ENGINEERING TRIPOS PART IA

Wednesday 4th June 2008 9 to 12

Paper 1

MECHANICAL ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 **(short)** The necked-down section of the pipe flow shown in Fig. 1 develops a low pressure which can be used to aspirate fluid upward from a reservoir. The liquids in the reservoir and the pipe are the same, and have density ρ.

(a) Show that the gauge pressure at section 1 is given by

$$p_1 = -\frac{1}{2}\rho V_2^2\left[(A_2/A_1)^2 - 1\right],$$

where A_1 and A_2 are the cross-sectional areas of the pipe at sections 1 and 2, and V_2 is the exit speed. [6]

(b) If $A_2/A_1 = 4$, and the height of the pipe above the surface of the reservoir is h = 10 cm, find the exit speed which is just sufficient to cause the reservoir fluid to rise up in the tube to section 1. (Take the gravitational acceleration to have a value of $g = 9.81$ m s^{-2}.) [4]

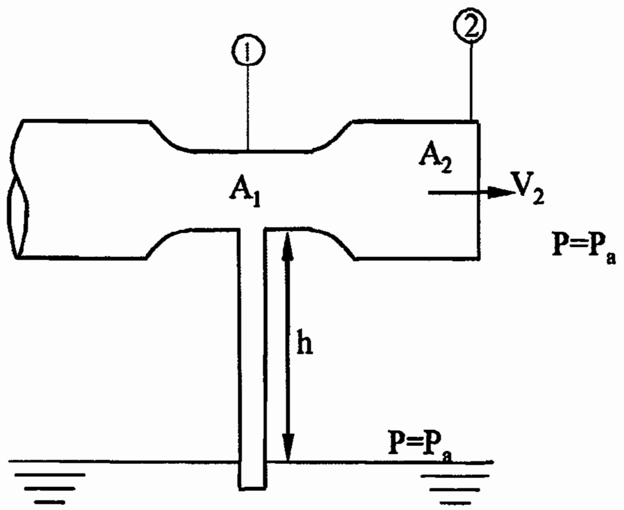

Fig. 1

2 (short) Flow in a channel undergoes an hydraulic jump from depth h_1 and speed V_1 to depth h_2 and speed V_2. The velocity may be considered uniform at sections 1 and 2 and the surface horizontal, as indicated in Fig. 2.

(a) Why is the pressure distribution hydrostatic at sections 1 and 2? [2]

(b) Why is Bernoulli's equation not valid between sections 1 and 2? [2]

(c) Show that the upstream and downstream conditions are related by

$$\frac{1}{2}\rho g h_1^2 - \frac{1}{2}\rho g h_2^2 = \rho h_1 V_1 [V_2 - V_1]$$

where g is the gravitational acceleration. Indicate clearly the control volume used in your analysis. [6]

