Engineering Tripos Part 1A 2008

Paper 1: Mechanical Engineering

Section A – Prof PA Davidson

1. (a) -
   (b) $V_2 = 0.362$ m/s

2. (a) -
   (b) -
   (c) -

3. (a) -
   (b) $T_2 = 391K$

4. (a) $\rho_1 = 2.49 \quad kg/m^3$
   $\dot{m} = 5.97\, kg/s$

   (b) $\dot{W} = 3.58 \quad MWatts$
   $p_2 = 1.41\, x\, 10^5 \quad N/m^2$

5. (a) -
   (b) -
   (c) $V_2 = 5$ m/s, $H_1 - H_2 = 12{,}500$ N/m^2
   (d) $\dot{W} = 125 \quad Watts$
   (e) -

6. (a) T, F, F
   (b) -
   (c) -

Section B – Dr D J Cole

7 (a) $\mathbf{v} = 40\mathbf{e_t}$ m/s

$$\mathbf{a} = -\frac{5\sqrt{3}}{2}\mathbf{e_t} + \frac{5}{2}\mathbf{e_n} \text{ m/s}^2$$

(b) 640 m

8 (a) $\omega_{AB} = \frac{\omega}{3\sqrt{2}}$ clockwise

$\omega_{BC} = \frac{\omega}{3\sqrt{2}}$ anticlockwise

(b) $3\sqrt{2}T$ clockwise

9 (a) $9R$

(b) $\frac{V}{4}$

10 (a) $0;\ \sqrt{\frac{3k}{2m}}$

11 (a) (i) $\frac{gmvt}{L} + \dot{m}v$ for $0 < t < \frac{L}{v}$ where $\dot{m} = \frac{mv}{L}$

(b) (ii) $\frac{3}{2}g^2\frac{m}{L}t^2$ for $0 < t < \sqrt{\frac{2L}{g}}$

12 (a) 1 kN; 0.01 mm

(b) $\omega_n = \sqrt{\frac{k}{m}};\quad \zeta = \frac{\lambda}{2\sqrt{km}};\quad x = \frac{b}{m}e^{i\omega t}$

(c)
$$\frac{\frac{b}{m}\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

(d) 5.59 N

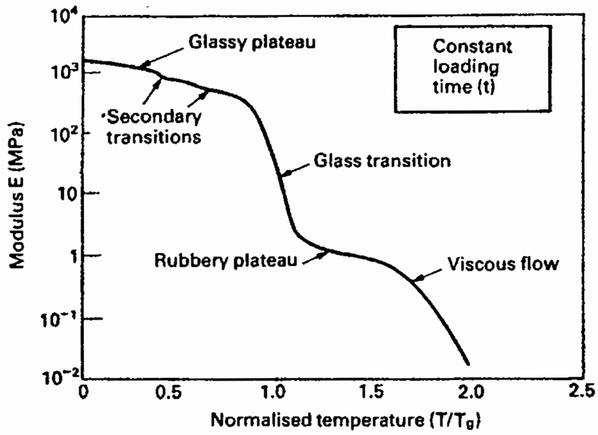
ENGINEERING TRIPOS PART IA

Thursday 5^{th} June 2008 9 to 12

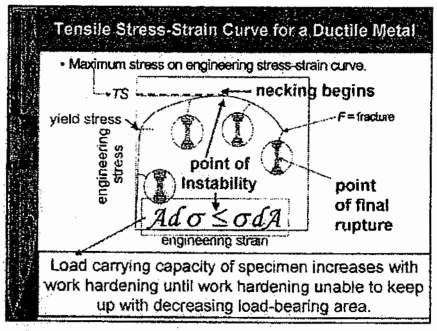
Paper 2 STRUCTURES AND MATERIALS

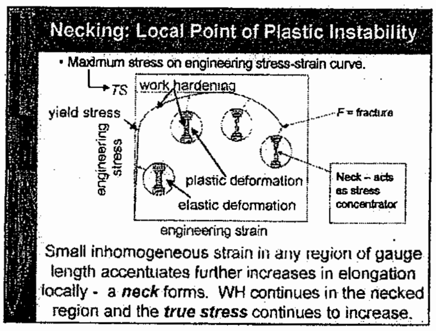
Section B

7.



