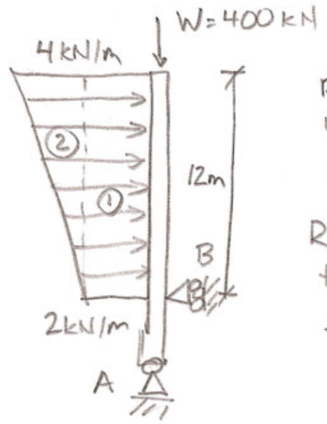
PAPER 2 - STRUCTURES 2008/09

EXTENDED CRIB

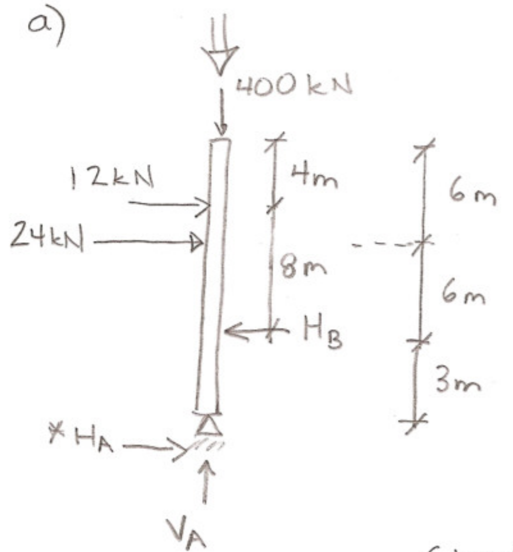
1.



Resultant force from uniform component (1)
$= 2\text{kN/m} \times 12\text{m} = 24\text{kN}$

Resultant force from triangular component (2)
$= \frac{(4-2)}{2}\text{kN/m} \times 12\text{m} = 12\text{kN}$

a)



$\Sigma M_B = 0$

$12 \times 8 + 24 \times 6 = H_A \times 3 \quad \therefore \; \underline{\underline{H_A = 80\text{kN}}}$

$\Sigma V = 0$

$V_A - 400 = 0 \quad \therefore \; \underline{\underline{V_A = 400\text{kN}}}$

$\Sigma H = 0$

$12 + 24 + H_A = H_B \quad \therefore \; \underline{\underline{H_B = 116\text{kN}}}$

Check $\Sigma M_A = 0$

$12 \times 11 + 24 \times 9 - H_B \times 3 = 0$ ✓

* A number of students did not include H_A

** A common mistake was to not use the projected length

b) Tower cross-section

Bearing pressure ** (acts perpendicularly)

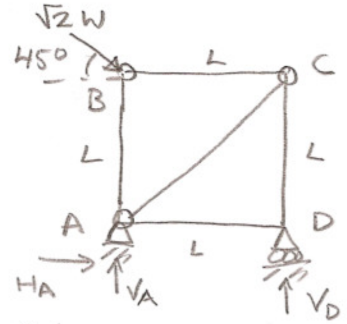
Resultant horizontal component

$H_{max} = P_{max} \cdot h \cdot l$

$$H_{max} = \underset{\text{allowable pressure}}{10\text{ MPa}} \times \underset{\text{height}}{100\text{ mm}} \times \underbrace{\frac{200\text{ mm}}{\cos 45°}}_{\text{projected length}} = 282843\text{ N} = 283\text{ kN}$$

$$\text{Factor of safety} = \frac{H_{max}}{H_B} = \frac{283\text{kN}}{116\text{ kN}} = 2.44$$

2.



Overall equilibrium

$\Sigma M_A = 0 \quad V_D \cdot L = \sqrt{2}W\cos 45^\circ \cdot L$

$\therefore V_D = W$

$\Sigma V = 0 \quad V_A + V_D - \sqrt{2}W\sin 45^\circ = 0$

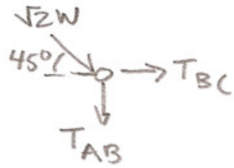
$V_A = 0$

$\Sigma H = 0 \quad H_A + \sqrt{2}W\cos 45^\circ = 0$

$H_A = -W$

a) Bar forces

Joint B



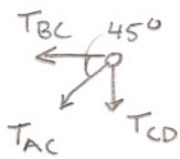
$\Sigma H = 0$

$\sqrt{2}W\cos 45^\circ + T_{BC} = 0 \quad \therefore \underline{\underline{T_{BC} = -W}}$

$\Sigma V = 0$

$-\sqrt{2}W\sin 45^\circ - T_{AB} = 0 \quad \therefore \underline{\underline{T_{AB} = -W}}$

Joint C



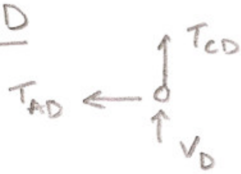
$\Sigma H = 0$

$-T_{BC} - T_{AC}\cos 45^\circ = 0 \quad \therefore \underline{\underline{T_{AC} = \frac{W}{\cos 45^\circ}}} = \sqrt{2}W$

$\Sigma V = 0$

$-T_{AC}\sin 45^\circ - T_{CD} = 0 \quad \therefore \underline{\underline{T_{CD} = -W}}$

Joint D



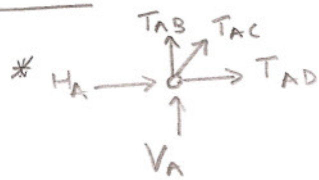
$\Sigma H = 0$

$-T_{AD} = 0 \quad \therefore \quad \underline{\underline{T_{AD} = 0}}$

Check $\Sigma V = 0$

$T_{CD} + V_D = 0$ ✓

Joint A (check)



$\Sigma H = 0$

$H_A + T_{AD} + T_{AC}\cos 45^\circ = 0$ ✓

$\Sigma V = 0$

$V_A + T_{AB} + T_{AC}\sin 45^\circ = 0$ ✓

Note for completeness all joints have been calculated in the crib but only a subset of these calculations are required to find the bar forces

* A common mistake was to forget H_A at Joint A and as a result calculate a non-zero value for T_{AD}. But this mistake should have been easily identified since by inspection at Joint D there is no horizontal reaction so T_{AD} must be zero.

