ENGINEERING TRIPOS PART IA

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Thursday 7 June 2007 9 to 12

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Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

*Answers to questions in each section should be tied together and handed in separately.*

*There are no attachments.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

SECTION A

1 **(short)** A long line of tall vertical cylindrical telegraph poles is straight in plan, with the poles uniformly spaced 20 m apart. A uniform continuous flexible cable of weight 9 N $m^{-1}$ is attached to each pole at the same level, and dips 0.5 m in each span. The weight of the cable may be treated as being uniformly distributed over the horizontal, and the diameter and weight of each pole may be regarded as small.

(a) Find the tension in the cable at midspan of a typical span, and evaluate the force(s) applied to a typical pole. [4]

(b) The straight line of poles ends, and another such line begins, at one such pole, where the cable deviates through an angle of 45 degrees in plan, all other conditions remaining unchanged. Find the stress-resultants (i.e. all the internal 'force' quantities) in this corner pole at a cross-section 7 m below the point where the cable is attached. How would you calculate the expected deflection of this pole, assuming elastic behaviour? [6]

2 **(long)** The plane **pin-jointed** truss shown in Fig. 1 is simply supported over the horizontal span AF, with joint A fixed but joint F free to displace horizontally. All the angles of the truss are either 45 or 90 degrees.

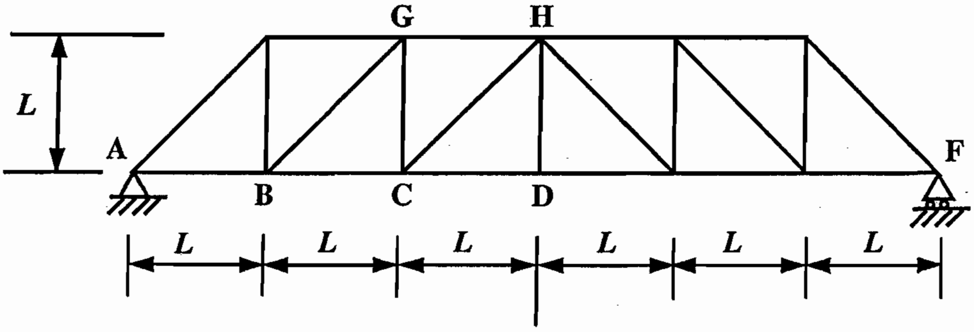


Fig. 1

(a) By the method of sections, find the forces in members BC, BG, CG and GH due to a downwards load $W$ applied at C. [8]

(b) In certain circumstances, the members BC, BG, CG and GH extend by amounts $+\Delta$, $-\Delta/\sqrt{2}$, $+\Delta/2$ and $-2\Delta$ respectively, the lengths of all other members remaining unchanged. The ratio $\Delta/L$ is small.

(i) Find the displacement of joint C vertically in these circumstances. [8]

(ii) Hence, noting that the lengths of many members do not change, find by inspection the vertical displacement of joint D. [4]

(c) In other circumstances, all the top horizontal members extend by $-\delta$, all the bottom horizontals by $+\delta$, all the diagonals by $-\delta/\sqrt{2}$ and all the verticals by $+\delta/2$, where $\delta/L$ is small. By symmetry, the vertical displacement of joint D *relative* to joint G is affected only by the extensions of a few bars near midspan of the truss. By drawing a displacement diagram, or by virtual work, show that this *relative vertical* displacement is $3\delta$ downwards. [10]

(TURN OVER

3 **(short)** A light uniform horizontal beam is to be placed symmetrically on to two point supports as shown in Fig. 2. The supports allow free rotation but prevent any vertical displacement; horizontal displacement is prevented at one of the supports but may occur freely at the other. The separation $h$ between the supports may be varied.

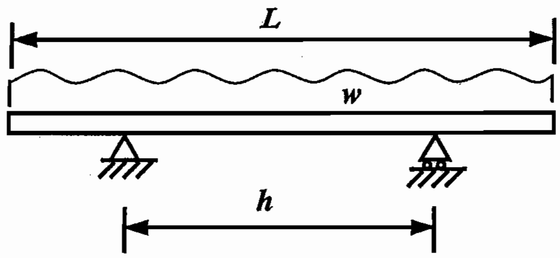


Fig. 2

(a) For the ratio $h/L$ equal to 0.6, sketch (giving salient values) the bending moment diagram for the beam due to a uniform downward load $w$ per unit length applied over the entire length $L$. [3]

