ENGINEERING TRIPOS PART IA

Wednesday 6 June 2007 9 to 12

Paper 1

MECHANICAL ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 **(short)** Water flows under a sluice gate as shown in Fig. 1. At sections 1 and 2 the flow is uniform and horizontal. The density of the water is $\rho = 10^3$ kg m^{-3} and the gravitational acceleration is $g = 9.81$ m s^{-2}.

(a) Why is the pressure distribution hydrostatic at sections 1 and 2? [2]

(b) Show that the force per unit width, F, required to hold the gate in place is related to the depth h and speed V of the fluid by

$$F = \tfrac{1}{2}\rho g h_1^2 - \tfrac{1}{2}\rho g h_2^2 + \rho h_1 V_1^2 - \rho h_2 V_2^2 .$$ [6]

(c) If $h_1 = 3$ m, $h_2 = 0.5$ m and $V_1 = 1$ m s^{-1}, find the force F. [2]

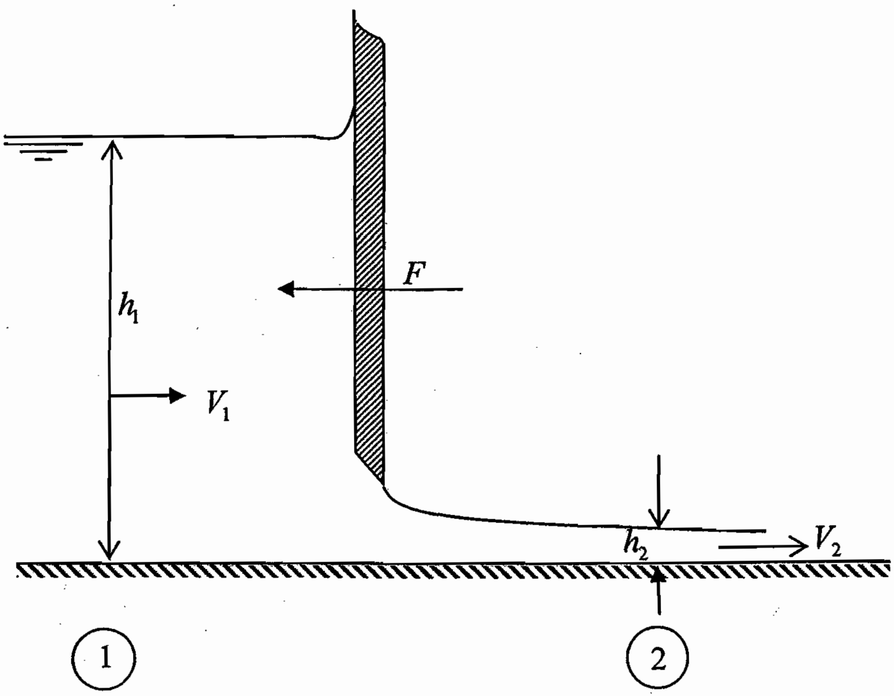

Fig. 1

2 **(short)** A piston pushes slowly downward on water held in a cylindrical tank. The water escapes from the tank through a nozzle, as shown in Fig. 2. The piston is located a height h above the nozzle, the force applied to the piston is F, the speed of the piston is V_1, and the cross-sectional areas of the cylinder and nozzle are A_1 and A_2 respectively. The pressure on the top surface of the piston is atmospheric.

(a) Use Bernoulli's equation to show that

$$F/A_1 = \tfrac{1}{2}\rho(V_2^2 - V_1^2) - \rho g h,$$

where ρ is the density of the water, g the gravitational acceleration, and V_2 the speed of the fluid leaving the nozzle. You may neglect friction, but you must state any other assumptions that you make. [7]

(b) Find the force F for the case where $A_1 = 1\ \text{m}^2$, $A_2 = 0.001\ \text{m}^2$, $h = 1.5$ m, $\rho = 10^3$ kg m^{-3}, $g = 9.81$ m s^{-2} and $V_1 = 0.01$ m s^{-1}. [3]

