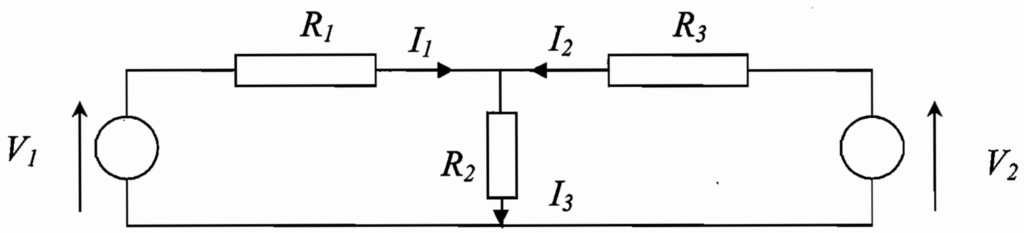
Part 1(a) paper 3 2007 crib

SECTION A

1 **(long)** (a) Explain briefly how the techniques of *mesh current analysis* and *loop current analysis* are used in d.c. and a.c electrical circuits.

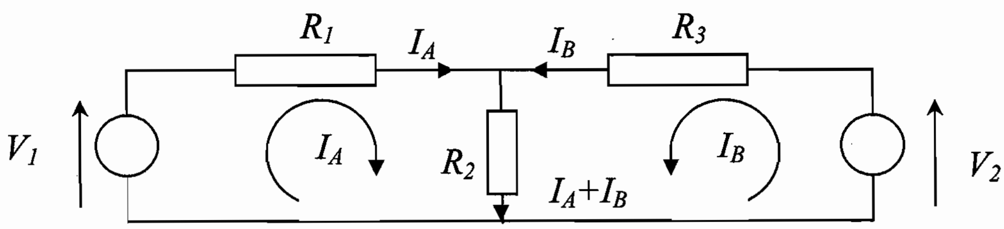
Mesh/loop current analysis refers to the application of Kirchhoff's voltage law to solve for unknown currents in a circuit. In mesh analysis the unknown currents flow through the individual circuit elements, and Kirchhoff's current law needs to be applied to enforce current conservation. In loop analysis, the unknown currents flow around each circuit loop, and the currents through the individual elements are appropriate sums of these loop currents. The number of equations and unknowns is equal to the number of independent circuit loops.

Examples:



Mesh analysis:

KCL $\Rightarrow$ $I_3=I_1+I_2$
Then solve KVLx2



Loop Analysis
Solve KVLx2

(b) Figure 1(a) shows the circuit for an a.c. bridge. The voltage source supplies a sinusoidal waveform of frequency ω. At balance the current through the meter *M* is zero. Find the condition for balance in terms of R_1, R_2, R_3, R_4 and the ratio *L/C*.

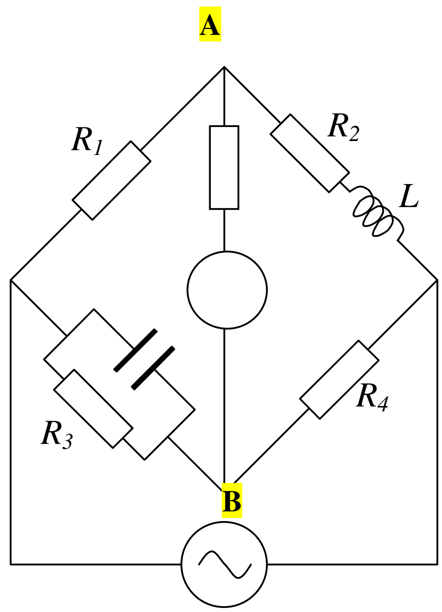


Fig. 1(a)

Different approaches are possible. Here we give one possible route.

At balance the current though meter M is zero. Thus the voltages at corners **A** and **B** must be the same. This gives:

$R_1R_4=(R_2+j\omega L)(R_3/j\omega C)/(R_3+1/j\omega C)$

$R_1R_4R_3j\omega C+R_1R_4=R_2R_3+j\omega LR_3$

$L/C=R_1R_4$

$R_1R_4=R_2R_3$

(c) Consider the bridge in Fig. 1(b). Find an expression for the frequency ω at which the bridge balances, in terms of R_1, R_2, R_3, R_4 and C.