8.





9. (c) Zn forms $Zn^{2+} + 2e$

•

If 1 mol of zinc dissolves, the number of electrons released is: 6.022 x 10^{-23} x 2.

The charge released is 6.022 x 10^{23} x 2 x 1.602 x 10^{-19} C = 1.93 x 10^{-5}C.

The number of coulombs liberated by the anode is:

(136 kg)/(0.0654 kg) x 1.93 x 10^{5} C = 4.01 X 10^{8} C.

If the current is 1A, 1C passes per second. Therefore, the time is:

= (4.01 x 10^{8})/(2 x 60 x 60 x 24 x 365) = 6.36 years.

10. (a) Survival probability of the larger component V (the compressor blade) is given by: $P_S(V) = exp\{-(V/V_O)\ (\sigma/\sigma_O)^m\}$

Taking natural logsequating *survival probabilities* of the 2 samples of *different size* and re-arrange the design equation to show: $(\sigma_{comp} / \sigma_{test}) = (V_{test} / V_{comp})^{1/m}$

where $\sigma_{testpiece}$ is 250MPa and $V_{testpiece}$ is 0.1 the volume of the blade.

For the sprobability of survival of 95% and 10 fold increase in volume, the strength of the blade is reduced from about 250 to 200 MPa.

11. (a) $\sigma=pR/t$ = (1.5 x 0.84)/0.014 = 90MPa

(b) (i) $\sigma_f = K_{Ic}/(\pi a)^{1/2} = 45/(\pi 0.01)^{1/2} = 257$ MPa

$p = \sigma t/R$ = (257 x 0.014)/0.84 = 4.3MPa

This is nearly 3 times greater than the allowable design pressure.

(c) Relief safety valve must have become inoperative and the vessel was over-pressurized.

(d) Leak before break

(e) This condition is achieved by setting $a = t$:

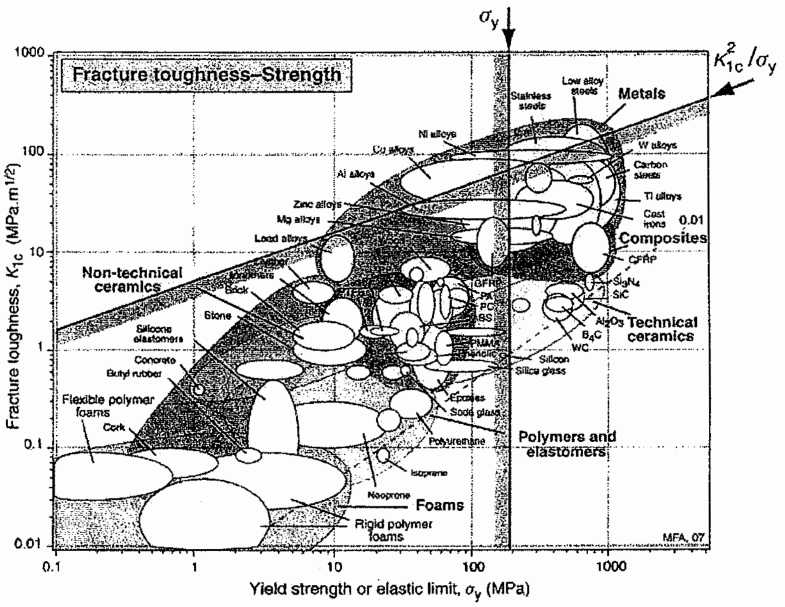
$$\sigma = \frac{K_{IC}}{\sqrt{\pi t}}$$

Furthermore, the wall thickness t of the vessel must be sufficiently thick that pressure p is contained without the vessel yielding: $t \geq \frac{p\mathcal{R}}{2\sigma_y}$

Substituting this equation into the one above: $p \leq \frac{1}{\pi\mathcal{R}}\left(\frac{\mathcal{K}_{IC}^{\ 2}}{\sigma_y}\right)$

(f) The pressure carried most safely is when the material selected has the greatest value of: $\mathcal{M}_I = \frac{\mathcal{K}_{IC}^2}{\sigma_y}$

(g)