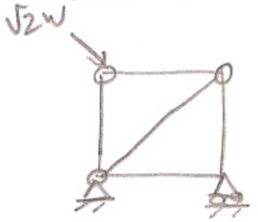
2 (continued)

b) - virtual work solution

$$\overbrace{P^* \cdot \delta = \Sigma T^*}^{\text{virtual force system}} \cdot e$$

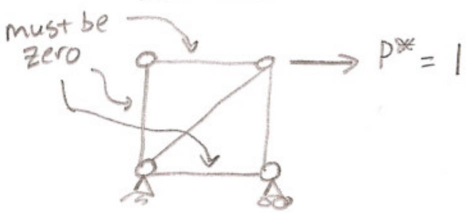
$$P^* \cdot \underbrace{\delta = \Sigma T^* \cdot e}_{\text{real displacement system}}$$

Real



extensions + displacements

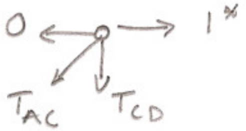
Virtual



bar forces + applied load

For virtual force system apply unit horizontal force at C

Joint C



$\Sigma H = 0$

$-T_{AC} \cos 45° + 1 = 0 \quad \therefore T_{AC} = \sqrt{2}$

$\Sigma V = 0$

$-T_{CD} - T_{AC} \sin 45° = 0 \quad \therefore T_{CD} = -1$

Bar	force	length	$e\ (WL/EA)$	T^*	$T^* e\ (WL/EA)$
AB	$-W$	L	-1	0	0
BC	$-W$	L	-1	0	0
CD	$-W$	L	-1	-1	1
AC	$\sqrt{2}W$	$\sqrt{2}L$	2	$\sqrt{2}$	$2\sqrt{2}$
AD	0	L	0	0	0

$\Sigma T^* \cdot e = 1 + 2\sqrt{2} \left(\frac{WL}{EA}\right)$

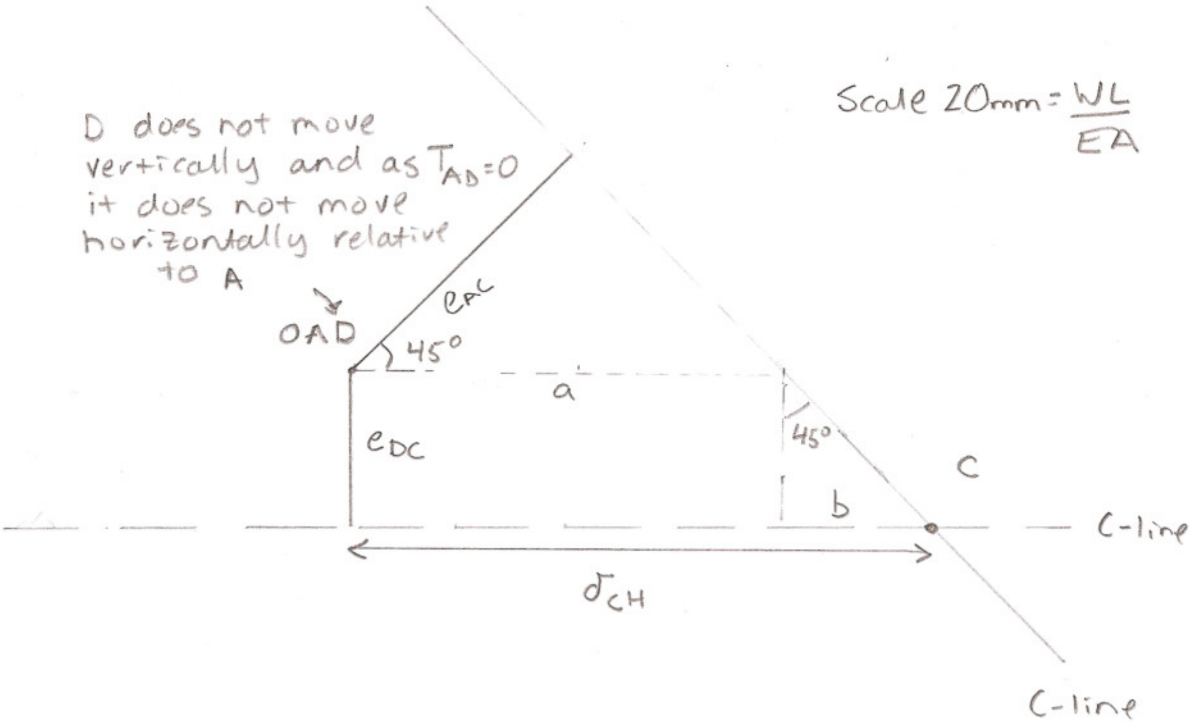
$P^* \cdot \delta_{CH} = \Sigma T^* \cdot e$

$1 \cdot \delta_{CH} = (1 + 2\sqrt{2})\left(\frac{WL}{EA}\right)$

$\delta_{CH} = (1 + 2\sqrt{2})\left(\frac{WL}{EA}\right) \rightarrow$

2b) - Solution using displacement diagram

extensions calculated on previous page



- measured

$$\delta_{CH} = \frac{77\,mm}{20\,mm}\frac{WL}{EA} = 3.85\frac{WL}{EA} \rightarrow$$

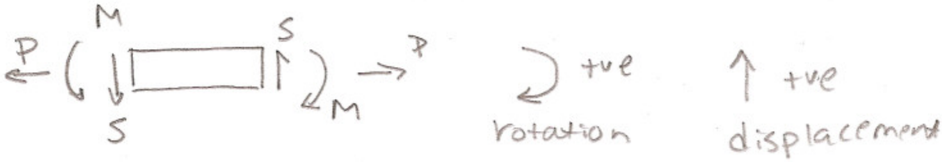
- using trig

$$\delta_{CH} = a + b = \frac{e_{AC}}{\cos 45°} + e_{DC} = \sqrt{2}\cdot\frac{2WL}{EA} + \frac{WL}{EA}$$

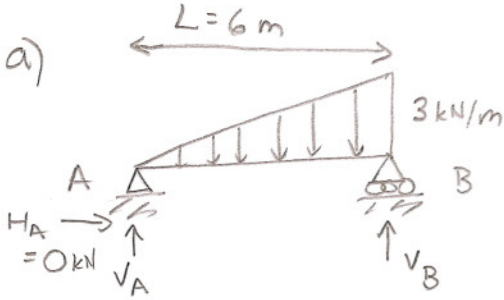
$$= (1 + 2\sqrt{2})\frac{WL}{EA}$$

3

Sign convention (use databook convention)



a)



