Dr A Ferrari Sec A
Dr T Flack Sec B

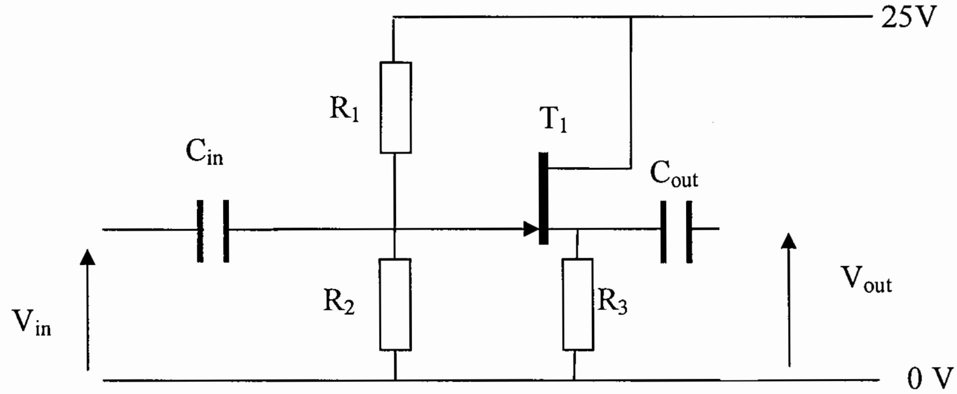
Engineering Tripos Part 1A

Paper 3

Section A: Dr A Ferrari

1 **(short)** In the circuit of Fig. 1, R_1= 3MΩ and C_{out} may be assumed to be large. Transistor T_1 has small signal parameters g_m=7mS and r_d= 60kΩ. The operating point for T_1 is defined as V_{DS}= 7V, I_{DS}= 2.8 mA, V_{GS}= -5.5 V

(a) Calculate values for R_2 and R_3



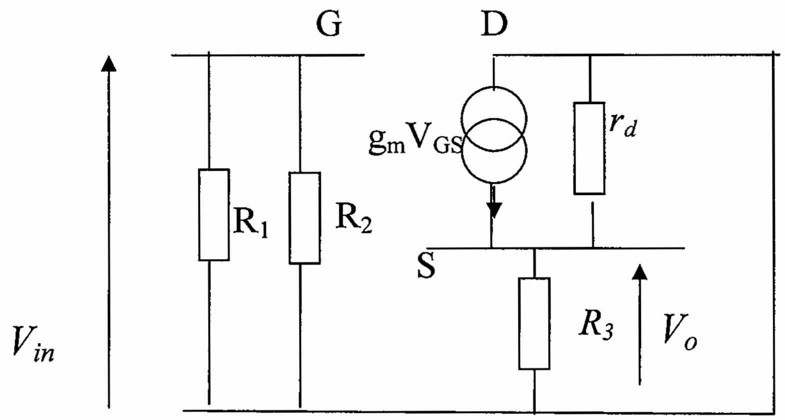
There are 18V across R_3

$18V/R_3 = I_{DS} = 2.8$ mA

⇒ R_3= 6.43 kΩ

V_{G0}= (18-5.5)V= 12.5 V
⇒ R_1=R_2= 3MΩ

(b) Calculate the small signal gain and input and output impedances at mid-band frequencies



At midband C_{out} is negligible

$R' = r_d \parallel R_3 = 5.7\ k\Omega$
$Vo/V_{in} = g_m R' / (1+g_m R') = 0.976$

$I_{out}(short) = g_m V_{in}$

$V_o\ (open) = g_m R' V_{in} / (1+g_m R')$

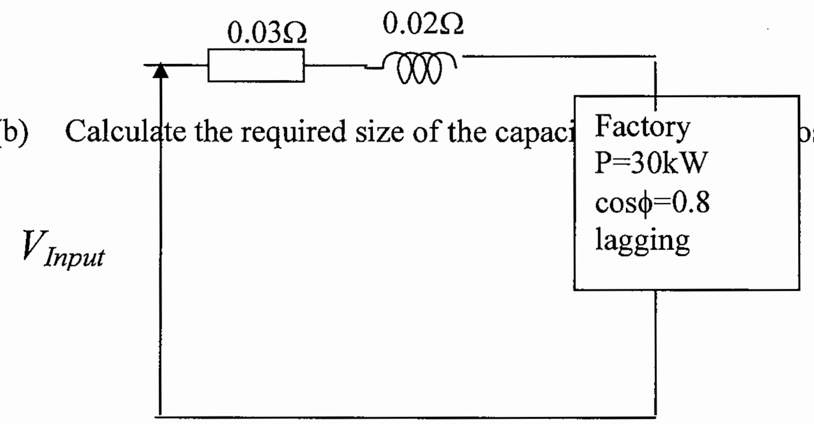
Output impedance R_{out}:

$R_{out} = V_o(open)/I_{out}(short) = R'/(1+g_m R') = 139.4\ \Omega$

$R_{in} = R_1 \parallel R_2 = 1.5\ M\Omega$

2 **(short)** A small factory consumes 30kW at 240 V, with a lagging power factor of 0.8. The supply line has impedance $Z_s = 0.03 + j0.02\ \Omega$

(a) Draw a circuit diagram and calculate the power lost in the line and the voltage supplied by the power station.



(b) Calculate the required size of the capaci oss the factory terminals in order

$240 \cdot I \cdot \cos\phi = P$

$\Rightarrow I = 30000/(240 \cdot 0.8) = 156.2$ A

$Q_{Load} = P \tan\phi = 22500$ VAR

$P_{Line} = I^2 0.03 = 732$ W
$Q_{Line} = I^2 0.02 = 488$ VAR

$P_{Input} = 30000W + 732W = 30732W$
$Q_{Input} = 22500VAR + 488VAR = 22988VAR$

$VA_{Input} = (P_{Input}^2 + Q_{Input}^2)^{0.5} = 38378$ VA

$V_{Input} = VA_{Input}/I = 245.7$ V

(b) Calculate the required size of the capacitor connected across the factory terminals in order to correct the power factor to unity.

$X_C = V_{Load}^2 / Q_{Load} = 240^2/22500 = 2.56\Omega$

$C = 1/(2\pi f X_C) = 1.243$ mF

Inserting the numerical values

C=1.7 mF

3 **(short)** A transformer with a primary:secondary turns ratio of 10:1 has a load of impedance $(0.5+j0.9)\Omega$ connected to its secondary winding, and its primary winding is connected to a 240V rms supply. The equivalent circuit parameters of the transformer referred to the primary are:
$R_1 = 3\Omega$; $R_2' = 2\ \Omega$; $X_1 = 4\ \Omega$; $X_2' = 3.5\ \Omega$
The magnetising resistance and iron loss resistance are large enough to be ignored.