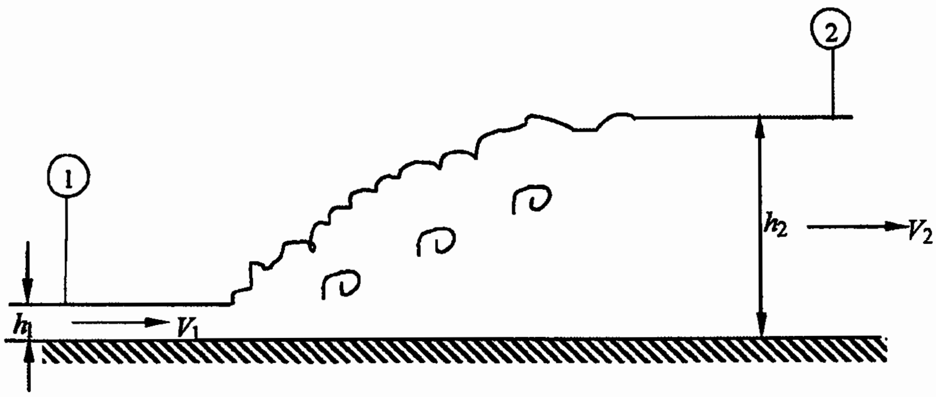

Fig. 2

3 (**short**) Methane is held in a large reservoir tank at a pressure of p_1 and a temperature of $T_1 = 300$ K. A bottle is connected to the reservoir via a delivery valve (Fig. 3). The walls of the bottle are insulated and the methane can be treated as a perfect gas with a specific heat $c_v = 1.71$ kJ kg^{-1}K^{-1} and gas constant $R = 0.520$ kJ kg^{-1}K^{-1}. Initially the valve is closed and the vessel evacuated. The valve is then opened so that methane flows into the bottle and the valve is shut when the pressure in the bottle equals that in the reservoir. We wish to determine the final temperature of the gas, T_2, in the bottle.

(a) Consider the system shown in Fig. 3, composed of the gas in the reservoir which ends up in the bottle, as well as the evacuated bottle. Calculate the displacement work performed on the system as the gas fills the bottle, and hence use the first law to show that

$$mp_1v_1 = mc_v(T_2 - T_1),$$

where m is the mass of the gas entering the bottle and v_1 is the specific volume of the gas in the reservoir. [6]

(b) Find T_2. [4]

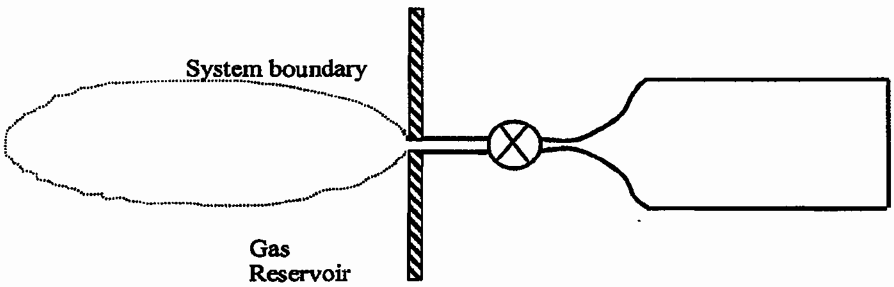

Fig. 3

4 (**short**) A perfect gas enters a turbine at a temperature of $T_1 = 1400$ K, a pressure of $p_1 = 10^6$ N m^{-2}, and a speed of $u = 4$ m s^{-1}, through an inlet area of 0.6 m^2. The exit temperature is $T_2 = 800$ K. Changes in potential and kinetic energy may be neglected, the flow may be treated as reversible, and there is negligible heat transfer to the surroundings. The gas has specific heats of $c_p = 10^3$ J kg^{-1} K^{-1} and $c_v = 713$ J kg^{-1} K^{-1}, and a gas constant of $R = 287$ J kg^{-1} K^{-1}.

(a) Calculate the inlet density and the mass flow rate through the turbine. [3]

(b) Find the power output from the turbine and the outlet pressure, p_2. [7]

5 **(long)** Water flows through an abrupt expansion in a pipe, as shown in Fig. 4. The flow is uniform upstream and downstream of the expansion (sections 1 and 2), and the pressure on the back-face of the expansion is uniform and equal to the upstream pressure, p_1 . The cross-sectional areas upstream and downstream of the expansion are A_1 and A_2.

(a) Consider the control volume whose inlet and outlet are sections 1 and 2 respectively. Use the steady-flow momentum equation to show that

$$p_2 - p_1 = \rho V_2 [V_1 - V_2],$$

where V is the speed of the fluid at any one section. [7]

(b) Derive an expression for the change in Bernoulli's constant between sections 1 and 2 in terms of ρ, V_1 and V_2. [7]

(c) If V_1 = 10 m s^{-1}, ρ = 10^3 kg m^{-3} and $A_2/A_1 = 2$, calculate the difference in Bernoulli's constant between sections 1 and 2. [5]