Overall equilibrium

$\Sigma M_A = 0$

$$-V_B \times 6\text{m} + \frac{3\text{kN/m} \times 6}{2} \times \frac{2}{3} \times 6\text{m} = 0$$

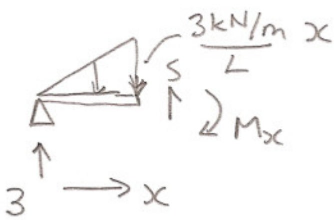
$$\therefore \underline{\underline{V_B = 6\text{ kN}}}$$

$\Sigma V = 0$

$$V_A + V_B - \frac{3\text{kN/m} \times 6}{2} = 0$$

$$\underline{\underline{V_A = 3\text{ kN}}}$$

FBD (L = 6m)



$$M_x + 3x - \underbrace{\frac{\cancel{3}x}{L} \cdot x \cdot \frac{1}{2}}_{\text{resultant force}} \cdot \underbrace{\frac{x}{\cancel{3}}}_{\text{location}} = 0$$

$$\therefore M_x = \frac{x^3}{12} - 3x$$

b) $$M = EI\left(-\frac{d^2v}{dx^2}\right)$$

$$\frac{d^2v}{dx^2} = -\frac{1}{EI}\left(\frac{x^3}{12} - 3x\right)$$

$$\frac{dv}{dx} = -\frac{1}{EI}\left(\frac{x^4}{48} - \frac{3x^2}{2}\right) + C$$

$$v = -\frac{1}{EI}\left(\frac{x^5}{240} - \frac{x^3}{2}\right) + Cx + D$$

A common mistake was to forget constants of integration

Boundary conditions $v = 0,\ x = 0 \rightarrow D = 0$

$v = 0,\ x = L$

3 b (continued)

$$0 = -\frac{1}{EI}\left(\frac{L^5}{240} - \frac{L^3}{2}\right) + C.L$$

$$\therefore\ C = \frac{1}{EI}\left(\frac{L^4}{240} - \frac{L^2}{2}\right)$$

$$V = -\frac{1}{EI}\left(\frac{x^5}{240} - \frac{x^3}{2}\right) + \frac{1}{EI}\left(\frac{L^4}{240} - \frac{L^2}{2}\right)x$$

at $x = 2$ m and where $L = 6$ m

$$V = -\frac{1}{EI}\left(\frac{2^5}{240} - \frac{2^3}{2}\right) + \frac{1}{EI}\left(\frac{6^4 . 2}{240} - \frac{6^2}{2} . 2\right)$$

$$= \frac{1}{EI}\left(\frac{-32 + 960 + 2592 - 8640}{240}\right) = \frac{-5120}{240EI}$$

$$V = \frac{64}{3EI} \downarrow$$

units of kN, m

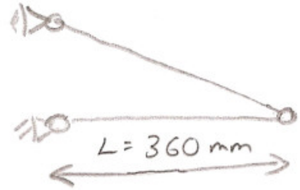
from sign convention a -ve displacement is downwards

4 a)

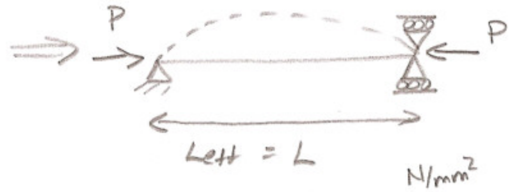
Consider the possibility of buckling about either the x-x or y-y axes

Need to check both x-x and y-y

x-x direction



side elevation



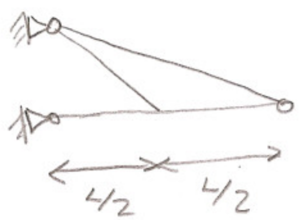
* Many students wrote down $P_{cr} = \pi^2 EI/L^2$ but then used L instead of L^2!

$$P_{cr_{x-x}} = \frac{\pi^2 E I_{xx}}{L_{eff}^2} = \frac{\pi^2 \cdot 70000 \times 6800}{(360)^2}$$

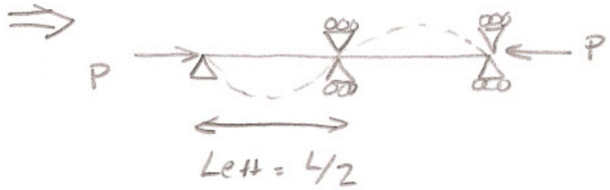
N/mm², mm⁴, mm

$$P_{cr_{x-x}} = 36249\ N = \underline{\underline{36.2\ kN}}$$

y-y direction



plan view

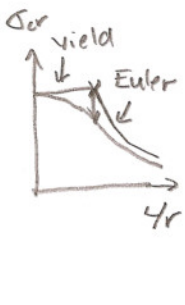


$$P_{cr_{y-y}} = \frac{\pi^2 E I_{yy}}{L_{eff}^2} = \frac{\pi^2 \cdot 70000 \cdot 1500}{(180)^2}$$

critical $\rightarrow$ $P_{cr_{y-y}} = 31985\ N = \underline{\underline{32.0\ kN}}$