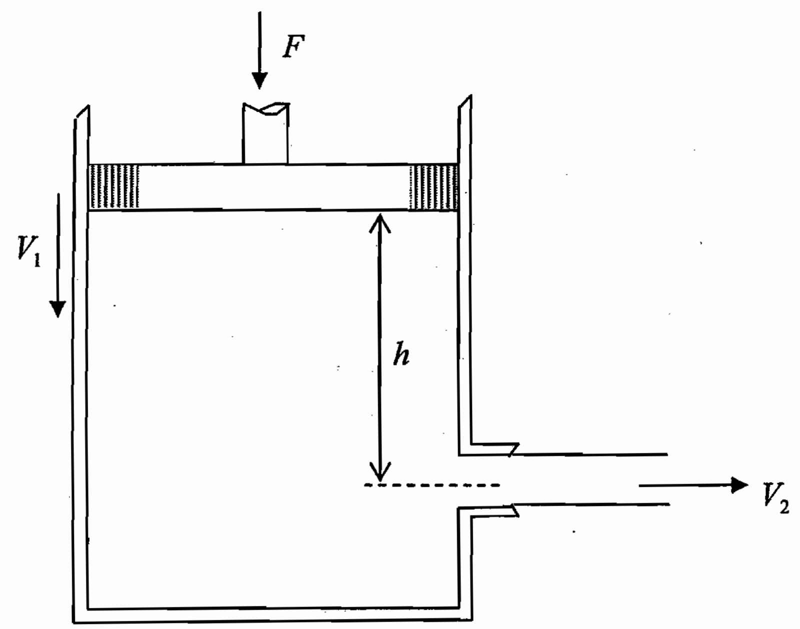

Fig. 2

(TURN OVER

3 (short)

(a) Use the first law, in the form $dq = du + pdv$, to show that, for a reversible adiabatic process, the incremental change in specific enthalpy is $dh = vdp$. Hence show that, when an incompressible fluid undergoes a reversible adiabatic process, the total change in specific enthalpy is given by

$$\Delta h = \Delta p / \rho ,$$

where ρ is the density of the fluid. [3]

(b) Water flows through an insulated pump. On the assumption that the flow is reversible, use the steady-flow energy equation to show that the pump power is given by

$$\dot{W}_p = (\dot{m}/\rho)\left[(p_2 - p_1) + \tfrac{1}{2}\rho(V_2^2 - V_1^2)\right] ,$$

where $\dot{m}$ is the mass flow rate, p_1 and V_1 are the inlet pressure and speed, and p_2 and V_2 are the outlet pressure and speed. You may neglect changes in potential energy. [5]

(c) Why will the flow not be reversible in practice? [2]

4 (short) A rigid tank of volume 0.25 m^3 contains methane at 773 K and 600 kPa. The tank is then cooled to 193 K. The methane may be treated as a perfect gas with specific heats c_p= 2.23 kJ kg^{-1} K^{-1} and c_v= 1.71 kJ kg^{-1} K^{-1}, and a gas constant of R = 0.520 kJ kg^{-1} K^{-1}.

(a) Find the mass of gas and the final pressure. [6]

(b) What is the heat transfer to the surroundings? [4]

5 (long) The jet pump shown in Fig. 3 operates by injecting water through a nozzle into a larger duct, also containing water. At section 1 the speed of the fluid leaving the the jet is uniform and equal to V_J, while the speed of the surrounding fluid is also uniform and equal to V_1. The two flows are fully mixed at the downstream location 2, where the fluid speed is uniform and equal to V_2. The flow is turbulent but steady-on-average and we wish to calculate the pressure rise Δp between sections 1 and 2, and the corresponding pressure-rise coefficient, $c = \Delta p / \frac{1}{2}\rho V_J^2$.

(a) Why is the pressure at the exit of the jet equal to that of the surrounding fluid? [3]

(b) Consider the control volume whose inlet and outlet are sections 1 and 2 respectively. Use the steady-flow momentum equation to show that

$$\Delta p = \rho\lambda V_J^2 + \rho(1-\lambda)V_1^2 - \rho V_2^2 .$$

Here $\lambda = A_J / A_2$ where A_J is the cross-sectional area of the nozzle and A_2 is the cross-sectional area of the duct. [8]

(c) If $V_J = 4$ m s^{-1}, $V_1 = 1$ m s^{-1}, $\rho = 10^3$ kg m^{-3} and $\lambda = 0.25$, find V_2 and hence the pressure rise Δp and pressure-rise coefficient, c. [5]