(a) Find the impedance of the load referred to the primary

$Z_L' = (10/1)(0.5+j0.9)\Omega = (50+j90)\Omega$

(b) Determine the load current

$I_L' = 240V/(R_1+R_2'+jX_1+JX_2'+Z'_L) = 240/(55+j97.5) = 2.14$ A

$I_L = 10 \cdot 2.14$ A $= 21.4$ A

(c) Find the load power and the transformer power loss and efficiency

$P_L = I_L^2\ R_L = 229$ W

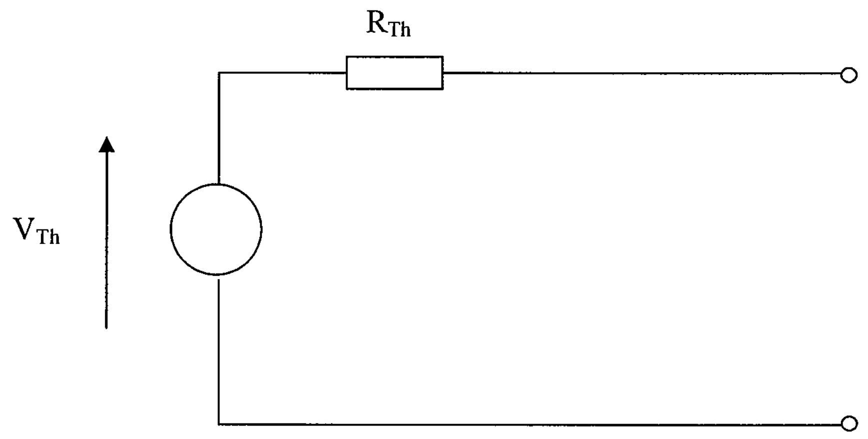
$P_{Loss} = I_L'^2 (R_1+R_2') = 23$ W

$P_{in} = P_L + P_{Loss} = 252$ W

Efficiency $= P_L/P_{in} = 91\%$

4 **(long)** (a) Describe Thevenin's and Norton's Theorems. Illustrate these by deriving an equivalent model in each case for the circuit shown in Fig 2(a)

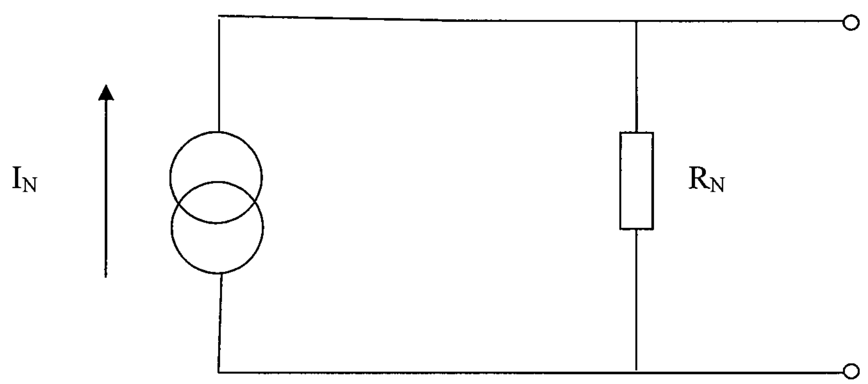
Thevenin: Any linear circuit may be represented as:



$V_{Th}=V_{Open\ Circuit}$

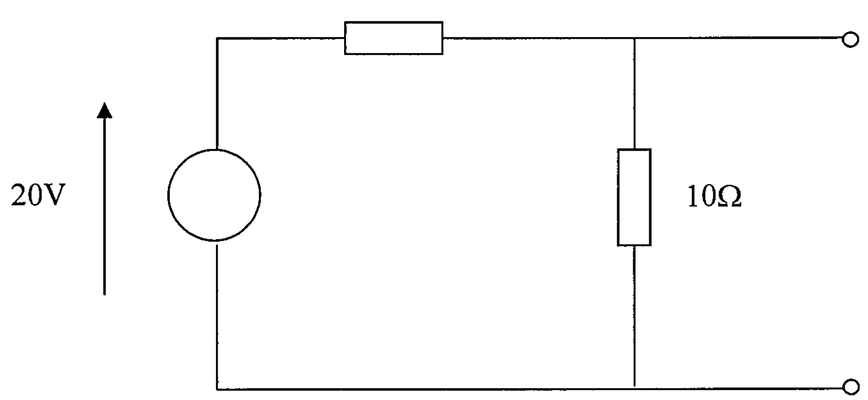
$R_{Th}=V_{Open\ Circuit}/I_{Short\ Circuit}$

Norton: Any linear circuit may be represented as:



$I_N=I_{Short\ Circuit}$
$R_N=V_{Open\ Circuit}/I_{Short\ Circuit}$

Figure 2(a):