12. (d) Particle volume fraction = area fraction = $\frac{\pi d^2/4}{\ell^2} = 0.05$

$$\bar{\ell} = \frac{\pi^{1/2}}{2(0.05)^{1/2}} d = 198 \times 10^{-6} = \ell + 2d$$

Therefore, the separation distance, ℓ, is $\ell = \ell' - 2d = 198\mu m - 100\mu m = 98\mu m$

Now:

$$\sigma_y = 2\tau_y = \frac{4Gb}{\ell}$$

$$\sigma_y = \frac{4 \times 140 \times 10^9 \times 3 \times 10^{-8}}{98 \times 10^{-6}}$$ = 170MPa approximately.

Thus, total yield strength is: σ_y = 50MPa +170MPa = 220MPa.

Engineering Tripos Part 1A

Paper 3

Section A. Linear Circuits

1 (a) R_3= 6.43 kΩ; R_1=R_2= 3MΩ; (b) 0.976; R_{out}=139.4 Ω; R_{in}=1.5 MΩ
2(a) P_{Line}= 732 W; Q_{Line}=488 VAR; V_{Input}= 245.7 V; (b) C=1.243 mF
3(a) Z_L'=(50+j90)Ω; I_L=21.4 A; (c) P_L=229 W; P_{Loss}= 23W; Efficiency=91%
4(b) I_{xx}=2.235 A; (c) I_{xx}=1.97 A; (d) I_{xx}=0.685 A
5(b) R_{in}=2075Ω; G=-7.19; (c) v_0=-3.66 v_S; F_{3dB}=531kHz

Section B Digital

6 (a) $S_1 = \overline{A.B}$ $S_0 = \overline{A.\bar{B}.\bar{A}.B}$ (b) 2 gates are needed, an AND gate and an XOR gate.

7 (a) Capacity in bits is $2^{14} \times 2^3 = 2^{17}$.

8 (b) The H, C and Z flags will be set, the N and V flags will not be set. (c) 12 clock cycles.

9 (b) 4 states so minimum number of bistables is 2.

Section C Electromagnetics

10 (a) 1.02 T (b) 0.306 mWb (c) 5.4×10^4 Am^{-1}

11 (b) B = 0.133 mT at 1.5 mm, B = 0 T at 3 mm (no nett current enclosed).

12 (d) Force is 0.077 N.

Dr Tim Flack

Engineering Tripos Part IA 2008

Paper 4: Mathematical Methods

Short Answers

Section A

Q1: $\begin{pmatrix} 0 \\ 1/3 \\ 4/3 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix}$

Q2: $y = Ae^{x} + Be^{-4x} + 5xe^{x}$

Q3: (b) $x_n = n\dfrac{21}{10}$

Q4: (a) circle centred at (5,0) with radius 6; (b) (i) 0; (ii) 1/2; (c) $z = 3$ or -3

Q5: (a) eigenvalues 1, 3, -1; normalised eigenvectors $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}, \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{pmatrix}$;

(c) $\begin{pmatrix} \frac{1}{2}(3^{10}+1) & 0 & \frac{1}{2}(3^{10}-1) \\ 0 & 1 & 0 \\ \frac{1}{2}(3^{10}-1) & 0 & \frac{1}{2}(3^{10}+1) \end{pmatrix}$

Section B

Q6: $x(t) = -\dfrac{5}{4}e^{-5t} + \dfrac{13}{4}e^{-t}$

Q7: Step response $y = \dfrac{1}{14} + \dfrac{1}{35}e^{-7t} - \dfrac{1}{10}e^{-2t}$;

Impulse response $\dfrac{dy}{dt} = -\dfrac{1}{5}e^{-7t} + \dfrac{1}{5}e^{-2t}$

Q8: (a) 0.0410; (b) 0.0311

Q9: $V(t) = \frac{V_0}{\pi} + \frac{V_0}{2}\sin\omega t - \frac{V_0}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n+1)(2n-1)}\cos 2n\omega t$

Q10: (a) saddle point at (0,0), minimum at (1,-1), minimum at (-1,1)

Section C

Q11: (b) produces an infinite loop

Q12: (b) (i) exchange sort; (ii) Quicksort