(d) Confirm that Bernoulli's equation is not satisfied on the centre-line of the duct between sections 1 and 2. Why is this? [6]

(e) Given that Δp is a function of the five independent variables ρ, V_J, V_1, A_J, A_2, use dimensional analysis to show that the pressure-rise coefficient, c, can be expressed as a function of $\lambda = A_J / A_2$ and V_J / V_1 only. What happens to the value of Δp if V_J and V_1 are both doubled? [8]

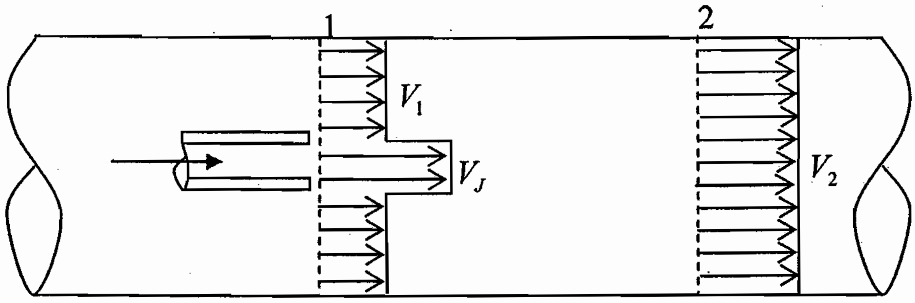

Fig. 3

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6 (long)

(a) A cyclic heat engine operates between a hot thermal reservoir at temperature T_H and a cold thermal reservoir at temperature T_C. Use the Clausius inequality to derive an expression for the maximum efficiency of the heat engine in terms of T_H and T_C. [8]

(b) The working fluid in a particular power station is water / steam. It undergoes a cyclic process, receiving heat at a rate of 4000 MW in the boiler, at an average temperature of 360 °C, and rejecting heat at a rate of 2300 MW from the condenser at 30 °C. All other processes in the cycle are adiabatic. Calculate the power output and the efficiency of the cycle. Calculate also the maximum possible efficiency of the cycle based on the given temperatures, and explain briefly why this is different from the actual efficiency. [7]

(c) Figure 4 shows part of an energy storage device, comprising a cyclic heat engine operating between a finite hot thermal reservoir and an infinite cold thermal reservoir. The hot reservoir has a thermal capacity of 1 MJ K^{-1} and its initial temperature is 600 K. The cold reservoir is at a constant temperature of 300 K. Calculate the maximum possible work output that can be produced by the device. [12]

(d) The processes in the heat engine are now reversed so that it acts as a heat pump. What is the minimum work input required to return the hot reservoir to its original state? [3]

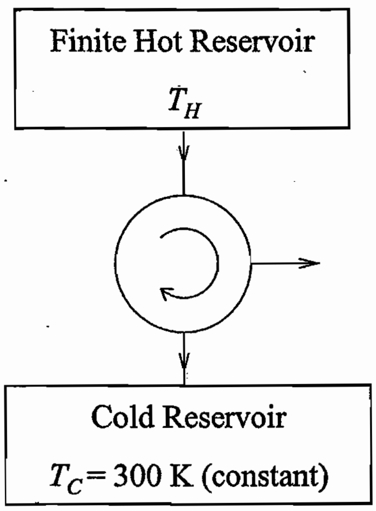

Fig. 4

SECTION B

7 **(short)** A uniform rod AB of length a and mass m rotates freely in a horizontal plane about a fixed point at A, with angular velocity ω, as shown in Fig. 5.

(a) What is the moment of momentum of the rod about point A? [4]

(b) The point B now strikes a stationary particle, also of mass m, which sticks to the rod. What is the angular velocity of the rod-mass system, immediately after the impact? [3]

(c) What fraction of the system's initial kinetic energy is lost in the impact? [3]

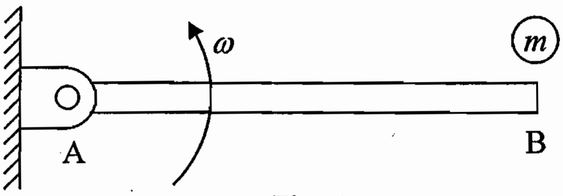

Fig. 5

8 **(short)** A mass m moves freely in a circle of radius r on a frictionless horizontal table, as shown in Fig. 6. It is attached to a scale pan, also of mass m, by a light, inextensible string which passes smoothly through a small hole at the centre of the circle.