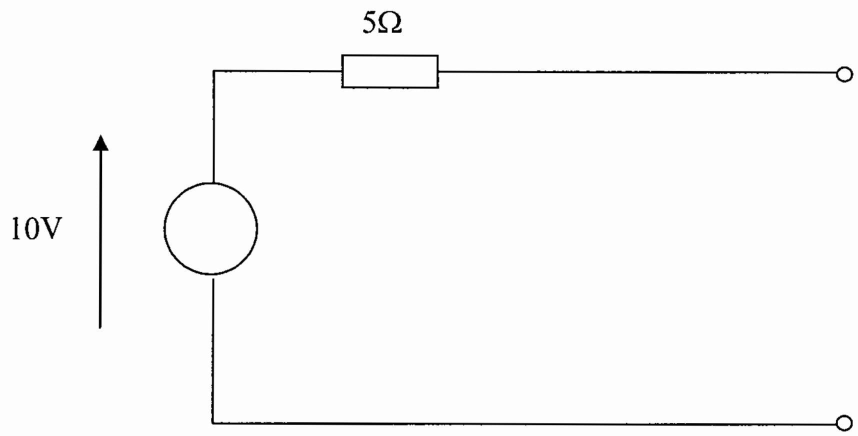


$V_{Open\ Circuit}=20V\cdot 10/(10+10)=10V$

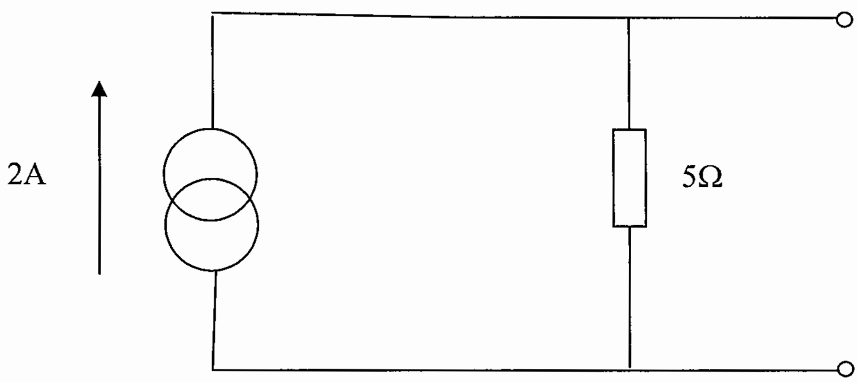
$I_{Short\ Circuit} = 20V/10\Omega = 2A$

$\Rightarrow$

Thevenin equivalent of Fig. 1(a)

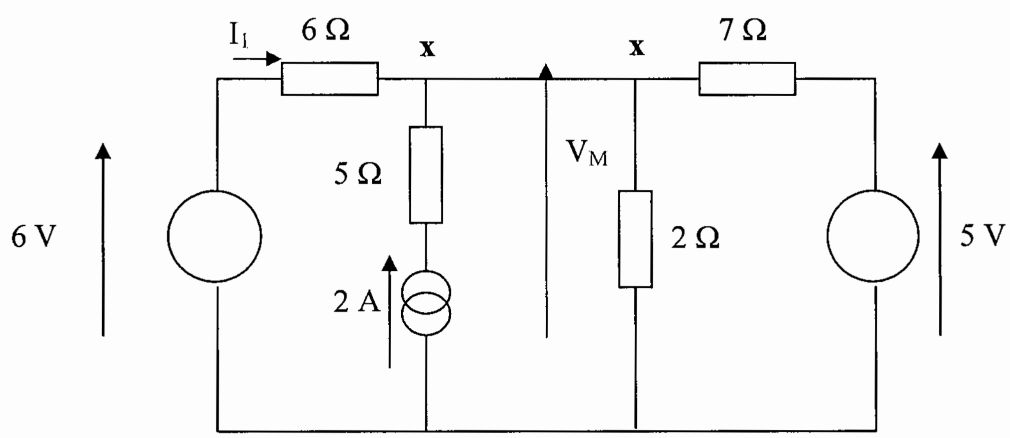


Norton equivalent of Fig. 1(a)



(b) Calculate the current in the link x-x for the circuit shown in Fig. 2(b), using Norton's and Thevenin's theorems to simplify the circuit.

Figure 2 (b)



Using Nodal Voltage Analysis

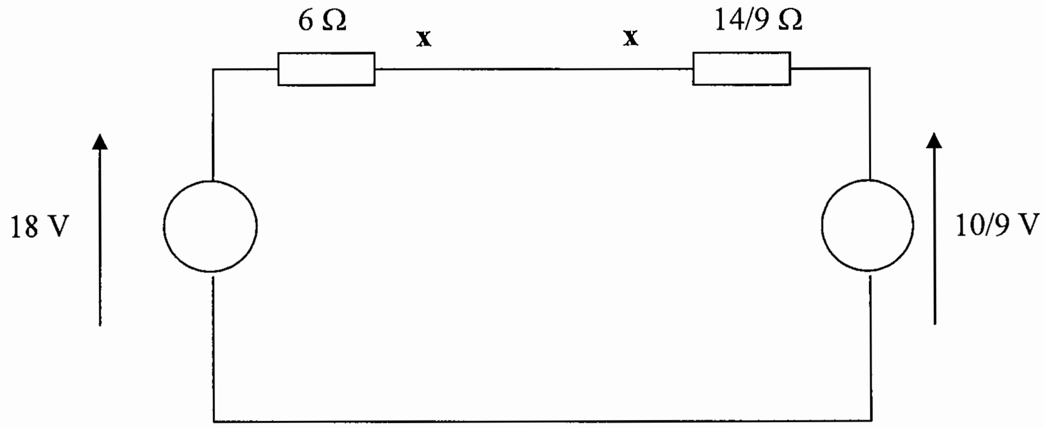
$(6V-V_M)/6\Omega+2A+(5V-V_M)/7\Omega=V_M/2\Omega$

$\Rightarrow V_M=4.59$ V

$I_1=(6V-V_M)/6\Omega=0.235$ A

$I_{xx}=I_1+2A=2.235$ A

Alternatively, the circuit in Fig 2b can be converted to



Then

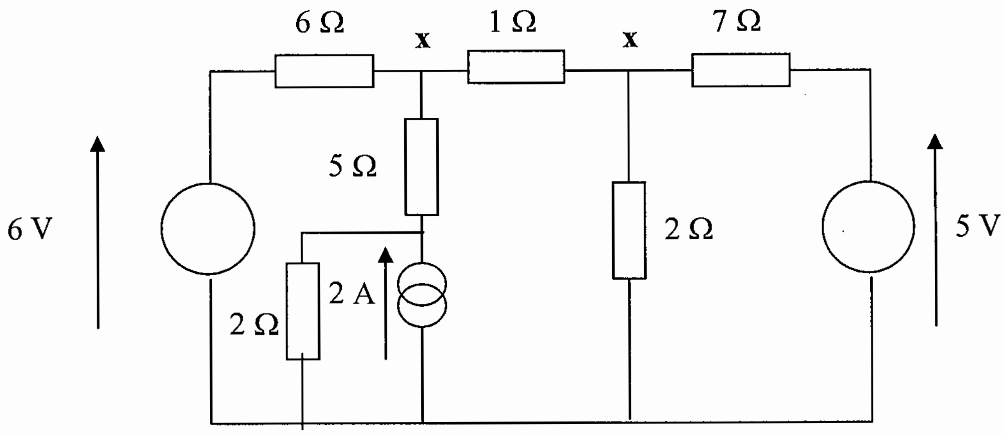
$I_{xx}=(18V-10/9V)/(6\Omega+14/9\Omega)=2.235A$

(c) If a resistor of value 1Ω is connected in place of the link x-x, what current would pass through it?

