ENGINEERING TRIPOS PART IA

Wednesday 3rd June 2009 9:00 to 12:00

Paper 1

MECHANICAL ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 (**short**) A pipe discharges water (density 1000 kg m^{-3}) to the atmosphere from a very large tank, which is open to the atmosphere as in Fig. 1. The cross sectional area of the duct at the exit, station 1, is 0.01 m^2, and at station 2 it is 0.02 m^2. Consider the flow to be frictionless.

(a) Neglecting the effect of the air density and velocity at station 3, find:

(i) the velocity at station 1; [4]

(ii) the gauge pressure at station 2. [4]

(b) Including the effect of the air density, which is 1.2 kg m^{-3}, what is the percentage change in the velocity at station 1? [2]

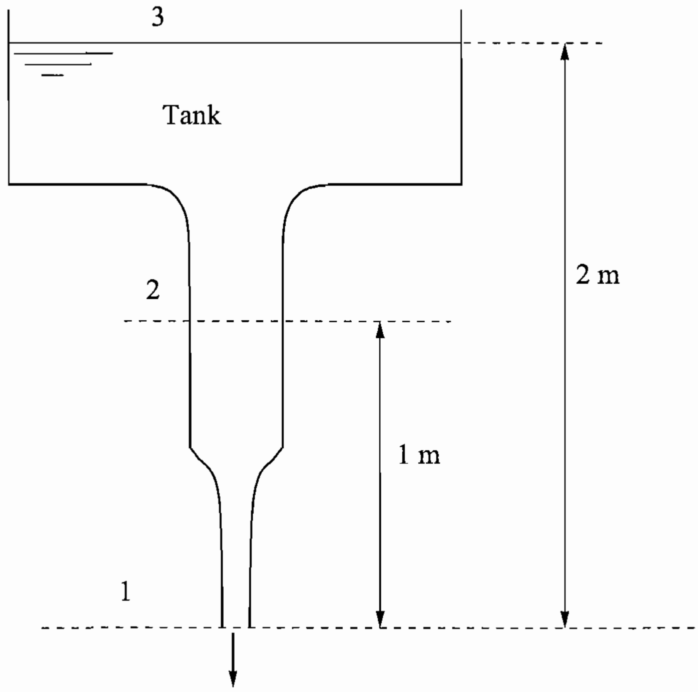

Fig. 1

2 (**short**) A water jet having a velocity $V = 10\ \mathrm{m\,s^{-1}}$ and a cross-sectional area of $0.002\ \mathrm{m^2}$ impacts on a stationary plate and it is deflected radially as shown in Fig. 2. The flow may be considered to be frictionless and the effects of gravity can be neglected. The flow may be considered to be a purely radial flow above a certain radius, r_o. The plate radius R is 0.05 m and $R > r_o$. The water density is $1000\ \mathrm{kg\,m^{-3}}$.

(a) Why is the radial velocity equal to V for radius $r \geq r_o$? [2]

(b) What is the thickness t of the water at the edge of the plate? [2]

(c) What is the force on the plate? [4]

(d) What is the force exerted on the plate when it moves at $5\ \mathrm{m\,s^{-1}}$ in the x direction? [2]