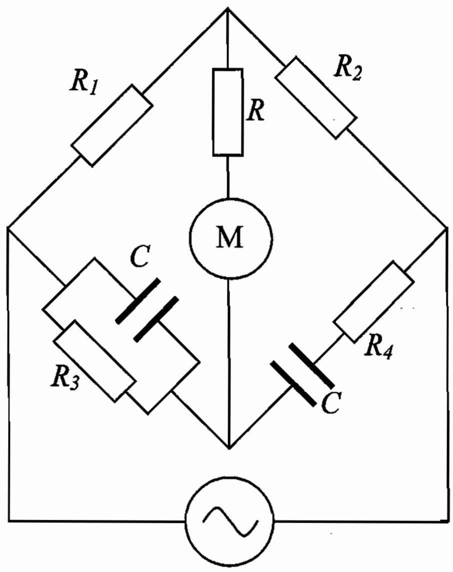


Fig. 1(b)

Again, different approaches are possible. Using the same as 1(b):

$R_1(R_4+/j\omega C)=R_2(R_3/j\omega C)/(R_3+1/j\omega C)$

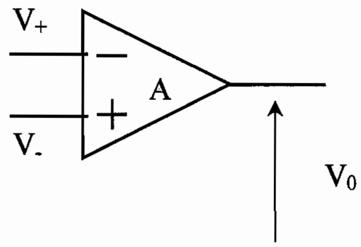
$-R_1R_4R_3j\omega^2C^2+(R_4+R_3)R_1\omega C+jR_1=R_3R_2\omega C$

The real part does not give any indication on ω

Form the imaginary part we get

$\omega^2=1/C^2R_3R_4$

2 **(long)** (a) Explain what is meant by an ideal operational amplifier, with particular reference to gain, input resistance and output resistance. Explain how the ideal op-amp approximations may be used to simplify the analysis of op-amp circuits. [5]



An ideal op-amp has infinite voltage gain A_0, infinite input resistance R_i, and zero output resistance, R_0.

The simplifications that the ideal assumptions allow are

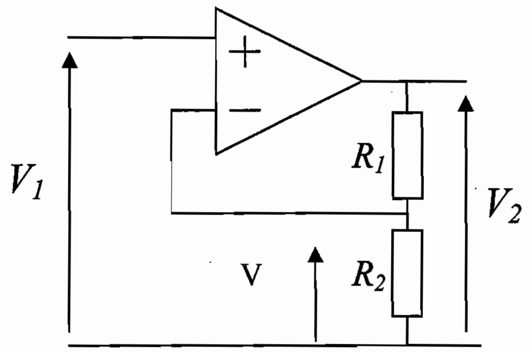
1) $A \rightarrow \infty \Rightarrow V_+ = V_-$ Virtual earth principle

2) $R_i \rightarrow \infty \Rightarrow i_+ = i_- = 0$ No current flows into or out of + or –

3) $R_0 \rightarrow 0 \Rightarrow V_0$ is a perfect voltage source, with zero internal resistance. Unaffected by connection of arbitrary loads.

(b) The op-amp in Fig. 2(a) may be assumed as ideal, except for a finite gain A. Derive an expression for the voltage gain V_2/V_1 in terms of R_1, R_2 and A. Calculate V_2/V_1 if $R_1 = 150\text{k}\Omega$, $R_2 = 2\text{k}\Omega$ and $A = 10^5$. [10]

Fig. 2(a)



$V_1\text{-}V=V_2/A$ (2b.i)

$V=R_2/(R_1+R_2)V_2$ (2b.ii)

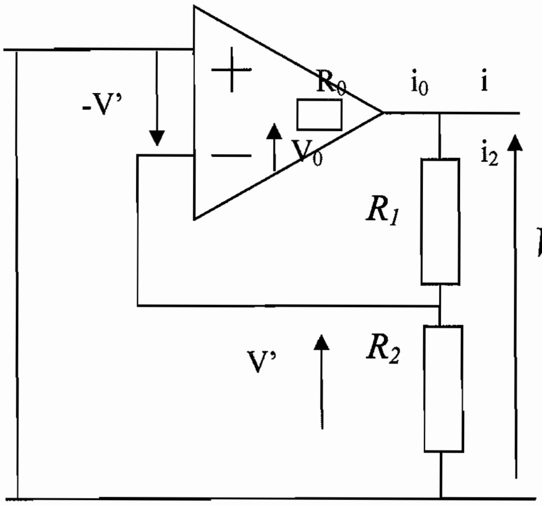
From (2.i)+(2.ii) we get

$V_2/V_1=A(R_1+R_2)/(R_1+R_2+AR_2)$ (2b.iii)

Inserting the numerical values in (2b.iii) we get

$V_2/V_1=75.9$

(c) If instead the op-amp has a finite output resistance $R_0 = 150\ \Omega$, show that R_{out}, the output resistance of the circuit of Fig. 2(a), is approximately equal to $R_0\,(R_1+R_2)\,/\,(AR_2)$ and calculate R_{out}.