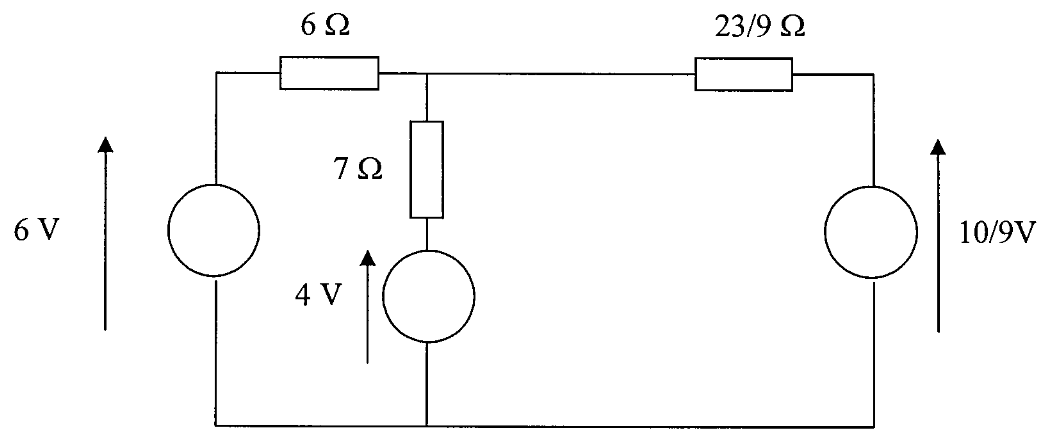
Using the above graph, this is simply answered as

$I_{xx}=(18V-10/9V)/(6\Omega+14/9\Omega+1\Omega)=1.97$ A

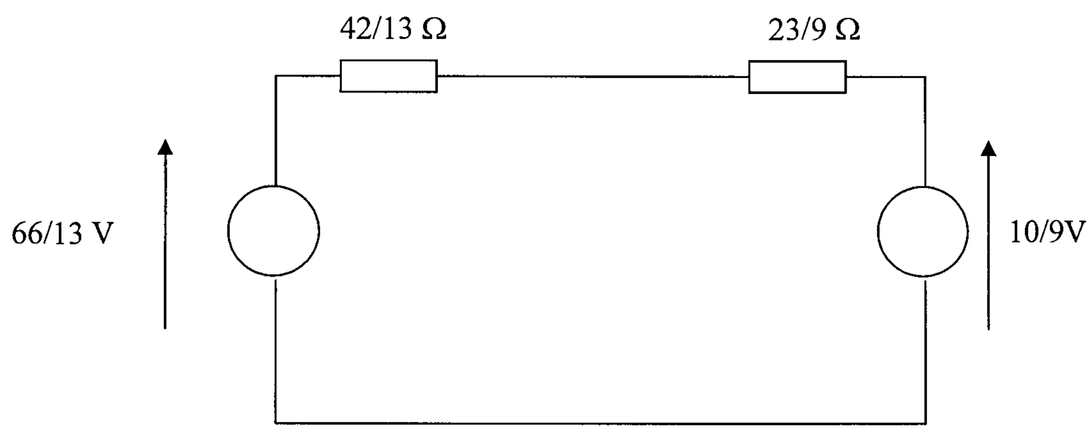
(d) If a resistor of value 2 Ω is connected in parallel to the current source in Fig. 2(b), what is the new value of current passing through the 1 Ω resistor connected in x-x?



This can be converted via the following steps



Then



I_{xx}=(66/13V-10/9V)/(42/13Ω+23/9Ω)=0.685 A

5(Long) (a) What are the parameters that describe an ideal operational amplifier and under what circumstances may real amplifiers be considered as ideal?