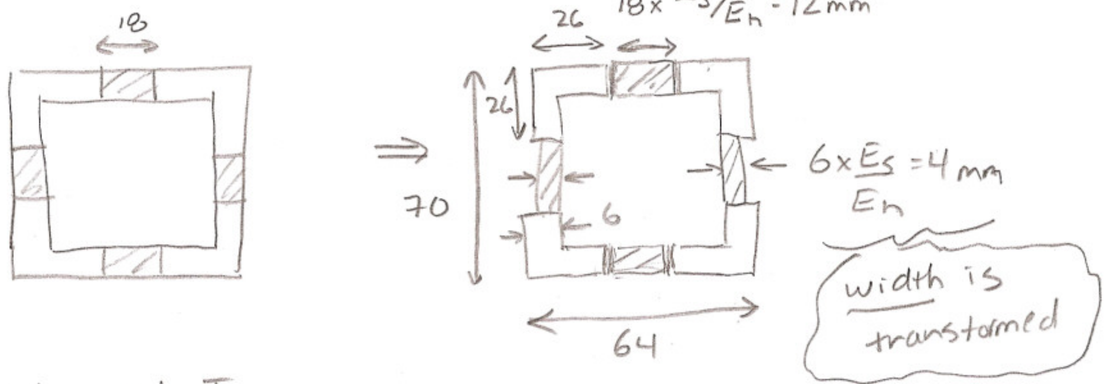
b) $P_y = A_{TOT}\,\sigma_y = 120\ mm^2 \times 260\ MPa = 31200\ N = 31.2\ kN$

c) Although P_y is slightly less than P_{cr}, P_{cr} was calculated assuming no imperfections. A particular concern is that since $P_{cr} \approx P_y$, this is a region of behaviour that will be particularly imperfection sensitive. Therefore we would expect the structure to fail due to buckling at a load less than 32 kN.



Q5 (7)
1/2

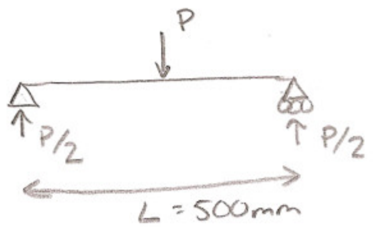
5 a) transform
softwood to hardwood



transformed I_h

$$I_h = \frac{64 \times 70^3}{12} - \frac{52 \times 58^3}{12} - \frac{2 \times 18^3}{12} \times 2 = 981904 \text{ mm}^4$$

(Note - could also transform hardwood to softwood to give $I_s = 1.473 \times 10^6 \text{ mm}^4$)



$S = -P/2 = 1$ kN

$M_{max} = \frac{P}{2} \cdot \frac{L}{2} = 250$ kN.mm

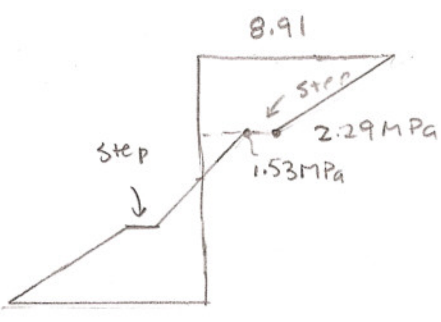
i) hardwood - maximum stress

$$\sigma_h = \frac{M y_{max}}{I_h} = \frac{250\,000 \times 35}{981904 \text{ mm}^4} = 8.91 \text{ MPa}$$

ii) softwood

$$\sigma_s = \sigma_h \times \frac{E_s}{E_h} = 8.91 \times \frac{8}{12} = 5.94 \text{ MPa}$$

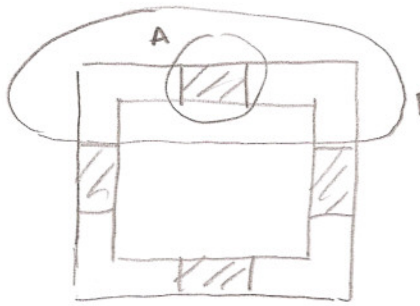
b)



$\sigma_h (y=9) = \frac{M \cdot 9}{I_h} = 2.29$ MPa

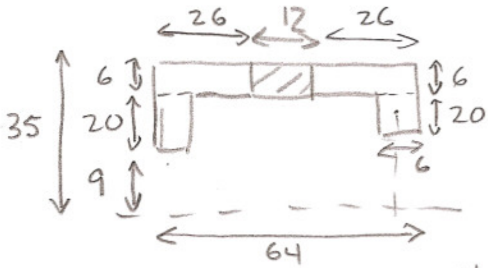
$\sigma_s (y=9) = 2.29 \times \frac{8}{12} = 1.53$ MPa

5c)



two possibilities in terms of checking the adhesive shear stress however the stress will be higher at B so will control

make cut - use hardwood transformed properties



$$A c\bar{y} = 64 \times 6 \times 32 + 2 \times 20 \times 6 \times 19$$
$$= 16848 \text{ mm}^3$$

$$q = \frac{S A c\bar{y}}{I_h} = \frac{\overset{N}{1000} \cdot \overset{mm^3}{16848}}{981904 \text{ mm}^4} = 17.16 \text{ N/mm}$$

$$\tau = \frac{q}{a} = \frac{17.16}{\underbrace{2 \times 6}_{\text{two sides}}} = 1.43 \text{ MPa}$$

length of cut

d) - longitudinal stress scales linearly with P

hardwood $\frac{\sigma_{ALLh}}{\sigma_h} = \frac{40}{8.91} = 4.49$

softwood $\frac{\sigma_{ALLS}}{\sigma_S} = \frac{16}{5.94} = 2.69$ ← controls

$\therefore$ $P_{ALLS} = 2\text{kN} \times 2.69 = 5.39 \text{ kN}$

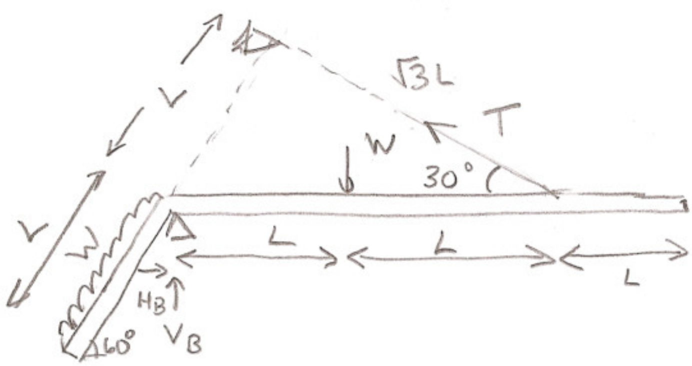
- shear stress scales linearly with P

$$\frac{\tau_{ALL}}{\tau} = \frac{8}{1.43} = 5.59$$

$\therefore$ $P_{ALL\tau} = 2\text{kN} \times 5.59 = 11.2 \text{ kN} > P_{ALLS}$

$\therefore$ Maximum applied point load = 5.39 kN and the longitudinal stress in the softwood is critical

6a)