In order to find R_{out}, we need to short the input of the op-amp and apply a voltage source V to the output

V_0=-AV'

$i = i_0+i_2= (V+AV')/R_0+(V-V')/R_1$

$V'/R_2=(V-V')/R_1$

We get

$R_{out}= V/i= R_0/[1+AR_2/(R_1+R_2)+R_0/(R_1+R_2)]$

However: $AR_2/(R_1+R_2)>>R_0/(R_1+R_2)+1$, thus

$R_{out}\approx R_0/A(1+R_2/R_1)$ (2c.i)

Substituting the numerical values in (2c.i) we get:

$R_{out}\approx 0.114\Omega$

(d) A load capacitor C, as shown in Fig. 2(b), is connected to the circuit described in part (c). Calculate C so that V_C drops 3 dB from its mid band value at 10 kHz.

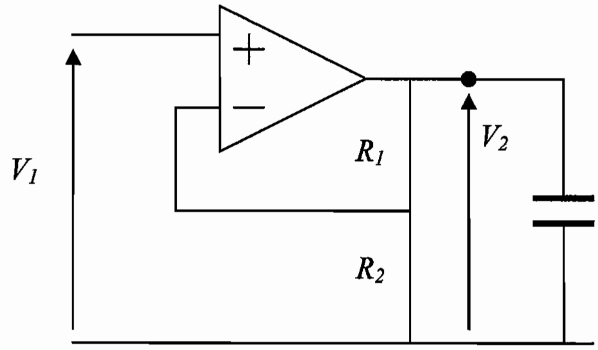
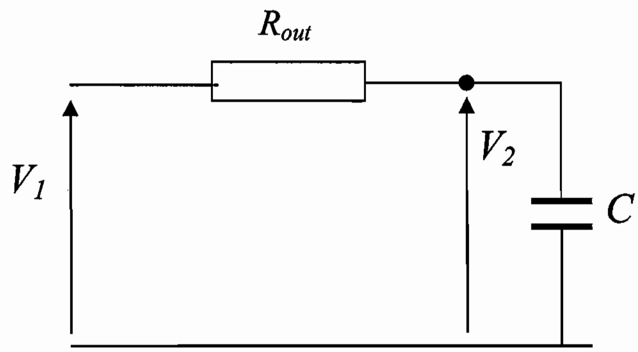


Fig 2(b)

The circuit is equivalent to



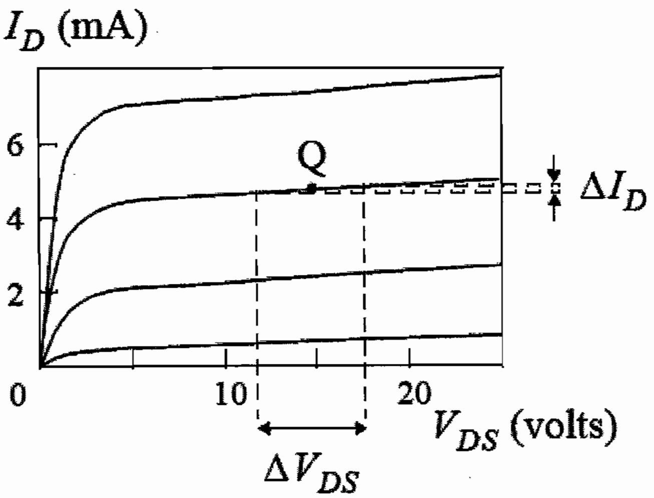
$V_C = V_1/j\omega C/(R_{out}+1/j\omega C) = V_1/(1+j2\pi fCR_{out})$

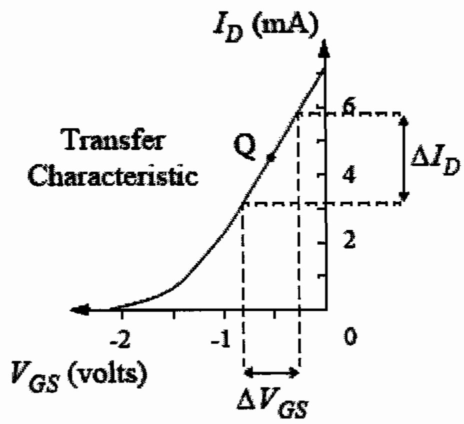
For –3dB we need $2\pi fCR_{out}=1$

Thus

$C=1/2\pi fR_{out}$ (2d.i)
Substituting the numerical values in (2d.i) we get
$C=139.6\ \mu F$

3 **(short)** (a) Define r_d and g_m in the small-signal a.c. model of a JFET. [3]





$$\Delta I_D = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_{GS}} \cdot \Delta V_{DS}$$

$$\Delta I_D = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{DS}} \cdot \Delta V_{GS}$$

$$\Delta I_D = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_{GS}} \cdot \Delta V_{DS} + \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{DS}} \cdot \Delta V_{GS}$$

Defining $$\frac{1}{r_d} \equiv \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_{GS}}, g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{DS}}$$