Output Resistance R_o

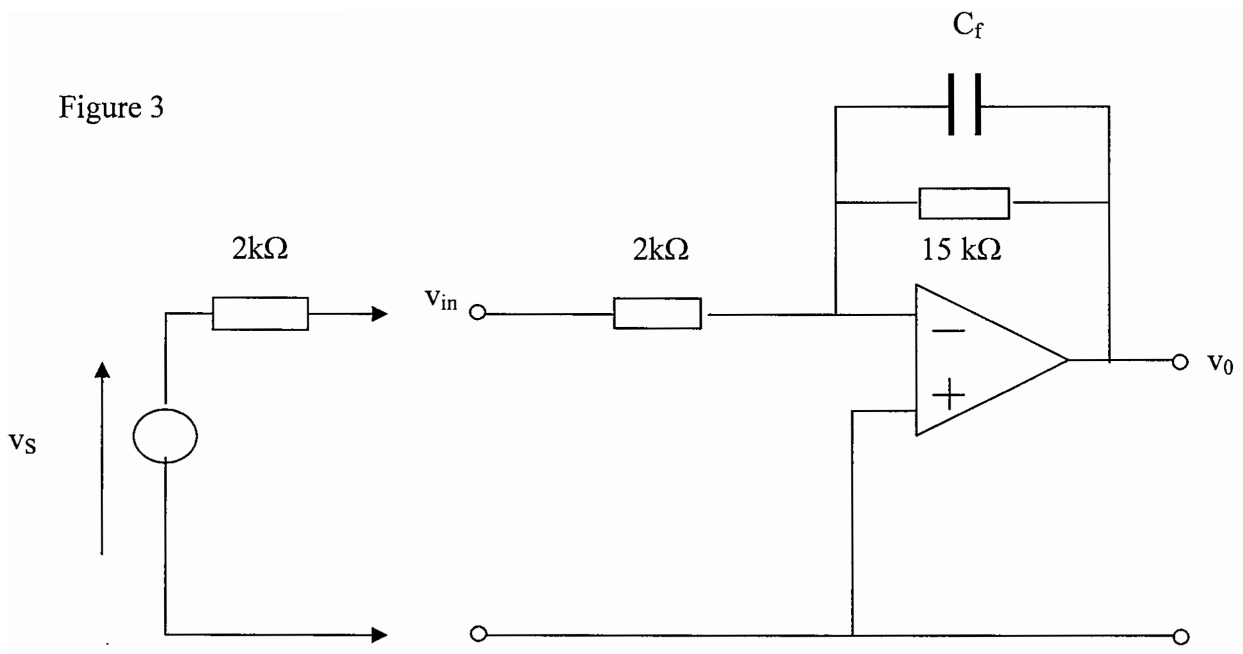
Input Resistance R_{in}

Gain A

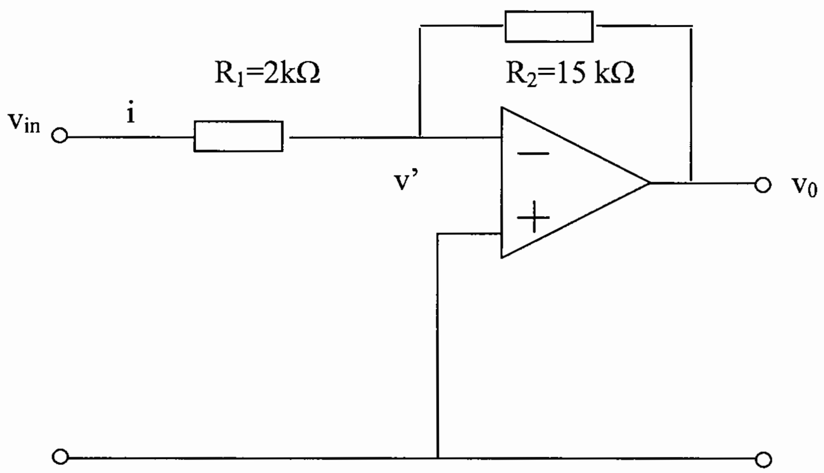
In the ideal case: $R_o=0$; $R_{in}=\infty$; $A=\infty$

(b) The op-amp in Fig. 3 may be assumed as ideal, except for the finite gain A of 200. Calculate the input impedance and voltage gain of the circuit, given that the signal source is not connected, and neglecting the effects of C_f.

Figure 3



If the signal source is not connected, and neglecting the effects of C_f, Fig 3 becomes:



$i= (v'-v_0)/R_2=(v'+Av')/R_2=v'(1+A)/R_2$

$v'=iR_2/(1+A)$

$v_{in}=iR_1+(iR_2)/(1+A)$

$R_{in}=v_{in}/i=R_1+R_2/(1+A)=2075\Omega$

$v_0=-Av'=-iAR_2/(1+A)$

$v_{in}=iR_{in}+(iR_2)/(1+A)$

$G=v_0/v_{in}=(-AR_2)/[R_1(1+A)+R_2]=-7.19$

(c) What is the output voltage v_0 in terms of v_S if the signal source in Fig. 3 is connected to the circuit?

$v_0=v_SG=-AR_2v_S/[R_1(1+A)+R_2]=-3.66\ v_S$

(d) With the signal source connected, and considering C_f=20pF, and treating the op-amp as ideal, what is the frequency at which then output voltage, v_0, falls to 70% of its low frequency value?

$v_0/v_S=-Z/R_1$

where

$Z=(R_2/j\omega C_f)/(R_2+1/j\omega C_f)=R_2/(1+j\omega C_fR_2)$

Thus

$v_0/v_S=-R_2/[R_1(1+j\omega C_fR_2)$

$\omega C_fR_2=1$ at 70%

F_{3dB}=531kHz

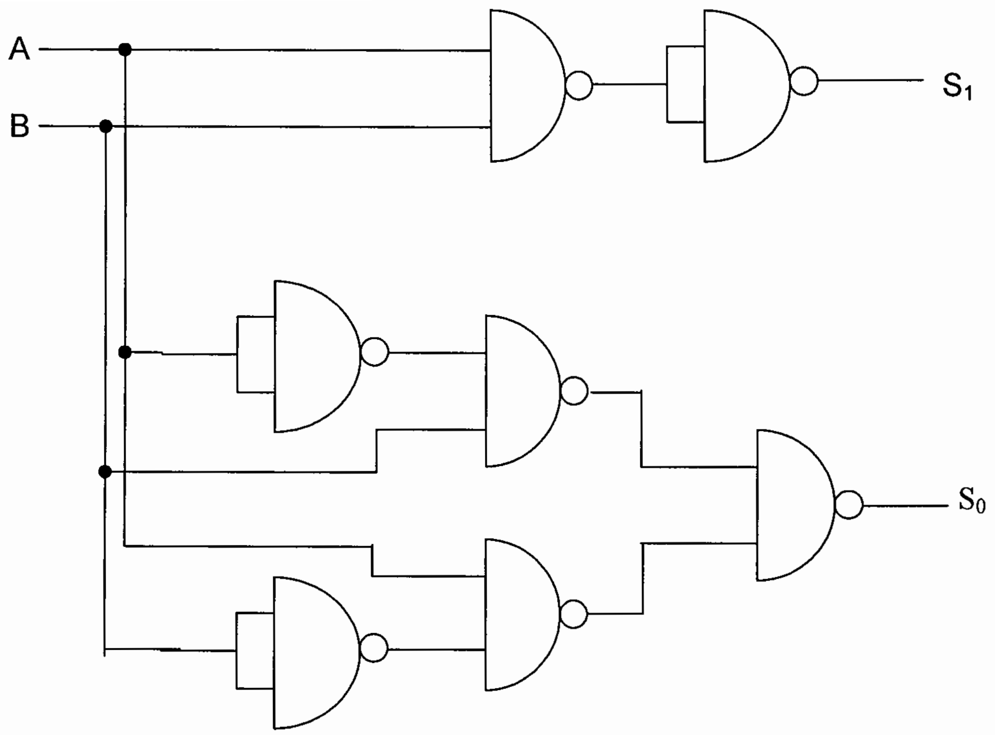
Section B Digital – Dr Tim Flack

6 (a) Truth table is:

A	B	S_1	S_0
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

By inspection $S_1 = A.B$ $\qquad S_0 = A.\bar{B} + B.\bar{A}$

NAND form $S_1 = \overline{\overline{A.B}}$ $\qquad S_0 = \overline{\overline{A.\bar{B}}.\overline{\bar{A}.B}}$



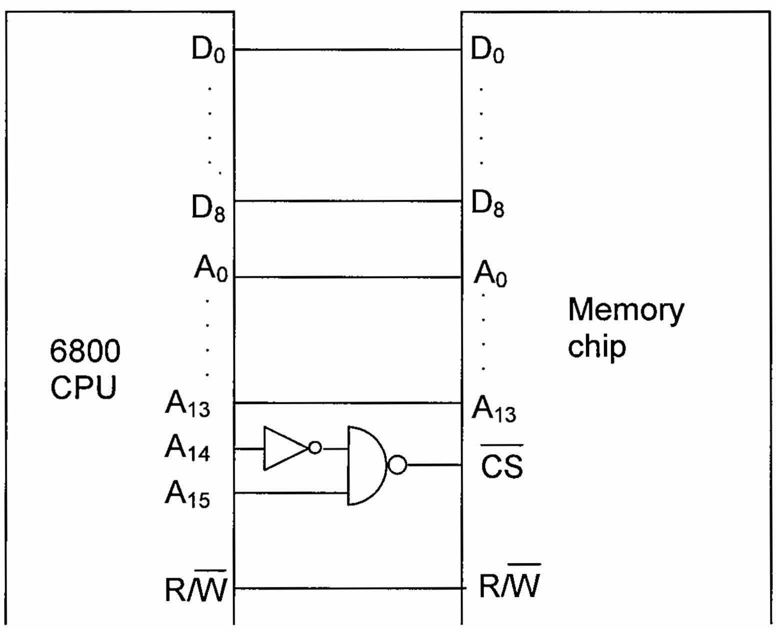
(b) 2 gates are needed, an AND gate and an XOR gate.