(a) Show that the speed of the mass is $\sqrt{gr}$. [4]

(b) A further mass M is now placed (quickly but gently) onto the scale pan. During the subsequent motion, the scale pan is observed to descend through a distance 0.5 r, before starting to rise again. How big is M in relation to m? [6]

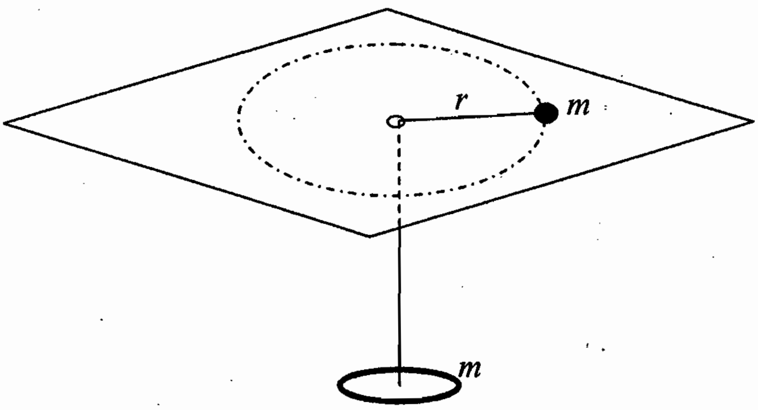

Fig. 6

9 **(short)** Figure 7 shows a jack being used to push a rigid bar CD into the vertical position. The jack is anchored at point A, and is attached to the midpoint of the bar at point B. At the instant shown, the point C is moving horizontally at 1 m/s, and the point D is constrained to move vertically.

(a) By means of a velocity diagram or otherwise, find the rate at which the jack is extending. [7]

(b) At what rate would the jack need to extend, to cause point D to move vertically at 1 m/s? [3]

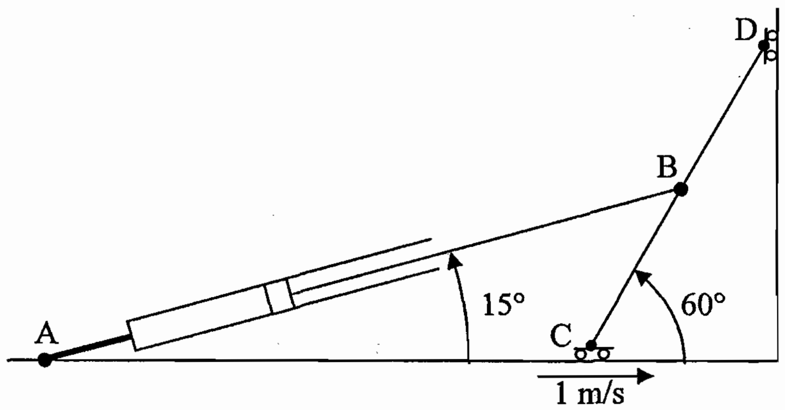

Fig. 7

10 **(short)** (a) For the circuit shown in Fig. 8, derive an expression of the form

$$T\,dv/dt + v = Ae$$

expressing the constants T and A in terms of C, R_1 and R_2. [6]

(b) After a long period during which $e = 1$ Volt, e suddenly changes to 2 Volts at time t_0. Sketch the resulting behaviour of the voltage v, showing salient values. [4]

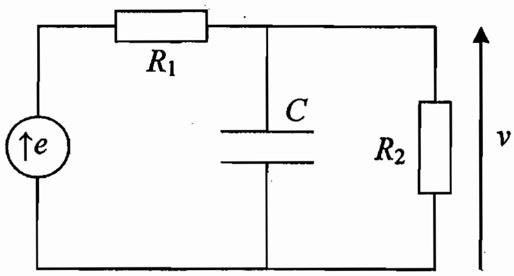

Fig. 8

11 (long) An industrial robot has been programmed to move a polishing head round a circle of radius 0.3 m at a constant peripheral speed of 1.5 m s^{-1}, as shown in Fig. 9. The robot consists of a radial arm hinged at a fixed point A and a sliding clamp C which holds the polishing head, with motors to control the angle θ of the arm and the position of the clamp along the arm. At the instant shown, the angle ABC is 90° (where B is the centre of the circle), and the distance AC (denoted by r) is 0.5 m.