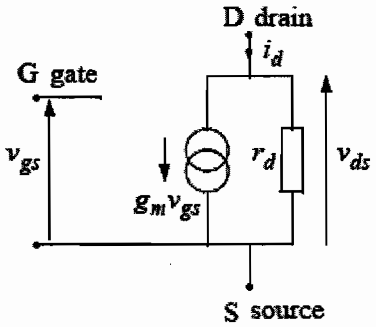
$i_d=\Delta I_D$ $\quad V_{DS}=\Delta V_{DS}$ $\quad V_{GS}=\Delta V_{GS}$

We get $i_d=V_{DS}/r_d+g_mV_{GS}$

r_d is the drain resistance

g_m is the mutual (or trans) conductance

This describes the following equivalent circuit



The use of the equivalent circuit greatly facilitates the small signal analysis of FET circuits.

(b) Consider the circuit in Fig. 3. Find values for R_1 and R_2 required to set the transistor d.c. operating point to V_{DS}= 14 V, I_D= 0.3 mA and V_{GS}= -3V. Assume an ideal transistor and zero gate current. [3]

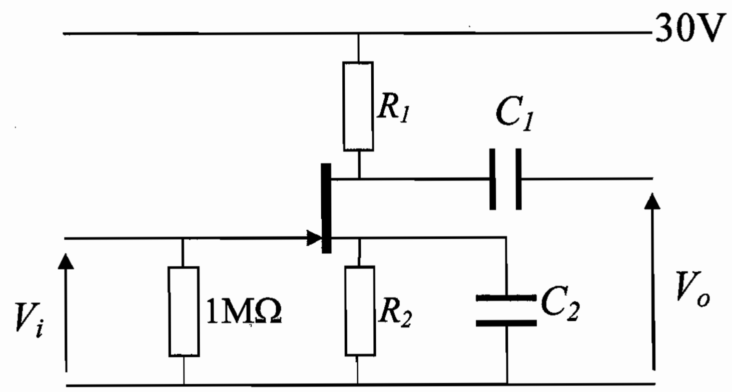
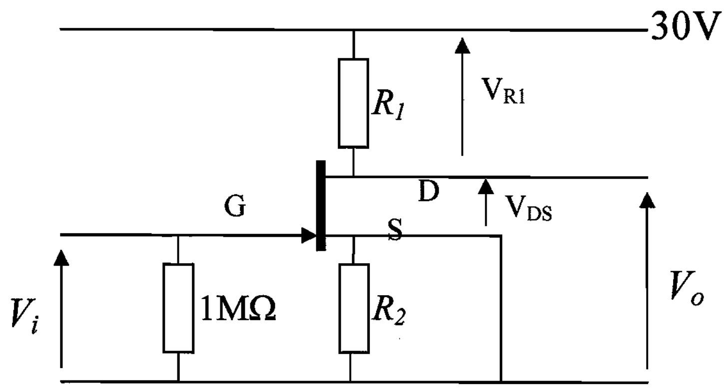


Fig. 3

DC equivalent circuit



Assuming ideal transistor and zero gate current

$R_2I_D=-V_{GS}$ (3b.i)

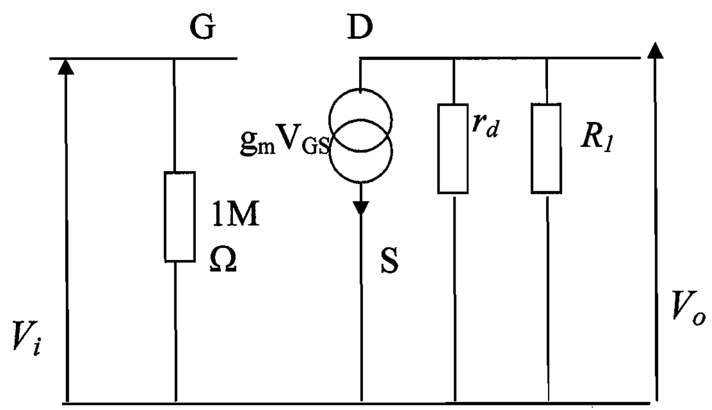
Substituting the numerical values in (3b.i) we ger

$R_2=10\text{k}\Omega$

$V_{R1}= 30-V_{DS}+V_{GS}=(30-14-3)\text{V}= 13 \text{ V}$

$R_1=V_{R1}/I_D=43.3 \text{ k}\Omega$

(c) Draw the small-signal circuit valid for mid-band frequencies, and find the mid-band small-signal gain, V_o/V_i. For the JFET small-signal parameters take $r_d = 30\text{k}\Omega$ and $g_m = 5\text{mS}$.