7 (a) 8 data lines means there are $8 = 2^3$ bits stored in every memory location. 14 address lines means there are 2^{14} memory locations. Thus the capacity in bits is $2^{14} \times 2^3 = 2^{17}$.

(b) 8000 and BFFF are shown below in binary:

1000 0000 0000 0000
1011 1111 1111 1111

As expected the 14 least significant bits change state in going from the lowest to the highest memory address and as address lines A_0 - A_{13} of the 6800 should be connected directly to address lines A_0 - A_{13} of the memory chip. Bits A_{14} and A_{15} of the 6800 address lines must be connected to the chip select input of the memory chip such that it is low when $A_{15}A_{14}$ is 10 and high otherwise.



8 (a) A table showing the instructions, the contents of accumulators A and B and the memory location $001B, and the clock cycles taken for the instructions is below.

	A	B	$001B	Clock cycles
LDAA #B3	B3	00	4D	2
LDAB $1B	B3	4D	4D	3
ABA	00	4D	4D	2
STAA $001B	00	4D	00	5

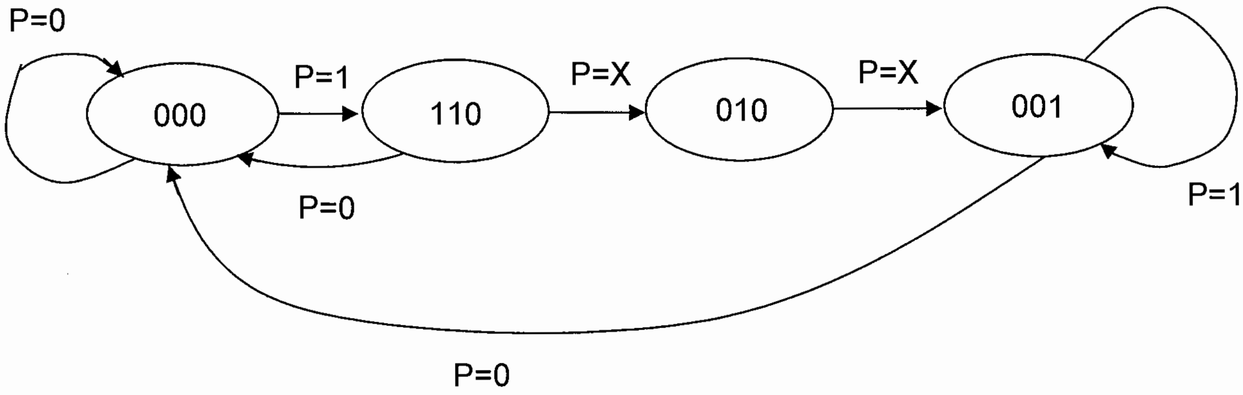
(b) $B3 and $4D are being added:

10110011
01001101

Adding these bit by bit shows that there is a half carry, a full carry and the result is 00. Thus the H, C and Z flags will be set. The MSB of the result is zero and so the N flag will not be set. In 2's complement $B3 is -77 and $4D is +77 and so the result is correct. Thus the V flag is not set.

(c) Summing the clock cycles taken for each instruction gives 12 clock cycles.

9 (a) The state diagram for the system is shown below.



(b) There are 4 states. Minimum number of bistables N given by $2^N \geq N_{states}$ and so N = 2.

(c) Denote the 4 states S0, S1, S2 and S3, represented by the outputs of the 2 J-K bistables Q_A and Q_B, and relate these to the outputs required to drive the motor(M), heater(H) and alarm(A) first:

State	Q_A	Q_B	M	H	A
S0	0	0	0	0	0
S1	0	1	1	1	0
S2	1	0	0	1	0
S3	1	1	0	0	1

By inspection $M = \overline{Q_A} \cdot Q_B$ $\quad H = \overline{Q_A} \cdot Q_B + \overline{Q_B} \cdot Q_A = Q_A \oplus Q_B$ $\quad A = Q_A \cdot Q_B$

State transition table is:

	Present state		Next state					
P	Q_A	Q_B	Q_A	Q_B	J_A	K_A	J_B	K_B
0	0	0	0	0	0	X	0	X
1	0	0	0	1	0	X	1	X
0	0	1	0	0	0	X	X	1
1	0	1	1	0	1	X	X	1
0	1	0	1	1	X	0	1	X
1	1	0	1	1	X	0	1	X
0	1	1	0	0	X	1	X	1
1	1	1	1	1	X	0	X	0

(d) First, find combinational logic expressions for J_A, K_A, J_B, K_B in terms of Q_A, Q_B and P. Karnaugh maps are below.