(b) For the same applied load, derive (but do not solve) an equation for the ratio $h/L$ at which the maximum sagging and maximum hogging bending moments in the beam are equal in magnitude. [3]

(c) Sketch the diagram of shear force in the beam for $h/L = 0.6$, and find the maximum shear force. [2]

(d) State briefly, giving reasons, whether your answers would be affected if (i) the beam cross-section was not uniform or (ii) the beam was not light. [2]

4 **(long)** The light horizontal beam of span $L$ shown in Fig. 3 has uniform relevant bending stiffness $EI$ and is fixed against vertical displacement and rotation at both ends, while horizontal displacement can occur freely at one end only. The beam is initially unstressed, and a uniform downward load $w$ per unit length is then applied to the entire length of the beam.

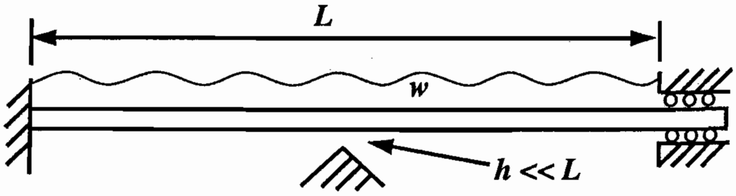


Fig. 3

(a) Assume linear-elastic behaviour and small deflections, and note that the hogging bending moments at the ends of this beam are then $wL^2/12$. By using suitable coefficients from the Structures Data Book, or by solving the differential equation for deflection of the beam, show that the beam deflects

(i) at midspan by $wL^4/384\,EI$, and [6]

(ii) by $3\,wL^4/2048\,EI$ at points $L/4$ from either end. [8]

(b) The beam deflects a small distance $h$ at midspan, but then comes into contact with a short rigid stop which prevents further deflection there – and loading continues beyond that stage. Describe briefly the boundary conditions on the further deflection of each half of the beam, for the further loading applied after the beam hits the stop.

(i) Sketch a graph of the reaction from the rigid stop versus the total load on the beam, giving salient values. [3]

(ii) Sketch graphs of deflection/span (on the vertical axis, up to about $1.2h/L$) against a suitable dimensionless measure of the applied load, for points on the beam at midspan and $L/4$ from either end. [10]

For what value of load does the deflection at $L/4$ from either end reach $h$? [3]

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5 **(short)** Figure 4 shows the cross-section of a composite T-beam, made of concrete and timber firmly connected together. The beam is loaded so as to be bent about a horizontal axis. The effective elastic modulus $E_c$ of the concrete is 20 GPa and the elastic modulus $E_w$ of the timber is 8 GPa. Linear-elastic behaviour may be assumed, and the materials are initially unstressed.

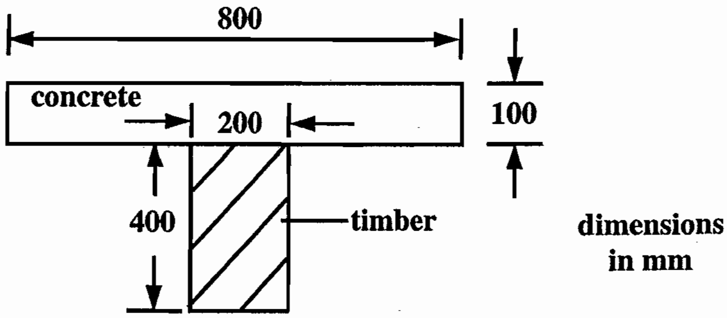


Fig. 4

(a) Find the longitudinal tensile stress at the bottom of the timber when the beam is subjected to a sagging bending moment of 200 kN m. [7]

(b) Determine the longitudinal shear force per unit length of beam transmitted across the interface between the timber and the concrete, when the transverse shear force in the beam is 70 kN. [3]

6 **(short)** The uniform linear-elastic strut shown in Fig. 5 is constrained to deflect only in the plane of the figure, with relevant flexural stiffness $EI$. Both ends of the strut are free to rotate; one end is completely fixed against displacement, while the other end is free to move only along the straight line between the ends of the strut. Both halves of the strut are initially straight, but the strut is crooked, with initial displacement (from the straight line joining its ends) of magnitude $e$ ($<< L$) at its centre $x = L/2$ , where the two halves are rigidly joined. The strut is subjected to equal inward forces $P$ as shown, acting along the straight line joining its ends.