(d) If the flow rate is 10 kg s^{-1}, calculate the rate of dissipation of mechanical energy in Watts. [5]

(e) We wish to introduce a dimensionless measure of the pressure rise, $\Delta p = p_2 - p_1$. Suggest a suitable form for this dimensionless pressure rise coefficient and use the results of (a) to express it in terms of the area ratio A_2/A_1 . Why is there an advantage in working with a dimensionless measure of the pressure rise? [6]

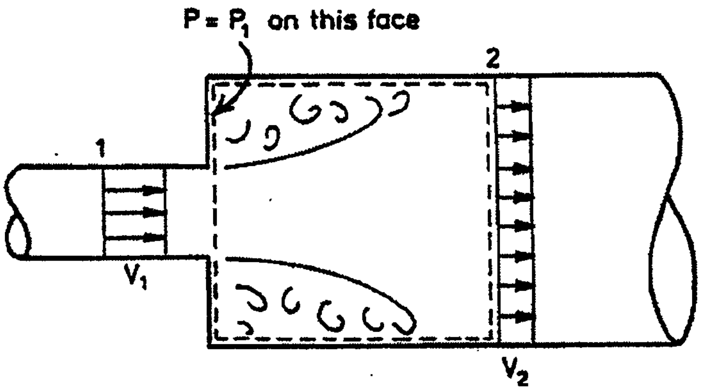

Fig. 4

(TURN OVER

6 (**long**) (a) State whether each of the following propositions is true or false, providing a brief explanation:

(i) internal energy is a property of a system;

(ii) in taking a system between two fixed states, the heat transfer between the system and the surroundings is independent of the path taken;

(iii) if a system undergoes an irreversible process, it cannot be returned to its initial state. [9]

(b) Consider a heat engine operating between two constant temperature reservoirs T_1 and T_2, where T_1 is the higher temperature. Use the Clausius inequality to show that $Q_1/T_1 \leq Q_2/T_2$ where Q_1 and Q_2 are the heat input and output respectively. Hence derive a relationship between the maximum efficiency of the engine and the temperatures T_1 and T_2.

[7]

(c) A heat engine operates between two reservoirs whose temperatures are fixed at T_1 and T_2 (Fig. 5). It is used to power a heat pump which takes heat from the lower reservoir at temperature T_2, and provides a heat flow to a third reservoir at temperature T_3 . The heat input to the engine is Q_1 and the output Q_2, while the heat input to the heat pump is Q_2^*. The exchange of work between the engine and heat pump is W. Show that the maximum amount of heat, Q_3, which can be delivered by the heat pump is determined by,

$$Q_3 = W + Q_2^* = Q_1[1 - T_2/T_1] + Q_3[T_2/T_3].$$ [8]

(d) Hence, or otherwise, find an expression for Q_3/Q_1 in terms of T_1, T_2 and T_3. Under what circumstances can Q_3 exceed Q_1? [6]

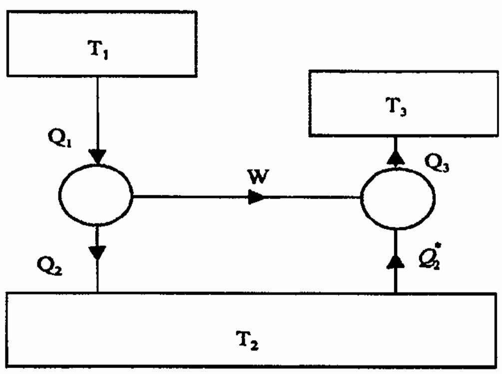

Fig. 5

SECTION B

7 **(short)** A racing car is instrumented to measure the magnitude and direction of the acceleration and velocity in a horizontal plane. At one instant the velocity and acceleration are as shown in Fig. 6, where the directions are defined with respect to the longitudinal axis of the car.