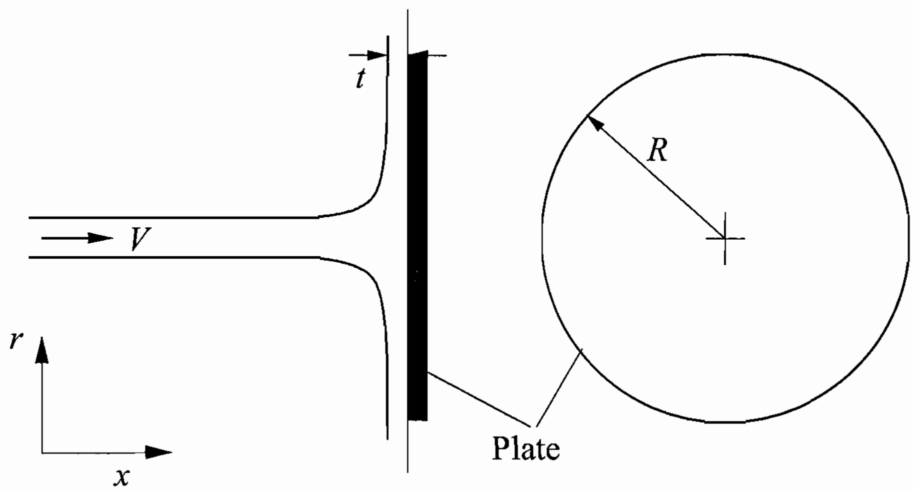

Fig. 2

3 (**short**) Consider a closed system containing 1 m^3 of air at a pressure of 14 bar absolute and a temperature of 200 °C. The specific heat at constant volume of the air may be taken as 0.74 $\mathrm{kJ\,kg^{-1}\,K^{-1}}$.

(a) What is the mass of the air in the system? [2]

(b) If 1800 kJ of heat is added to the system while its volume remains constant, what is the final pressure and temperature? [3]

(c) If 1800 kJ of heat is added to the system while its pressure remains constant, what is the final temperature and volume? [3]

(d) Is there any process possible where the entropy of the system remains constant while receiving heat? Justify your answer. [2]

4 (**short**) Air is flowing in a well–insulated duct at a rate of 1 $\mathrm{kg\,s^{-1}}$. At some section A, the pressure is 1.5 bar absolute, the temperature is 460 K and the area is 0.01 m^2. At another section, B, the pressure is 1.3 bar absolute and the velocity is 250 $\mathrm{m\,s^{-1}}$. The specific heat at constant pressure of the air may be taken as 1.02 $\mathrm{kJ\,kg^{-1}\,K^{-1}}$.

(a) What is the density of the air and its velocity at section A? [3]

(b) What is the air temperature and area of the duct at section B? [4]

(c) In which direction is the air flowing? [3]

5 **(long)** Water (density 1000 kg m^{-3}) passes down a channel of uniform width and over a broad-crested weir of height H as in Fig. 3. The flow can be considered to be frictionless.

(a) By considering a streamline along the free surface between (1) and (2), show that the velocity of the water upstream of the weir is given by

$$V_1 = (h_2 - H)\sqrt{\frac{2g(h_1 - h_2)}{h_1^2 - (h_2 - H)^2}}$$

and hence calculate the velocities at stations (1) and (2). [9]

(b) What is the velocity at station (3), if h_3 is 0.267 m? [6]

(c) How does the pressure vary with depth at station (1)? Explain your answer. [4]

(d) What is the force per unit width acting on the weir and in which direction does it act? [11]