$V_i = V_{GS}$ (3c.i)

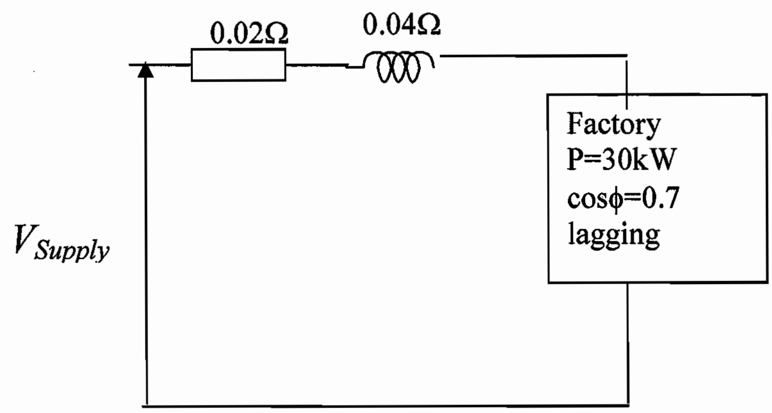
$V_0 = -g_m V_{gs} R_1 r_d / (R_1 + r_d)$ (3c.ii)

Substituting the numerical values in (3c.i) and (3c.ii) we get:

$V_0/V_i = -88.13$

4(short) A small factory consumes 30kW with a lagging power factor of 0.7 at the factory. When it is connected to a 50Hz supply the voltage across the factory terminals is 240V. The supply line has an impedance $Z_s = 0.02 + j0.04\ \Omega$.

(a) Draw the circuit diagram and calculate the real power lost in the line and the voltage supplied at the input. [5]



$P = VI\cos\phi$

$I = P/V\cos\phi = 178.6$ A

$Q = P\tan\phi = 30.61$ kVAR

$P_{lost} = I^2 0.02 = 638$ W

$Q_{lost} = I^2 0.04 = 1276$ VAR

$P_{in} = P + P_{lost} = 30.638$ kW

$Q_{in} = Q + Q_{lost} = 31.886$ kVAR

VA_{input}=44.220kVA

V_{supply}= 247.6 V

(b) Assuming that the voltage across the factory terminals continues to be 240V, calculate the required size of the capacitor connected across the factory terminals in order to correct the power factor to unity. [5]

$Q_C = V^2_{load} 2\pi f C$

Inserting the numerical values

C=1.7 mF

5 **(short)** A non-ideal transformer is short-circuit tested and open-circuit tested to measure its characteristics. The open-circuit test gives $V_{primary}$= 240V, $I_{primary}$= 0.2A, P = 15W, $V_{secondary}$= 120V. The short-circuit test gives $V_{primary}$= 40V, $I_{primary}$= 3A, P = 30W. Determine the values of the equivalent circuit parameters referred to the primary side of the transformer. (See page 20 of the Electrical and Information Data Book for the non-ideal transformer equivalent circuit) [10]

Open circuit:

Turns Ratio $N_1/N_2 = V_{primary}/V_{secondary}$= 2

$S = I_{primary}V_{primary}$= 48 VA

$Q = (S^2-P^2)^{0.5}$= 45.6 VAR

$R_0 = V^2_{primary}/P$=3.84 kΩ

$X_0 = V^2_{primary}/Q$=1.263 kΩ

Short Circuit

$S = V_{primary}I_{primary}$= 120VA

$Q=(S^2-P^2)^{0.5}$= 116.2 VAR

$R_{t1} = P/I^2_{primary} = 3.33\ \Omega$

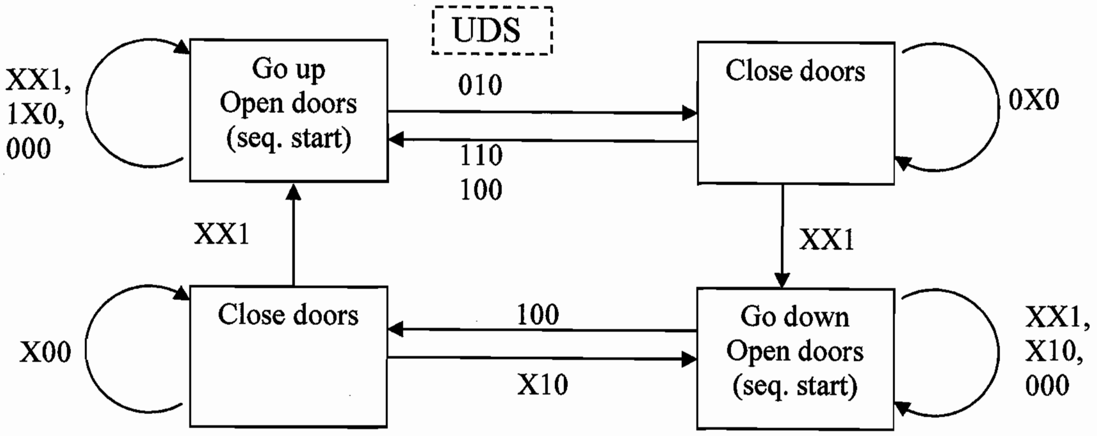
$X_{t1} = Q/I^2_{primary} = 12.91\ \Omega$.

