = \frac{(M+m)gr}{2}$

Since $V^2 = gr$,

$$\frac{3mgr}{2} = \frac{(M+m)gr}{2}$$

whence $M = 2m$

9 a) Velocity diagram (100mm = 1 m/s)



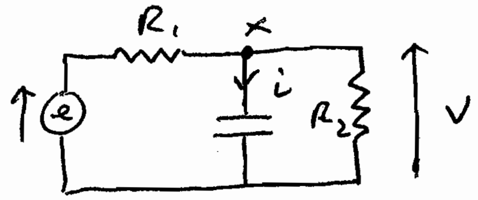
So jack is extending at $\underline{0.56 \text{ m/s}}$

b) D is currently moving at 0.58 m/s

To move D at 1 m/s, jack must extend

at $0.56 \times \frac{1}{0.58} = \underline{0.966 \text{ m/s}}$

10 a)



Sum currents at x

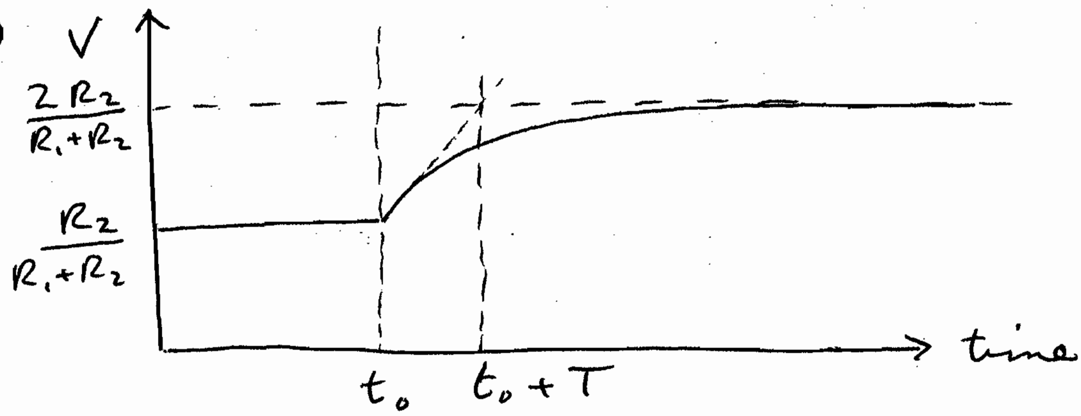
$$i = \frac{e - V}{R_1} - \frac{V}{R_2} \quad \text{and} \quad \frac{dV}{dt} = \frac{i}{C}$$

$$\text{So} \quad C\frac{dV}{dt} = \frac{e}{R_1} - V\left(\frac{R_1 + R_2}{R_1 R_2}\right)$$

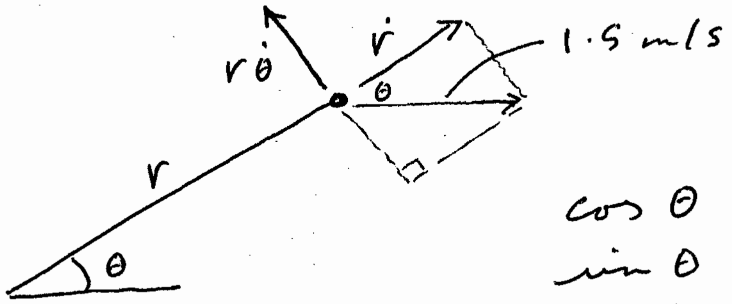
$$C\frac{R_1 R_2}{R_1 + R_2}\frac{dV}{dt} + V = \frac{R_2}{R_1 + R_2} e$$

Whence $T = \dfrac{C(R_1 R_2)}{R_1 + R_2}$, $A = \dfrac{R_2}{R_1 + R_2}$

10 b)



11 a)



$$\cos\theta = 0.8$$
$$\sin\theta = 0.6$$

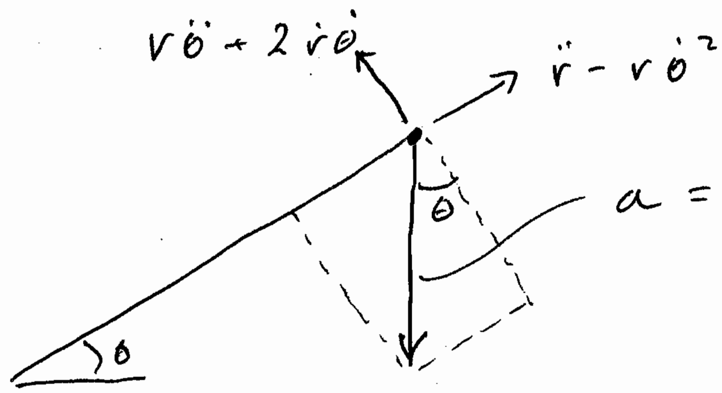
From diagram,

$$\dot{r} = 1.5\cos\theta = \underline{1.2 \text{ m/s}}$$

$$r\dot{\theta} = -1.5\sin\theta = -0.9 \text{ m/s}$$

$$\text{so } \dot{\theta} = \frac{-0.9}{0.5} = \underline{-1.8 \text{ rad/s}}$$

b)



$$a = \frac{V^2}{0.3} = 7.5 \text{ m/s}^2$$

$$r\dot{\theta}^2 - \ddot{r} = 7.5\sin\theta = 4.5 \text{ m/s}^2$$

$$\text{So } \ddot{r} = 0.5\times(-1.8)^2 - 4.5 = \underline{-2.88 \text{ m/s}^2}$$