(a) Express the velocity and acceleration in intrinsic coordinates. [5]

(b) Calculate the radius of curvature of the path of the vehicle. [5]

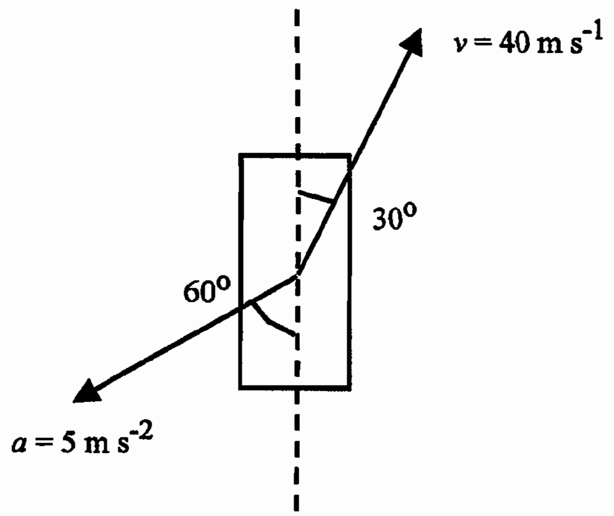

Fig. 6

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8 **(short)** A simple model of a cyclist's leg is shown in Fig. 7. The upper leg is AB, the lower leg is BC and the bicycle's crank is CD. AB and BC are each of length L and CD is of length $L/3$. At the instant shown the angle between AB and BC is 90° and the angle between CD and an imaginary line joining A and C is also 90°. CD rotates with angular velocity ω about D in the direction shown.

(a) Using the method of instantaneous centres, or otherwise, find the angular velocities of AB and BC. [6]

(b) A torque T resists motion of the crank CD. Determine the magnitude and direction of the torque that must act on AB. [4]

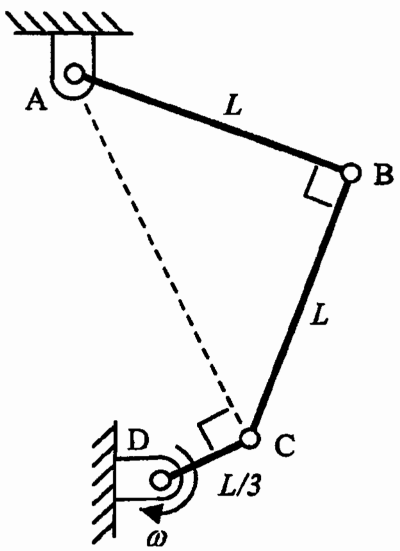

Fig. 7

9 **(short)** A satellite is in elliptical orbit around the earth. The eccentricity of the orbit is 0.6. The radius of the earth is R. The altitude at perigee of the satellite is $1.5\,R$.

(a) Find the altitude at apogee. [5]

(b) If the speed of the satellite at perigee is V, find the speed at apogee. [5]

10 **(short)** Two masses, m and $2m$, have displacements x and y as shown in Fig. 8. The masses are connected by a spring of stiffness k.

(a) Derive the equations of motion and hence calculate the natural frequencies of the system. [6]

(b) Without further calculation, sketch the mode of vibration for each natural frequency. [4]

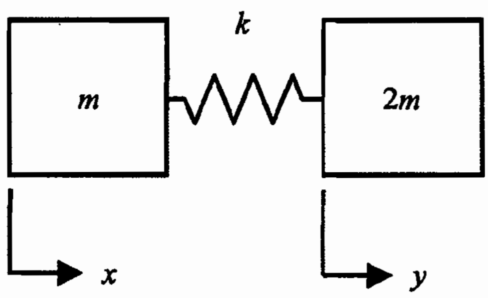

Fig. 8

(TURN OVER

11 **(long)** A chain has length L and mass m.

(a) The chain is initially at rest in a heap on the ground. At time $t = 0$ one end is lifted at constant speed v, as shown in Fig. 9a.