6 **(long)** There are 3 inputs:

U = call lift to the top floor,

D = call lift to the bottom floor,

S = doors shut



There are 4 states so we need 2 flip-flops. Let

state 0 = 00, state 2 = 10

state 1 = 01 state 3 = 11

U	D	S	Q_A	Q_B	Q_{An}	Q_{Bn}	J_A	K_A	J_B	K_B	
X	X	1	0	0	0	0	0	X	0	X	
1	X	0	0	0	0	0	0	X	0	X	
0	0	0	0	0	0	0	0	X	0	X	
0	1	0	0	0	0	1	0	X	1	X	
0	X	0	0	1	0	1	0	X	X	0	
X	X	1	0	1	1	0	1	X	X	1	
1	X	0	0	1	0	0	0	X	X	1	
X	X	1	1	0	1	0	X	0	0	X	
X	1	0	1	0	1	0	X	0	0	X	
0	0	0	1	0	1	0	X	0	0	X	
1	0	0	1	0	1	1	X	0	1	X	
X	0	0	1	1	1	1	X	0	X	0	
X	X	1	1	1	0	0	X	1	X	1	
X	1	0	1	1	1	0	X	0	X	1	

State table as above. Karnaugh maps have 5 inputs.

J_A

U,D \ S, Q_A	00	01	11	10
00	0	X	X	0
01	0	X	X	0
11	0	X	X	0
10	0	X	X	0

$Q_B = 0$

U,D \ S, Q_A	00	01	11	10
00	0	X	X	1
01	0	X	X	1
11	0	X	X	1
10	0	X	X	1

$Q_B = 1$

$$J_A = Q_B S$$

K_A

U,D \ S, Q_A	00	01	11	10
00	X	0	0	X
01	X	0	0	X
11	X	0	0	X
10	X	0	0	X

$Q_B = 0$

U,D \ S, Q_A	00	01	11	10
00	X	0	1	X
01	X	0	1	X
11	X	0	1	X
10	X	0	1	X

$Q_B = 1$

$$K_A = Q_B Q_A$$

J_B

U,D \ S, Q_A	00	01	11	10
00	0	0	0	0
01	1	0	0	0
11	0	0	0	0
10	0	1	0	0

$Q_B = 0$

U,D \ S, Q_A	00	01	11	10
00	X	X	X	X
01	X	X	X	X
11	X	X	X	X
10	X	X	X	X

$Q_B = 1$

$$J_B = U\overline{D}\,\overline{S}Q_A + \overline{U}D\overline{S}\,\overline{Q_A}$$

K_B

U,D \ S, Q_A	00	01	11	10
00	X	X	X	X
01	X	X	X	X
11	X	X	X	X
10	X	X	X	X

$Q_B = 0$

U,D \ S, Q_A	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	1	1	1	1
10	1	0	1	1

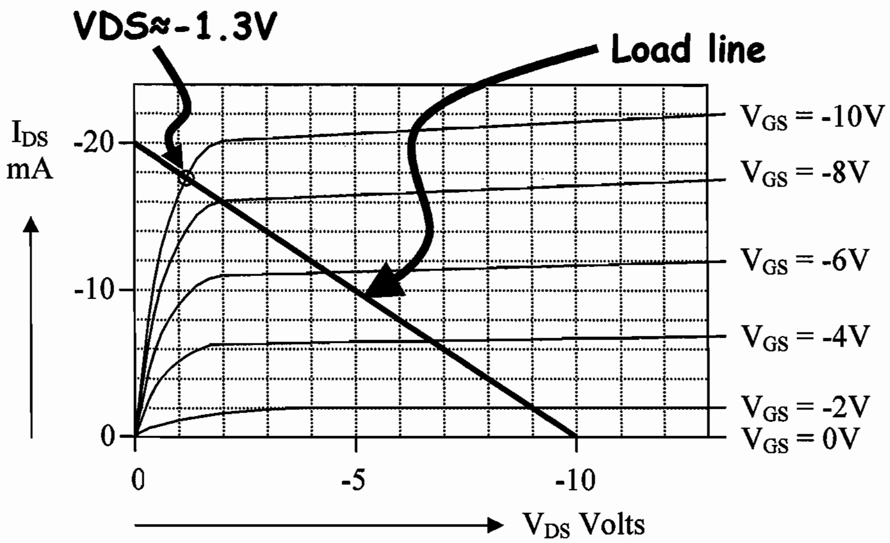
$Q_B = 1$

$$K_B = S + DQ_A + U\overline{Q_A}$$

Examiner's comments: Sensor S will always be off in states 0 and 2 so one can design the circuit in several ways. Some candidates considered systems with 6 states but realized that 2 are unnecessary when they came to draw up the state table. Some were discouraged by Karnaugh maps with 5 inputs but were able to demonstrate that they understood the principles. Average mark was 64%