(a) What are the values of $\dot{\theta}$ and $\dot{r}$ at the instant shown? [8]

(b) What are the values of $\ddot{\theta}$ and $\ddot{r}$ at the instant shown? [12]

(c) At what values of θ will the maximum and minimum values of $\dot{r}$ occur? [6]

(d) What will be the greatest value of $\dot{\theta}$, and where does it occur? [4]

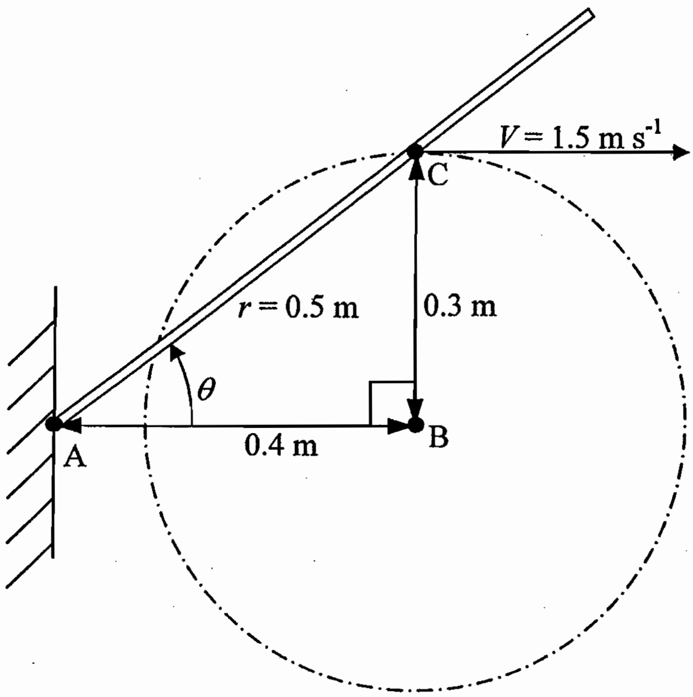

Fig. 9

(TURN OVER

12 **(long)** A sensitive laboratory instrument comprises a mass m supported by springs of total stiffness $2k$. It is placed on a lab bench which sometimes vibrates at a frequency of $\sqrt{3k/2m}$, with an amplitude of 0.1 mm.

(a) What will be the amplitude of the instrument's vibration, when the bench vibrates in this way? [6]

(b) To reduce this amplitude, the instrument is placed on a mount which consists of a mass $2m$ supported by springs of total stiffness k, as shown in Fig. 10. Write down the equation of motion of the new system in the form:

$$[M]\begin{bmatrix}\ddot{z}\\ \ddot{y}\end{bmatrix}+[K]\begin{bmatrix}z\\ y\end{bmatrix}=[K']x$$

identifying the values of the elements in the matrices $[M]$ and $[K]$, and in the vector $[K']$. [6]

(c) What is the new amplitude of vibration of the instrument, when the bench vibrates as described above? [12]

(d) What are the natural frequencies of the new system? [6]

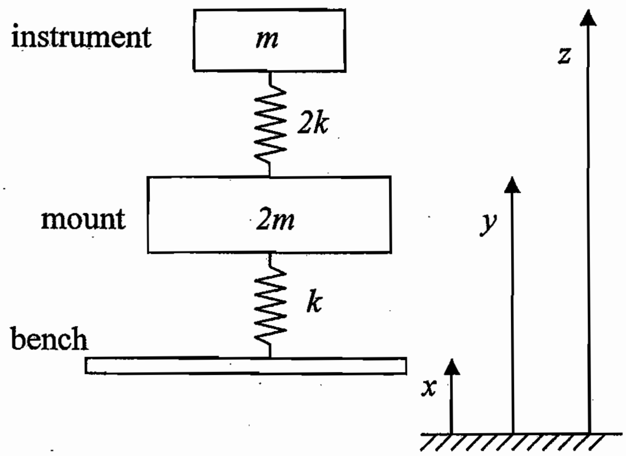

Fig. 10

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