(i) Derive an expression for the force F required to raise the chain as a function of time and hence show that the maximum force is $mg + (m/L)v^2$. Sketch a graph of force F versus time t and indicate the salient values on your sketch. [10]

(ii) Sketch also the force acting on the ground as a function of time. [5]

(b) One end of the chain is held at height L above the ground, with the other end just touching the ground, as shown in Fig. 9b. The chain is then released so that it falls freely under the action of gravity.

(i) Show that the maximum speed achieved is $\sqrt{2gL}$. [3]

(ii) Derive an expression for the force acting on the ground as a function of time and hence show that the maximum force is $3mg$. Sketch the force as a function of time and indicate the salient values on your sketch. [12]

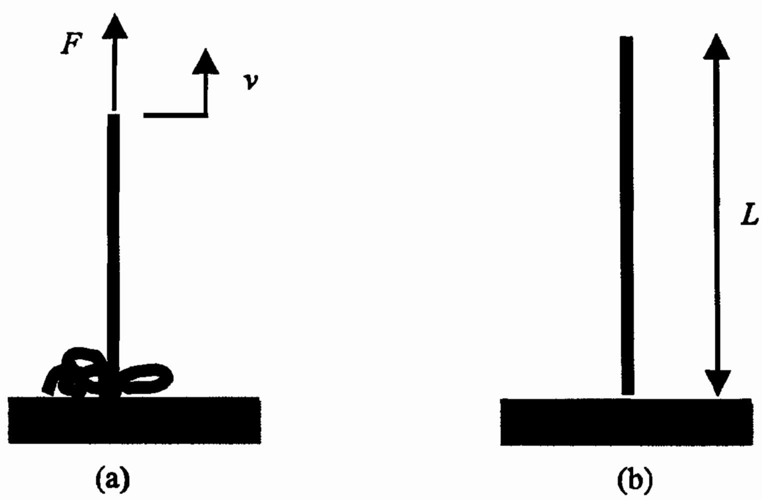

Fig. 9

12 **(long)** Figure 10 shows an engine of mass $m = 100$ kg supported on a flexible mounting with stiffness k and damping λ. Imbalance of the engine's rotating components causes a vertical sinusoidal force $f = \omega^2 b e^{i\omega t}$ to act on the engine mass, where ω is a frequency in rad s^{-1} and $b = 0.001$ kg m. Gravity can be ignored in all the calculations.

(a) If $\omega = 1000$ rad s^{-1}, what would be the amplitude of the force transmitted to the ground if the mounting were rigid? What would be the amplitude of the displacement if the mounting stiffness and damping were effectively zero? [5]

(b) Show that the equation of motion can be written in the form:

$$\frac{\ddot{y}}{\omega_n^2} + 2\zeta\frac{\dot{y}}{\omega_n} + y = -\frac{\ddot{x}}{\omega_n^2}$$

and find expressions for ω_n, ζ and x in terms of m, λ, k, b and $e^{i\omega t}$. [7]

(c) Find an expression for the amplitude of the displacement y, and sketch the amplitude as a function of frequency ω when $\zeta = 1$ and when $\zeta \ll 1$. Annotate the salient points on your sketch. [7]

(d) The mounting is specified to have stiffness $k = 250$ kN m^{-1} and damping $\lambda = 500$ N s m^{-1}. By means of a phasor diagram, or otherwise, find the amplitude of the force exerted by the mounting on the ground at a frequency of $\omega = 1000$ rad s^{-1}. [11]

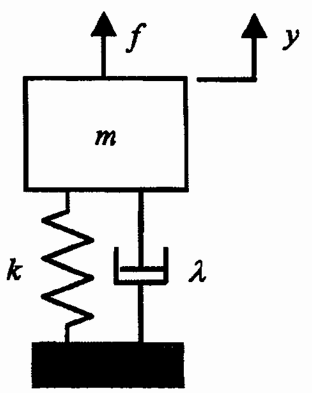

Fig. 10

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