J_A

P \ Q_AQ_B	00	01	11	10
0	0	0	X	X
1	0	1	X	X

$$J_A = P.Q_B$$

K_A

P \ Q_AQ_B	00	01	11	10
0	X	X	1	0
1	X	X	0	0

$$K_A = \bar{P}.Q_B$$

J_B

P \ Q_AQ_B	00	01	11	10
0	0	X	X	1
1	1	X	X	1

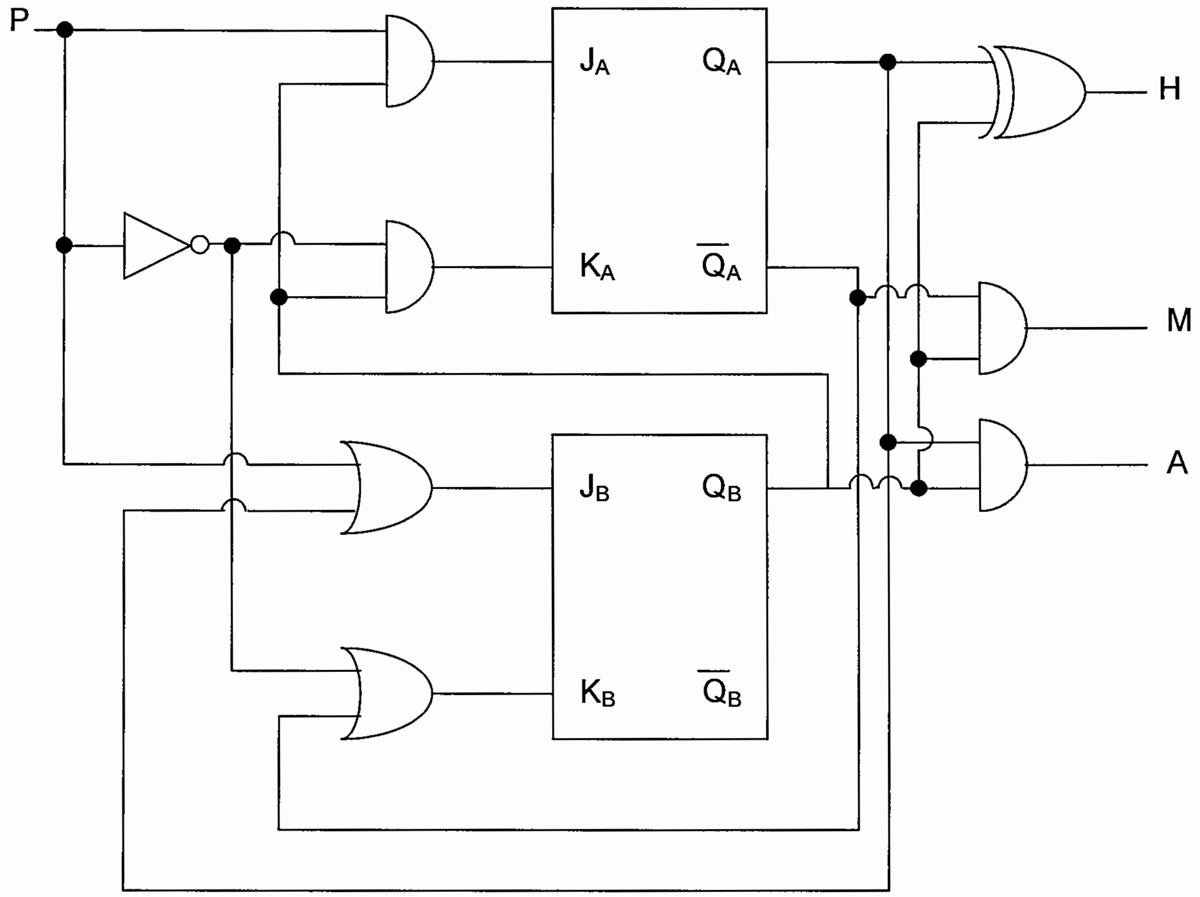
$$J_B = P + Q_A$$

K_B

P \ Q_AQ_B	00	01	11	10
0	X	1	1	X
1	X	1	0	X

$$K_B = \bar{P} + \overline{Q_A}$$

Circuit can now be drawn:



Examiner's note: There are numerous correct solutions to this question, depending on the choice of the mapping of the states to the Q_A and Q_B outputs. In fact, a more elegant solution than mine is obtained if the states follow a unit distance sequence ie 00, 01, 11, 10 as opposed to 00, 01, 10, 11.

Section C Electromagnetics – Dr Tim Flack

10 (a) Using subscript 'm' to denote quantities in the permanent magnet and 'g' in the air gap, and ignoring the soft iron since it is infinitely permeable:

Amperes Law gives: $H_m l_m + H_g l_g = 0$

Since the cross-sectional area throughout the magnetic circuit (ignoring fringing at the air gap) is constant then $B_g = B_m$

Furthermore $B_g = \mu_0 H_g$

Therefore

$$H_m l_m + \frac{B_m}{\mu_0} l_g = 0$$

Putting in numbers and rearranging:

$B_m = -1.88 \times 10^{-5} H_m$

Plotting this load line on the same axes as the B-H plot for Columax shows that the flux density is 1.02 T in the magnet, and therefore in the air gap too.

(b) Flux = B×A and A = 300 mm^2 = 3×10^{-4} m^2 giving flux = 0.306 mWb

(c) The load line plot from part (a) shows that the magnetic field intensity in the permanent magnet is 5.4×10^4 Am^{-1}

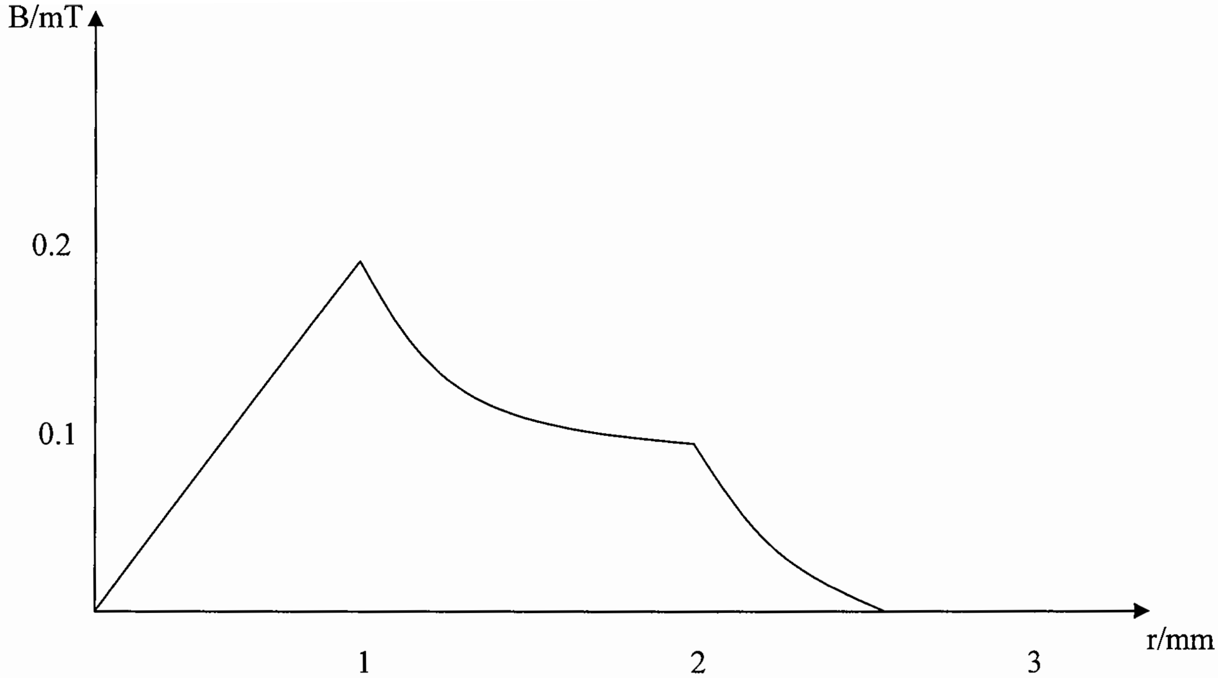
11 (a) Flux density always points in circumferential direction and at any given radius has constant magnitude. With current flowing in the positive z direction ie out of the paper flux density always points in anticlockwise direction. Thus, flux density vectors form circles. Note that there is no flux density outside the outer conductor, since net current is zero.