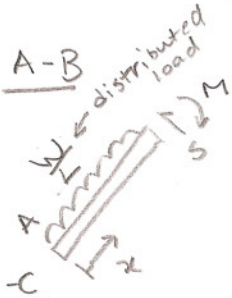
$\Sigma M_B = 0 \quad -T.(2L\cos 60°) + W.L - W.L/2 = 0 \quad \therefore \underline{\underline{T = \frac{W}{2}}}$

$\Sigma V = 0 \quad -W\sin 30° - W + T\sin 30° + V_B = 0 \quad \therefore \underline{\underline{V_B = \frac{5W}{4}}}$

(Not required)

$\Sigma H = 0 \quad H_B - T\cos 30° + W\cos 30° = 0 \quad \therefore H_B = -\frac{\sqrt{3}W}{4}$

b) A-B

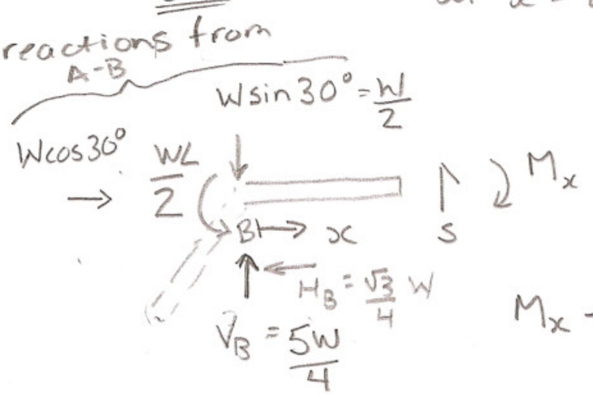


$-\frac{W}{L}x + S = 0 \qquad S = \frac{W}{L}x$

$M_x - \frac{W}{L}x.\frac{x}{2} = 0 \qquad M_x = \frac{Wx^2}{2L}$

at $x = L$ $\quad S = W, \; M = \frac{WL}{2}$

B-C



$-\frac{W}{2} + \frac{5W}{4} + S = 0 \quad \therefore S = -\frac{3W}{4}$

$M_x - \frac{WL}{2} - \frac{W}{2}x + \frac{5Wx}{4} = 0 \quad \therefore M_x = -\frac{3Wx}{4} + \frac{WL}{2}$

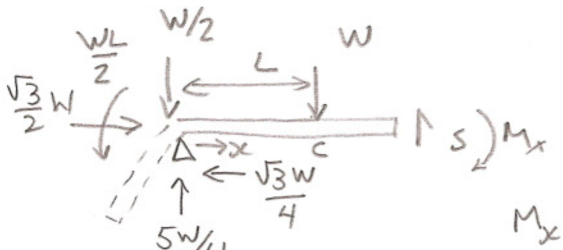
at $x = 0 \quad M = \frac{WL}{2}$

$x = L \quad M = -\frac{WL}{4}$

Note use of origin for x at B for ease of calculation

6 b) continued

C-D



$$-\frac{W}{2} + \frac{5W}{4} - W + S = 0 \quad \therefore \quad S = \frac{W}{4}$$

$$M_x - \frac{WL}{2} - \frac{W}{2}x + \frac{5W}{4}x - W(x-L) = 0$$

$$\therefore \quad M_x = \frac{Wx}{4} - \frac{WL}{2}$$

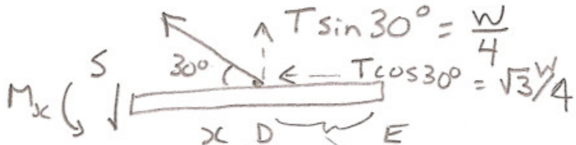
at $x = L$ $\quad M = -WL/4$ ✓

$x = 2L$ $\quad M = 0$ ✓

OR

can also make cuts from other end (simpler) e.g.

C-D



$-S + W/4 = 0 \qquad S = W/4$

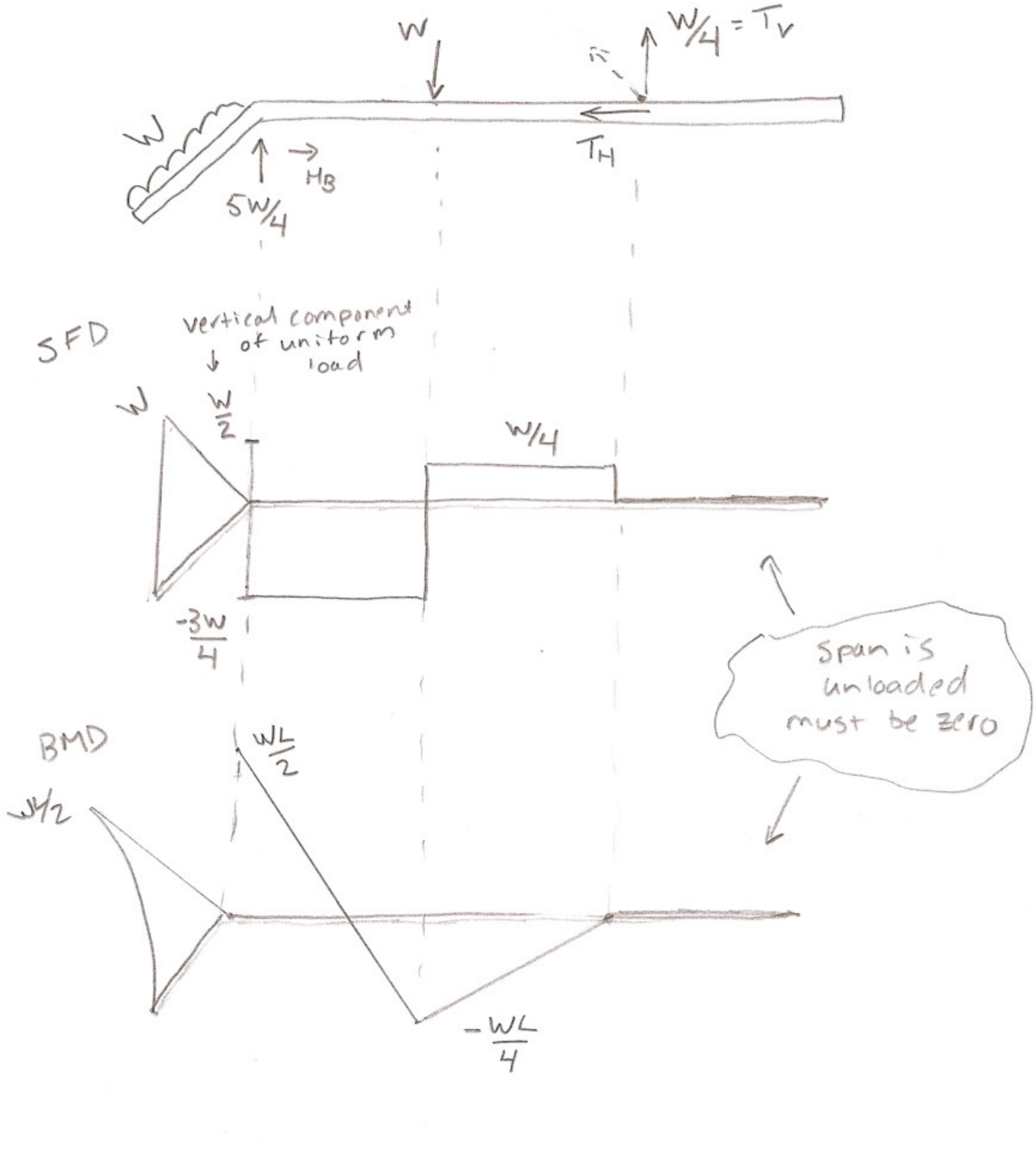
$M_x + \frac{W}{4}x = 0 \qquad M_x = -\frac{W}{4}x$