7 (long)

(a) If $I_{DS} = 0$, then $V_{DS} = -10$, while
if $V_{DS} = 0$, then $I_{DS} = -10/500 = -20$ mA
Plot this load line, and read off at $V_{GS} = -10$ V. We see that $\underline{V_{DS} = -1.3 \text{ volts}}$



(b) $I_{DS} \approx -17.5$ mA and $R = V_{out}/I_{DS} = 5/(-17.5\times10^{-3}) = \underline{286\ \Omega}$

(c) Vary the point at which the load line cuts the vertical axis by ±1 % and we see that the variation in current at the intersection with the line for $V_{GS} = -10$ V is also 1%, so the variation in output will be 1%. So when the gate voltages are as in (b), the error will be <u>0.05 V</u>

With least significant bit set, output is 5/128=0.039 V. Factor equals 0.05/0.039 = <u>1.28.</u>

Examiner's comment: Most candidates managed part (a) with ease but many found part (b) more difficult and there were few correct answers to part (c). Average mark was 44%.

8. **(short).**
(A + C).(B + D) = AB + BC + AD + CD

If A = $\overline{D}$ then

(A + C).(B + D) = AB + BC + C$\overline{A}$
= AB + C$\overline{A}$ (because the term BC is redundant)
= AB + CD

Alternatively, B = $\overline{C}$

Examiner's comments: Many candidates showed that the relationship is true if A = $\overline{D}$ and B = $\overline{C}$: few were able to show that either alone is sufficient. Average mark was 52%.

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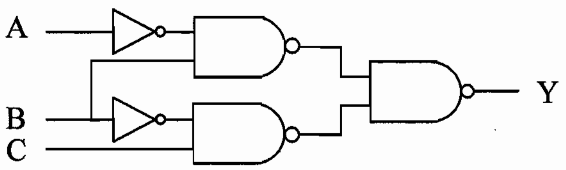
9 **(short)**



	$B\overline{C}$	BC	$\overline{B}C$	$\overline{B}\,\overline{C}$
A	0	0	1	0
$\overline{A}$	1	1	1	0

Figure 9

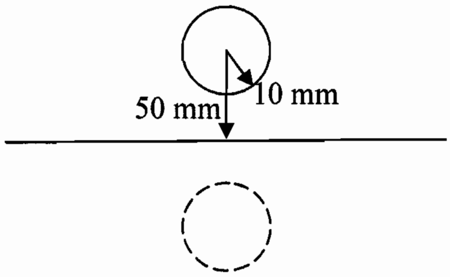
(a) The truth table implies that $Y = \overline{B}C + \overline{A}B$ but there is a hazard when A is off and C is on and B changes state. As B goes from 0 to 1, Y will briefly go from 1 to 0 and back, and it will do the same as B goes from 1 to 0.

(b) Add a third input to the right-hand NAND gate and join to this the output of an extra gate which takes the NAND of $\overline{A}C$.

Examiner's comments: Most candidates got this question correct. Average mark was 78%.

A R L Travis (examiner)

10 **(short)** A metal sphere of radius 10 mm is placed with its centre 50 mm above an infinite conducting plane and the sphere is then charged to 50 V. What is the approximate electric field at the conducting plane under the centre of the sphere?



First, find the charge stored on one sphere alone. Let this be Q. Then

$$D = \frac{Q}{4\pi r^2}$$

So:

$$E = \frac{Q}{\varepsilon_0 4\pi r^2}$$

So:

$$V = \int_R^\infty \frac{Q}{\varepsilon_0 4\pi r^2} dr = \frac{Q}{\varepsilon_0 4\pi R}$$

Where R is the radius of the sphere.

So

$$Q = V\varepsilon_0 4\pi R$$

Now use the method of images and add a sphere with charge $-Q$ whose centre is 100 mm from the centre of the first sphere. Summing the effects of both spheres:

So:

$$E = \frac{Q}{\varepsilon_0 4\pi r_1^2} - \frac{-Q}{\varepsilon_0 4\pi r_2^2}$$

$$= 2\frac{V\varepsilon_0 4\pi R}{\varepsilon_0 4\pi r^2}$$

Where r_1 is distance from 1st sphere, r_2 is distance from 2nd. Where R is the radius of the spheres and $r = r_1 = r_2$

$$= \frac{2VR}{r^2} = \frac{2\times 50\times 10\times 10^{-3}}{\left(50\times 10^{-3}\right)^2}$$

$$= \underline{400\text{V/m}}$$

Examiner's comment: Most candidates recognised that this should be solved by the method of images but then attempted very complicated ways of doing this which usually foundered in a sea of algebra. Average mark was 47%.