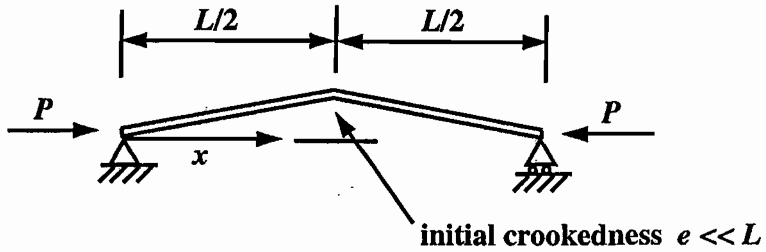


Fig. 5

(a) By considering equilibrium in the displaced position, with an upward transverse deflection $v(x)$ from the initial unloaded position, where $v << L$ and $x$ is measured from the left-hand end, show that the bending moment at any point on the strut with $x \leq L/2$ is $P(v + 2ex/L)$. [2]

(b) Hence derive a differential equation satisfied by $v(x)$, and show (by back substitution or otherwise) that its solution is, for $\alpha^2 = P/EI$, with $x \leq L/2$,

$$v = 2e\,\{\sin \alpha x / \alpha L \cos(\alpha L/2) - x/L\}.$$ [5]

(c) Sketch the graph of the deflection $v_c$ at the centre of the strut against $P$, and find the value of $v_c/e$ when $P = P_E/4$ where $P_E = \pi^2 EI/L^2$. [3]

SECTION B

7 **(long)**

(a) Sketch the force between two atoms as a function of their separation. Indicate by arrows the equilibrium spacing, the ideal maximum elastic tensile strain and a typical maximum elastic tensile strain in a real metal. [8]

(b) A strip of rectangular cross section *a* wide by *b* thick is bent through an arc in simple bending until the surface stress reaches the yield stress $\sigma_y$. State how the tensile strain varies across the bar and hence calculate the elastic energy stored per unit volume in terms of the yield stress and Young's Modulus. Compare this with the energy per unit volume which can be stored in simple tension. Why is it that in driving a clockwork mechanism we use bending rather than simple tension? Comment on the possibility of using torsion as an alternative to tension or bending. [12]

(c) Use the data book to choose a suitable material for a watch spring giving reasons for your choice. What factors, apart from energy density, need to be taken into account? [10]

8 **(long)**

(a) What is plastic deformation and how does it occur? What is meant by 'work hardening' and what causes it? Explain why the amount of work hardening for a given true strain is independent of whether the strain is tensile or compressive. Why do the nominal stress strain curves differ between compression and tension? [12]

(b) A bar is made of a material in which the curve of true stress against true strain can be approximated by $\sigma = \sigma_o \sqrt{\varepsilon}$ with $\sigma_0 = 800$ MPa. Elastic strains are small and can be ignored. The bar is put through the following sequence:

(i) It is stretched to a true strain of 2%.

(ii) The applied stress is removed.

(iii) Stress is reapplied to produce a further strain of 5% (i.e. a total of 7%).

(iv) The bar is compressed to its original length so that the total true strain is zero.

Sketch the graph of stress against strain for this sequence. What is the stress at the end of each of the four sections above? [12]

(c) A circus strong-man bends a bar of this material into an arc and asks a member of the audience to bend it straight again. How much stronger than the strong man must the member of the audience be to succeed? What effect other than work-hardening will make the task even more difficult? [6]

9 **(short)** What is the origin of friction between metal surfaces? How does the frictional force depend on the mean pressure between the surfaces, and how can this dependence be explained in terms of the effective area of contact and an oxide film between the surfaces? How would you choose two materials to minimise friction between them when they are in contact? [10]

10 **(short)** Explain how a proof test is used to avoid fatigue fracture in a pressure vessel. What other precautions should be taken in the design and manufacture of a structure to avoid fatigue fracture and other forms of failure? [10]

(TURN OVER

11 **(short)** What material property is used to describe each of the following colloquial terms?

(i) stiff
(ii) hard
(iii) brittle.

Describe briefly one method of measuring each property. How does cooling affect each of these parameters in metals? [10]

12 **(short)** When mild steel oxidises it is found to gain weight at a rate given by $m = \sqrt{\alpha t}$ where $m$ is the weight gain per unit area in kg $m^{-2}$, $t$ the time in seconds and $\alpha = 2\times10^{-8}$ $kg^2$ $m^{-4}$ $s^{-1}$ at 520°C. Find the depth of metal lost from the surface of a mild steel bar kept in a furnace at this temperature for a year. Assume the oxide formed is $FeO$ . [10]

**END OF PAPER**