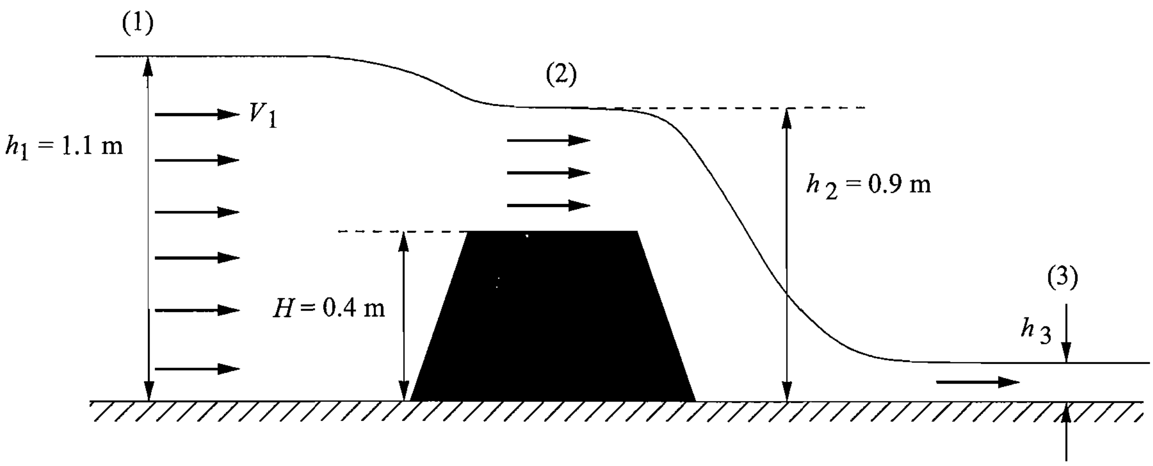

Fig. 3

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6 **(long)** (The three parts of this question are not related)

(a) It is desired to raise the temperature of water (specific heat capacity c_p) from T_0 to T_1 while it is flowing at a mass flow rate of $\dot{m}$ in a constant area duct as shown in Fig. 4:

(i) by using the SFEE, show that $\delta\dot{Q} = \dot{m}c_p\delta T$; [5]

(ii) if the environment is at T_0, show that using heat pumps the minimum power required is given by

$$\dot{W}_{\min} = \dot{m}c_p\left[(T_1 - T_0) - T_0 \ln\left(\frac{T_1}{T_0}\right)\right]$$

and determine the rate at which heat is extracted from the environment. [13]

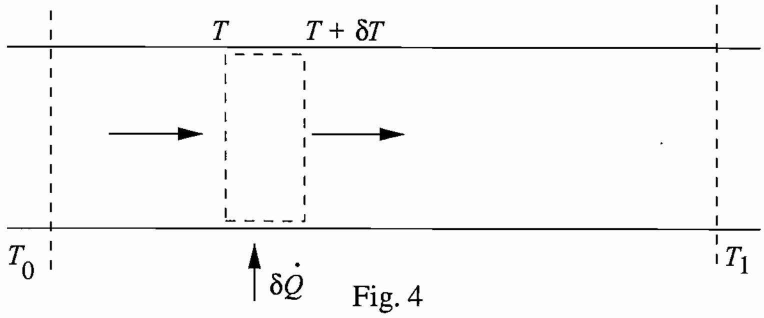

Fig. 4

(b) The Clausius Inequality states that for a system undergoing a cyclic process

$$\oint \frac{dQ}{T} \leq 0.$$

Hence or otherwise, show that for reversible processes between states 1 and 2 there exists a property whose change is given by

$$\int_1^2 \frac{dQ}{T}.$$

Why must this property have the same change between equilibrium states 1 and 2 even if the processes are irreversible? [6]

(c) One statement of the Second Law is that it is impossible to construct a device that can continuously extract heat from a single reservoir and convert it to work. Hence or otherwise prove that all reversible heat engines operating between two heat reservoirs with the same upper and lower temperatures have the same thermal efficiency and that no engine can have higher efficiency when operating between the same reservoirs. [6]

SECTION B

7 **(short)** Figure 5 shows a side elevation of a device for generating power from waves. A buoy is connected to the seabed by a rigid rod and a pinned slider at O. The buoy moves in the x-y plane under the action of the waves. At one instant the position, velocity and acceleration of the centre of the buoy A are as shown in the figure. Find $\dot{r}$, $\ddot{r}$, $\dot{\theta}$ and $\ddot{\theta}$. [10]