11 b) continued

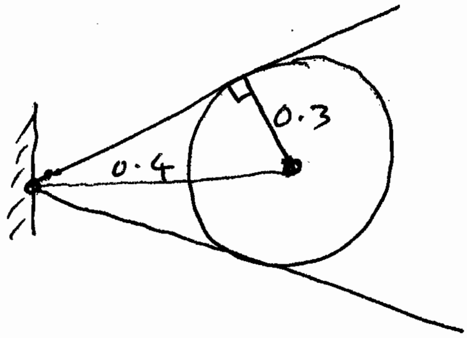
Also, $r\ddot{\theta} + 2\dot{r}\dot{\theta} = -7.5\cos\theta = -6$

so $r\ddot{\theta} = -6 - 2 \times 1.2 \times (-1.8)$

$= -1.68$

$\therefore \ddot{\theta} = \dfrac{-1.68}{0.5} = \underline{-3.36 \text{ rad/s}^2}$

c) Max and min values of $\dot{r}$ when V is along the rod:



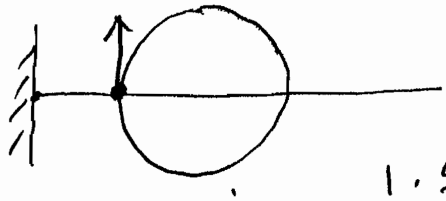
$\theta = \pm \sin^{-1}(3/4)$

$= \pm 48.6°$

So max $\dot{r}$ $(+1.5)$ @ $\underline{\theta = 48.6°}$

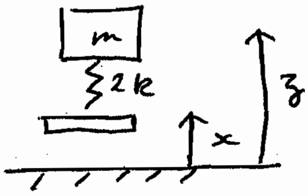
min $\dot{r}$ (-1.5) @ $\underline{\theta = -48.6°}$

d) Greatest value of $\dot{\theta}$ when V ⊥ rod, and r is a minimum:



Here, $\dot{\theta} = \dfrac{1.5}{0.1} = \underline{15 \text{ rad/s}}$

12 a) Without the mount,



$$m\ddot{z} = 2k(x - z)$$

For SHM

$$(2k - m\omega^2)z = 2kx$$

$$z = \frac{2k}{(2k - m\omega^2)}x$$

If $\omega = \frac{3k}{2m}$, $z = \frac{2k}{2k - \frac{3k}{2}}x = 4x$

So amplitude is <u>0.4 mm</u>

b) We can write

$$m\ddot{z} = 2k(y - z)$$

$$2m\ddot{y} = 2k(z - y) + k(x - y)$$

So $\begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix}\begin{bmatrix} \ddot{z} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} 2k & -2k \\ -2k & 3k \end{bmatrix}\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ k \end{bmatrix}x$

c) When $\ddot{z} = -\omega z$, and $\ddot{y} = -\omega y$, the equation above becomes

$$\underbrace{\begin{bmatrix} 2k - m\omega^2 & -2k \\ -2k & 3k - 2m\omega^2 \end{bmatrix}}_{[A],\ \text{say}}\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ k \end{bmatrix}x$$

Whence $\begin{bmatrix} z \\ y \end{bmatrix} = [A]^{-1}\begin{bmatrix} 0 \\ k \end{bmatrix}x$

12 c) continued

Alternatively -

$$(2k - m\omega^2)\,z = 2ky$$

$$(3k - 2m\omega^2)\,y = 2kz + kx$$

So $\dfrac{2k - m\omega^2}{2k}\,z = y = \dfrac{2kz + kx}{3k - 2m\omega^2}$

$$(2k - m\omega^2)(3k - 2m\omega^2)\,z = 4k^2 z + 2k^2 x$$

$$z = \frac{2k^2}{2k^2 - 7km\omega^2 + 2m\omega^4}\,x \qquad (1)$$

When $\omega^2 = \dfrac{3k}{2m}$,

$$z = \frac{2k^2}{2k^2 - 10.5k^2 + 4.5k^2}\,x = \frac{x}{2}$$

So amplitude is $\underline{0.05\text{ mm}}$ (8 times better)

d) Natural frequencies when $\omega_n = \alpha\sqrt{\dfrac{k}{m}}$, say

For $z/x = \infty$ in (1) above,

$$2k^2 - 7\alpha^2 k^2 + 2\alpha^4 k^2 = 0$$

$$\text{or } 2\alpha^4 - 7\alpha^2 + 2 = 0$$

whence $\alpha^2 = \dfrac{7 \pm \sqrt{49 - 16}}{4}$

So $\omega_n = 1.785\sqrt{k/m}$ or $0.560\sqrt{k/m}$