but note this is a different origin from above

at $x = 0$ $\quad M = 0$ ✓

$x = L$ $\quad M = -WL/4$ (as before)

6 b) continued



c) There are two aspects which contribute to the vertical displacement of E → the cable extension and the beam bending

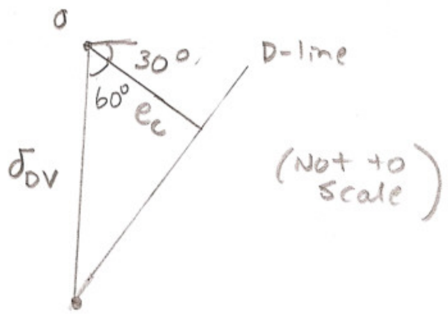
- cable

$$e_c = \frac{TL}{E_c A_c} = \frac{W\sqrt{3}\,L}{2E_c A_c}$$

6c) continued

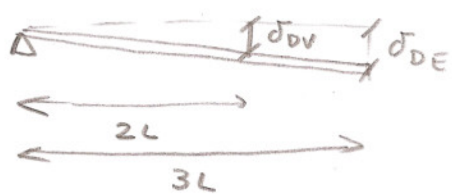
need to find vertical component of cable extension → can either use a displacement diagram or virtual work

e.g.



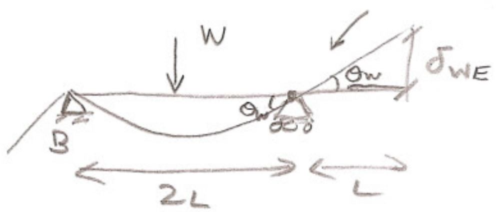
$$\delta_{DV} = e_c / \cos 60^\circ = \frac{\sqrt{3}\, WL}{E_c A_c}$$

to find displacement at E (for small angles)



$$\delta_{DE} = \frac{3}{2} \frac{\sqrt{3}\, WL}{E_c A_c} \downarrow$$

- beam - use databook cases, calculate assuming cable is inextensible



$$\theta_W = \frac{W(2L)^2}{16EI}$$

$$\delta_{WE} = \frac{WL^2}{4EI} \cdot L = \frac{WL^3}{4EI} \uparrow$$

$$\theta_M = \frac{\left(\frac{WL}{2}\right) \cdot 2L}{3EI} = \frac{WL^2}{3EI}$$

$$\delta_{ME} = \frac{\theta_M}{2} \cdot L = \frac{WL^3}{6EI} \downarrow$$

Total displacement

$$\delta_E = \frac{3}{2}\sqrt{3}\frac{WL}{E_c A_c}\downarrow + \frac{WL^3}{6EI}\downarrow - \frac{WL^3}{4EI}\uparrow$$

$$= \frac{3\sqrt{3}}{2}\frac{WL}{E_c A_c} - \frac{WL^3}{12EI} \downarrow$$

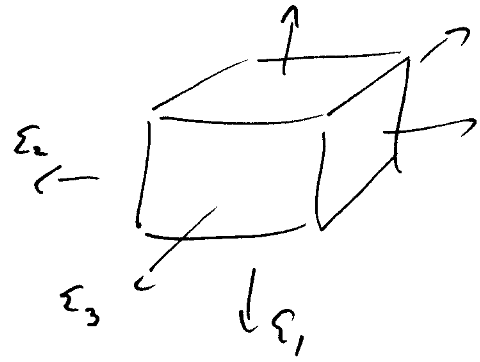
Q7/1

1A Paper 2 - Structures and Materials

Materials Crib 2009

MPF SUTCLIFFE

7 (a)



Apply uniaxial strain ε_1

By definition $\nu = -\frac{\varepsilon_2}{\varepsilon_1} = -\frac{\varepsilon_3}{\varepsilon_1}$

But by symmetry $\varepsilon_2 = \varepsilon_3$

and by conservation of volume $\frac{\varepsilon_1}{\varepsilon_1} + \frac{\varepsilon_2}{\varepsilon_1} + \frac{\varepsilon_3}{\varepsilon_1} = 0$

$\Rightarrow 1 - 2\nu = 0 \Rightarrow \nu = 0.5$ [4]

(b) $\sigma_z = -\frac{100}{40^2}$ MPa $\left(\frac{N}{mm^2} = M\right)$

$\varepsilon_y = 0$

$\sigma_x = 0$

$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} - \cancel{\nu\frac{\sigma_x}{E}}^{\,0} \Rightarrow \sigma_y = \nu\sigma_z$

