Engineering Tripos Part IA 2009

Paper 4: Mathematical Methods

Short Answers

Section A

Q1: $x - \frac{5}{3}x^3 + O(x^4)$

Q2: $y = \frac{3}{5} + \frac{2}{5}\exp(-3x)\cos x + \frac{11}{5}\exp(-3x)\sin x$

Q3: $y_n = -2\sqrt{3} + (1+2\sqrt{3})2^n$

Q4: (a) (i) $\frac{12}{7}$; (ii) -1; (iii) $\frac{1}{3}$;

(b) (i) $z = 2\exp(in\pi)$, n odd; (ii) $z = \exp\left(\pm i\frac{\pi}{6}\right)$ or $z = \exp\left(\pm i\frac{5\pi}{6}\right)$

Q5: (a) $-39x + 28y + 30k = -59$; (b) distance = 0;

(c) $\begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$; eigenvalue = -1; eigenvector $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

Section B

Q6: (a) a = 2; b = -2; (b) $f(x,y) = \frac{2x}{y} + \frac{1}{3}\ln(3x+2) + \frac{1}{2}\ln(y^2+9) + \text{const}$

Q7: $x(t) = -\frac{1}{30}\sin 3t - \frac{1}{15}\cos 3t - \frac{1}{12}\exp(-3t) + \frac{3}{20}\exp(-t)$

Q8: (a) 11440 ways; (b) 6952 ways

Q9: (a) Step response $y(t) = \frac{1}{4} + \frac{1}{12}\exp(-4t) - \frac{1}{3}\exp(-t)$;

Impulse response $y(t) = -\frac{1}{3}\exp(-4t) + \frac{1}{3}\exp(-t)$;

(b) $y(t) = -\frac{1}{3(4-\alpha)}[\exp(-\alpha t) - \exp(-4t)] + \frac{1}{3(1-\alpha)}[\exp(-\alpha t) - \exp(-t)]$;

Limit $\alpha \to 0$ produces the step response

Q10: (a) $x^2 = \frac{4}{3} + 16\sum_{n=1}^{\infty} \frac{(-1)^n}{\pi^2 n^2} \cos\frac{\pi n x}{2}$

Section C

Q11:

1	10000101	11011001000000000000000

Fractional error = 6.45×10^{-6} %

Q12: 1920 bytes;

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y_local = building.numbers[16].joints[1].y_coord;
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