11 **(short)** A straight infinitely long wire is placed in the plane of a square coil and parallel to one side as shown in Fig. 11. Each side of the coil is of length 5 mm. What is the flux through the coil if its centre is 100 mm from the wire which carries a current of 1 A? If the current in the wire alternates at 50 Hz, approximately how many turns should the coil have to generate an alternating potential of 1 μV across its terminals?

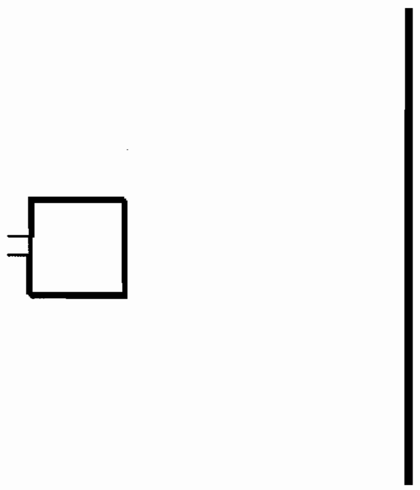


Fig. 11

$H = I/(2\pi r)$ $= 1/(2\times\pi\times 10^{-1}) = 1.59$ H/m

$B = \mu_0 H$ $= 4\pi \times 10^{-7} \times 1.59 = 2\times 10^{-6}$ T

$\phi_0 = B\times(5\times 10^{-3})^2$ $= 2\times 10^{-6}\times(5\times 10^{-3})^2 = \underline{50 \times 10^{-12}\text{ Wb}}$

$$V = n\frac{d}{dt}(\phi_0 \sin(2\pi f t))$$

Where n is the number of turns in the coil and f, the frequency, equals 50 Hz.

$$= n\phi_0 2\pi f \cos(2\pi f t)$$

$$V_p = n\phi_0 2\pi f$$

Where V_p equals the peak voltage

$$n = \frac{V_P}{\phi_0 2\pi f} = \frac{10^{-6}}{50\times 10^{-12}\times 2\pi \times 50}$$

$= \underline{\text{64 turns}}$

Examiner's comment: Most candidates were able to derive a formula for the magnetic field as a function of distance from the wire. The setter had (correctly) assumed that, given the coil was small compared to the distance from the wire, the magnetic field could be approximated as constant over the coil. Most students recognised that the field did vary and attempted to integrate the flux over the coil – most failing to set up the integral correctly. Both ways were allowed full marks (indeed only giving 0.1% difference in the answer). Most candidate's who got that far recognised that Faraday's law should be used for calculating the number of turns in the coil and could quote the law. However, the majority were unable to set up the expression for the alternating flux correctly and of those who could, few were able to differentiate correctly. Average mark was 54%.

12 **(short)** A 2 mm thick iron plate has a relative permeability of 1000. It is cut into a bar within an annulus, both of which are 10 mm wide as shown in Fig. 12. The annulus has an average diameter of 50 mm. If 0.5 A is passed through a coil of 300 turns round the bar and 2 mm gaps are cut between the top and bottom halves of the annulus either side of the coil, what is the magnetic field in the gaps? State any assumptions made.

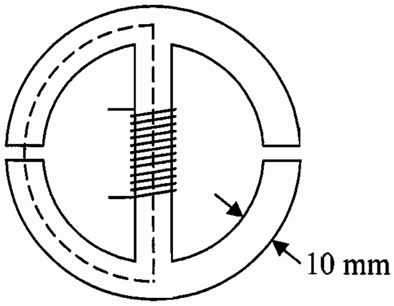


Fig. 12

Assume that H is uniform across the 10 mm width of the bar and similarly for the annulus. Integrate H round the dotted loop shown:

$$\Sigma HL = NI$$

$$(2B_A/\mu_0\mu_r)\times 50\times 10^{-3} + (B_A/\mu_0\mu_r)\,(\pi\times 25-2)\times 10^{-3} + (B_A/\mu_0)\times 2\times 10^{-3} = 300\times 0.5$$

$$B_A\times 2.18\times 10^{-3} = 150\ \mu_0$$

$$B_A = \underline{86\ \text{mT}}$$

Quite an easy question if the candidate knew how to do it. Those who did scored highly but there was a significant proportion who only had a basic idea and could not set up the equations correctly. A common error was not to recognise that the flux in the central bar was shared by the two arms. Average mark was 57%.

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