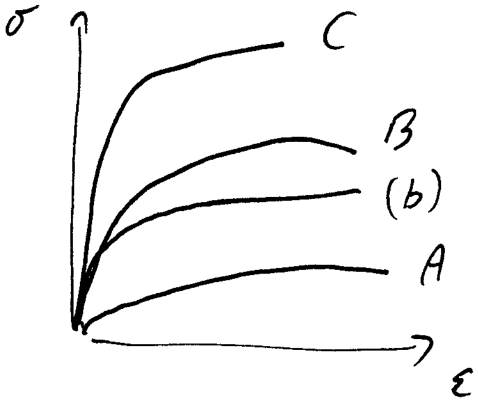
$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu\sigma_y}{E} - \cancel{\nu\frac{\sigma_x}{E}}^{\,0} = \frac{\sigma_z}{E}(1-\nu^2)$

$= \frac{100\ (0.75)\ \text{MPa}}{40^2 \cdot 3\ \text{MPa}} = 0.0156$

$\Delta t = t\,\varepsilon_z = 0.0156$ mm compression [6]

[not well answered - problems with (a) and omission of (b)]

Q8 (a)



III ≡ A - low yield/flow stress as absence of strengthening mechanisms.
Increasing σ due to increasing dislocation density associated with work hardening

II = B - As cast ⇒ high work hardening rate initially as low dislocation density.
- increased σ due to Mg solution strengthening

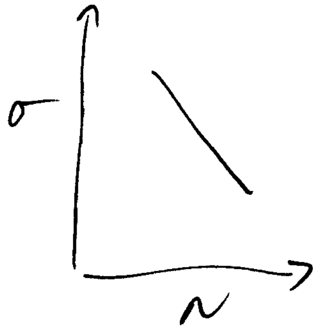
I = C - high σ_y as work hardening gives high initial dislocation density. Increasing σ_y as dislocations increase. Necking at higher σ [7]

(b) Now we truncate the first part of the curve as work hardening has been introduced before the test (see sketch above)

[3]

[well answered]

9 (a) <u>Design</u> - unnotched design -



use S-N curve to predict lifetime. Testing of coupon to find S-N curve. More realistic load case includes Miners and Goodmans rules for mean σ and load range changes.

Notched design - maybe allow some growth (eg. at welds) and calculate lifetime.

<u>Manufacture</u> - NDT to ensure flaws of acceptable size

- manufacture route to reduce flaws (eg casting may contain flaws) [5]

<u>Maintenance</u> - NDT to check flaws at regular intervals

[need to focus on fatigue in answer]

(b) $$\frac{da}{dN} = A\Delta K^n = A(\Delta\sigma\sqrt{\pi a})^n$$

$$\int dN = \int_{a_1}^{a_2} \frac{1}{A(\Delta\sigma\sqrt{\pi a})^n}\, da$$

① ≡ standard , ② ≡ reduced range

$$\frac{N_1}{N_2} = \frac{1}{2} = \frac{\cancel{\frac{1}{A}}\,\Delta\sigma_1^{-n}\,\cancel{\sqrt{\pi}^{-n}}\,\cancel{\int_{a_1}^{a_2}\sqrt{a}^{-n}\,da}}{\cancel{\frac{1}{A}}\,\Delta\sigma_2^{-n}\,\cancel{\sqrt{\pi}^{-n}}\,\cancel{\int_{a_1}^{a_2}\sqrt{a}^{-n}\,da}}$$

$$\Rightarrow \frac{\Delta\sigma_2}{\Delta\sigma_1} = \left(\frac{1}{2}\right)^{\frac{1}{n}}$$

$$\%\ \text{reduction} = 100 \times \left(1 - \frac{\Delta\sigma_2}{\Delta\sigma_1}\right) = 100\left(1 - \left(\frac{1}{2}\right)^{\frac{1}{n}}\right)$$

[5]

Q10

Identify materials used in construction:

- turbine
- dam
- housing
- infrastructure (road)
- transmission

Find energy and CO_2 associated with each item.

Include maintenance/decommissioning 'costs'.

Calculate energy production (river flow records…).

Calculate energy pay back time.

Need also to look at other environmental costs, e.g. disruption, pollution (physical, noise, pollution of raw materials), ecological cost of dam.

[environmental impact not, for example, financial viability] [5]

(b) Micro-architecture tends to confer advantages in terms of structural performance.

eg egg shell or cuttlefish

calcite crystals in egg increase toughness

wood

foam structure gives better bending performance

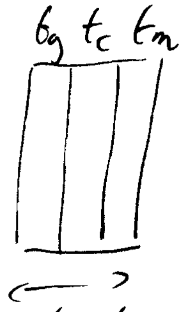
Also allows multifunctionality

e.g. wood again – sap channels along cells

- blood through channels in bone allows growth and healing

[Other examples and points valid.] [needs sketches] [5]

11 (a) Transverse



→ σ - same

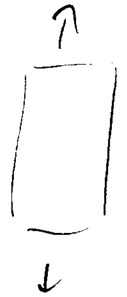
ε - averaged

$t = t_g + t_c + t_m$

$$\Delta t = \varepsilon_g t_g + \varepsilon_c t_c + \varepsilon_m t_m = \sigma\left(\frac{t_g}{E_g} + \frac{t_c}{E_c} + \frac{t_m}{E_m}\right) \text{ using } \varepsilon E = \sigma$$

$$\varepsilon = \frac{\Delta t}{t}, \quad E_T = \frac{\sigma}{\varepsilon} = \frac{t_g + t_c + t_m}{t_g/E_g + t_c/E_c + t_m/E_m}$$

Longitudinal



σ - averaged

ε - same

$$\frac{\text{Force}}{\text{unit depth}} = \sigma_g t_g + \sigma_c t_c + \sigma_m t_m = \varepsilon(E_g t_g + E_c t_c + E_m t_m)$$

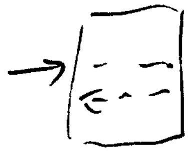
$$\sigma = \frac{\text{Force/unit depth}}{t}$$

$$E_L = \frac{\sigma}{\varepsilon} = \frac{E_g t_g + E_c t_c + E_m t_m}{t_g + t_c + t_m}$$

[10]

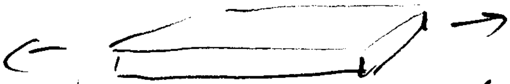
[need to derive these results, not write results down by inference from data book formulae]

11 (b) Sound waves



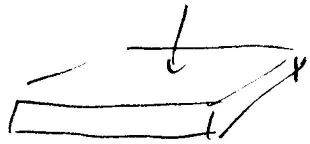
Time to travel inversely proportional to wave speed E/ρ.
Use ultrasound or impact to measure average wave speed.