(b) Ampere's Law gives $H.2\pi r = I$ and so $B = \mu_0 I/2\pi r$ in which I is the net current enclosed. At a radius of 1.5 mm total current enclosed is 1A and so:

$B = 4\pi\times10^{-7}\times1/(2\pi\times1.5\times10^{-3}) = 0.133$ mT

At a radius of 3 mm no net current is enclosed and so B = 0 T.

(c) As radius increases, and assuming uniform current density in the inner conductor, current enclosed increases with r^2, and so B increases linearly with r until r = 1 mm, at which point B = 0.2 mT. B then decreases with 1/r until r = 2 mm, and then to zero at r = 2.5 mm. This gives sketch shown below.



12 (a) A conducting grounded plane can only have field lines impinging on it normally since it is an equipotential. A set of image charges (that is, equal but opposite charge distribution placed symmetrically about the conducting plane) satisfies that condition at the conducting plane boundary, and the condition that the electric field tends to zero at infinity is still satisfied at the far boundaries. Thus, in the region above the image plane the charge distribution is still the same and the required boundary conditions are satisfied which means that the same solution should be obtained.

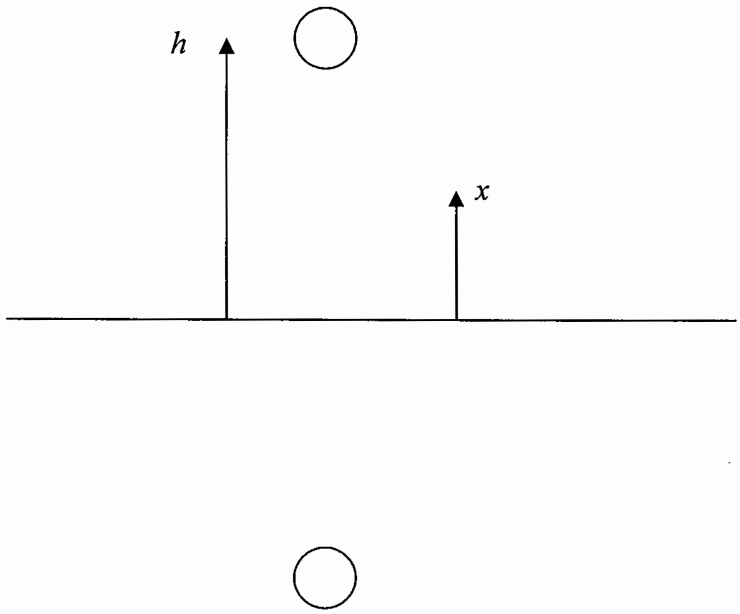
(b) Gauss' law states that the total electric flux passing out of a closed surface is equal to the charge enclosed. Consider a single charged sphere in free space. The electric flux is spherically symmetric about the centre of the sphere and so by Gauss' Law:

$$\oiint \mathbf{D}.d\mathbf{S} = Q$$

$$\mathbf{D} = \frac{Q}{4\pi r^2} \quad \text{so} \quad \mathbf{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

where r is the distance from the centre of the sphere. The image charge will produce an electric field which will superimpose with the field of the actual charge. Using the coordinate system shown below the total electric field is given by:

$$\mathbf{E}_T = \frac{Q}{4\pi\varepsilon_0(h-x)^2} + \frac{Q}{4\pi\varepsilon_0(h+x)^2}$$



The potential between the grounded plane and the sphere is found by integrating the total electric field between x = 0 and x = h-R:

$$V = \frac{Q}{4\pi\varepsilon_0}\int_0^{h-R} \frac{1}{(h-x)^2} + \frac{1}{(h+x)^2}dx = \frac{Q}{4\pi\varepsilon_0}[(h-x)^{-1} - (h+x)^{-1}]_0^{h-R}$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{(2h-R)}\right)$$

From C = Q/V:

$$C = \frac{4\pi\varepsilon_0}{\left(\frac{1}{R} - \frac{1}{(2h-R)}\right)} = \frac{2\pi\varepsilon_0 R(2h-R)}{(h-R)}$$

(c) The principle of virtual work states that the work done in causing an infinitesimally small displacement between two bodies which experience an electrostatic force is equal to the change in stored energy plus work done by any externally-connected energy source. Applying virtual work to the case where two bodies are electrically isolated means that there is no externally-connected energy source, and that the charge on the bodies must remain fixed. Thus, the work done on the system must be equated with the change in stored field energy:

$$F\delta x = \delta\left(\frac{1}{2}CV^2\right) = \delta\left(\frac{q^2}{2C}\right) = \frac{q^2}{2}\delta(C^{-1})$$

$$F = \frac{q^2}{2}\frac{\partial(C^{-1})}{\partial x} = \frac{q^2}{2}\frac{d(C^{-1})}{dC}\frac{\partial C}{\partial x} = -\frac{q^2}{2C^2}\frac{\partial C}{\partial x} = -\frac{1}{2}V^2\frac{\partial C}{\partial x}$$

(d) The force will be in the negative x direction, and applying virtual work means determining dC/dh:

$$\frac{dC}{dh} = 2\pi\varepsilon_0 R \frac{d}{dh}(2h - R)(h - R)^{-1} = 2\pi\varepsilon_0 R(2(h - R)^{-1} - (2h - R)(h - R)^{-2})$$

$$F = \frac{1}{2}V^2\frac{dC}{dh} == \frac{1}{2}V^2 2\pi\varepsilon_0 R\frac{-R}{(h - R)^2} = -\frac{V^2\pi\varepsilon_0 R^2}{(h - R)^2}$$

Putting in the numbers gives F = 0.077 N