Direct loading



Bending or axial. Measure force with load cell, ε with laser or strain gauge.

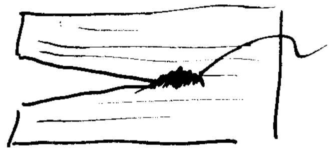
Vibration



Hit / vibrate, measure mode shape and resonant frequencies.

[generally well done, need to comment on accuracy of tensile test extension measurement - this can be difficult] [6]

(c) Splitting



- crack propagates between layers or in matrix
- low energy absorption due to low area

⇒ small K_{IC}

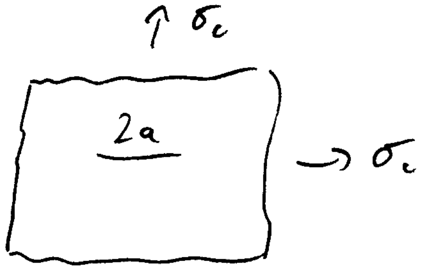
Transverse to fibres

- crack travels up and down in matrix between fibres (large area)
- need to break fibres

⇒ high energy absorption high toughness G_{IC} and hence high fracture toughness K_{IC}

[6]

11 (d)



$\sigma_c = fn\,(a, d, E_L, E_T, \sigma_f, K_{Ic})$ [this first step done poorly]

FL^{-2} $\quad L \quad L \quad FL^{-2} \quad FL^{-2} \quad FL^{-2} \quad FL^{-3/2}$

7 variables
2 dimensions } ⇒ At least 7 − 2 = 5 groups.

[many used MLT system to give at least 4 groups, and 'by inspection' only identified 4 groups]

$$\frac{\sigma_c}{\sigma_f} = fn\left(\frac{a}{d}, \frac{E_L}{E_T}, \frac{E_L}{\sigma_f}, \frac{K_{Ic}}{\sigma_f \sqrt{d}}\right)$$

[many other groups possible, these have a physical explanation]

$\frac{a}{d}$ – if the crack is of the order of the fibre size need to worry about crack location

$\frac{E_L}{E_T}$ – ratio of stiffnesses in two directions

$\frac{\sigma_c}{\sigma_f}$ – plate strength as proportion of uniaxial strength

$\frac{E_L}{\sigma_f}$ – 1/typical strain at failure

$\frac{K_{Ic}}{\sigma_f \sqrt{d}}$ – process zone size is $\frac{K_{Ic}^2}{\sigma_f^2}\frac{1}{\pi}$ so this gives ratio of process zone to fibre size [8]

[more detail given above than expected – any sensible comments scored well]

12 (a) (i) - light weight (carried around)
- non-toxic
- easily formed (key property)
- easy to remove
- strong, stiff enough to support bone, tough
- cheap enough

(ii) - cheap when sourced locally
- renewable / sustainable material
- excellent lightweight stiffness and strength (key)

(iii) - ductile and so absorbs a lot of energy (key)
- cheap - critical because of large volume
- easily manufactured (rolling) into appropriate shape
- weight not a problem

[8]

(b) (i) $\delta = \dfrac{WL^3}{8EI} = \dfrac{WL^3}{8E\beta t}$ (structures D.B 4.5.1)

$m = \rho A L = \rho \alpha t L$

Eliminate free variable t

$\Rightarrow m_\delta = \rho \alpha L \cdot \dfrac{WL^3}{8E\beta\delta} = \dfrac{\rho \alpha W L^4}{8E\beta\delta}$

Shortlist: Need to minimise mass $\Rightarrow$ maximise E/ρ

Use selection chart 3.1 to choose, ignoring ceramics as not appropriate being insufficiently tough:

CFRP, Al, bamboo, wood, steel, Mg, Ti, Ni

[eg generally well done]

[8]

12 (b)(ii) $\frac{\sigma}{y} = \frac{M}{I} \Rightarrow \sigma_m = \frac{y_n W L}{2\beta t}$ $\left(M_{max} = \frac{WL}{2} \text{ at root}\right)$

$y_{max} = y_n$

$m = \rho \alpha t L$ as before

Eliminate $t \Rightarrow$ ~~m_{σ}~~ $m_\sigma = \frac{\rho \alpha L^2 y_m W}{2\beta \sigma_m}$

$= \frac{\rho}{\sigma_m}\left(\frac{20 . 300^2 . 2.5}{2 . 40}\right) \frac{mm^4 . N}{mm^3}$

$= \frac{\rho}{\sigma_m} . 225 \ N\,mm$

$m_\delta = \frac{\rho}{E}\left(\frac{\alpha W L^4}{8\beta\delta}\right) \frac{mm . N . mm^4}{mm^3 \ mm}$

$= \frac{\rho}{E}\left(\frac{20 . 5 . (300)^4}{8 . 40 . 15}\right) N\,mm$

$= \frac{\rho}{E} \ 169 \times 10^3 \ N\,m$

	$\frac{\rho}{\sigma_m}\left(\frac{Mg/m^3}{MN\,m^{-2}}\right)$	$\frac{\rho}{E}\left(\frac{g\,m^{-3}}{N\,m^{-2}}\right)$	m_σ (g)	m_δ (g)	m_{max} (g)
Wood	0.04	4.0×10^{-5}	$\boxed{9.0}$	6.8	9.0
Steel	0.039	3.9×10^{-5}	$\boxed{8.78}$	6.6	8.78
Al	0.0135	3.86×10^{-5}	3.0	$\boxed{6.5}$	6.5

$\Rightarrow$ choose Al. [as long as consistent units are used it isn't essential to keep track of them]

[check $t = m/\rho\alpha L = 0.6$ mm $\Rightarrow$ OK] [11]

(iii) Other factors: corrosion, manufacture, joining, surface finish, aesthetics. Cost may not be an issue. [3]