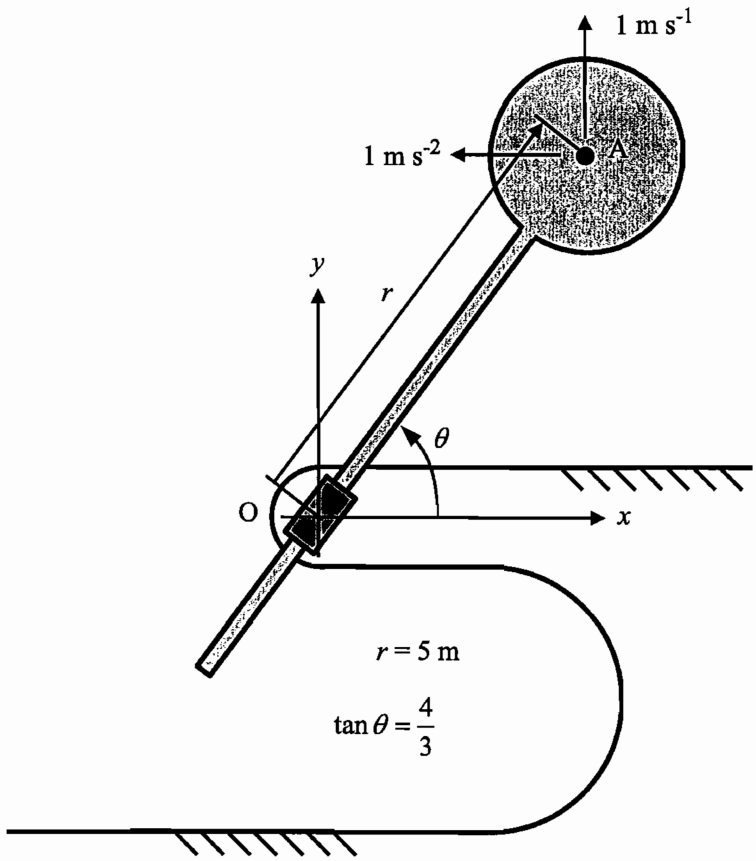

Fig. 5

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8 **(short)** A mass m travels at a velocity u on a frictionless surface as shown in Fig. 6. The mass m hits another mass M that is attached to ground by a spring of stiffness k. The two masses remain attached to each other after impact.

(a) What is the velocity immediately after the impact? [3]

(b) What is the maximum deflection of the spring? [4]

(c) Explain qualitatively how the answers to (a) and (b) would be affected if friction acted between the mass M and the ground. [3]

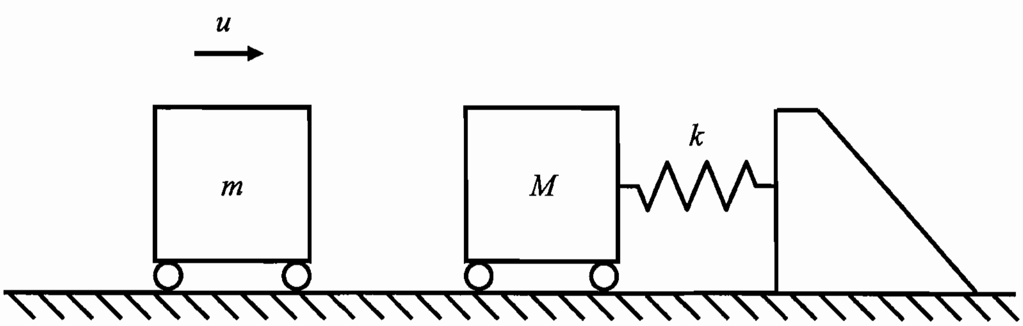

Fig. 6

9 **(short)** A uniform rectangular lamina of mass m is centred on x–y axes and has side length a along the x–axis and side length b along the y–axis.

(a) Prove that the radius of gyration of the lamina about the x–axis is $b/\sqrt{12}$. [5]

(b) The rectangular lamina now rotates about an axis that is normal to the plane of the lamina and passes through one corner of the lamina. Find the moment of inertia of the lamina about this axis. [5]

10 (**short**) Figure 7 shows an arrangement of two springs (stiffnesses $2k$ and k) and one damper (damping rate λ). One end of the arrangement is fixed to ground and a force $f(t)$ is applied to the other end. Show that the displacement $x(t)$ is related to $f(t)$ by

$$f + A\frac{df}{dt} = Bx + C\frac{dx}{dt}$$

and find expressions for A, B and C. [10]

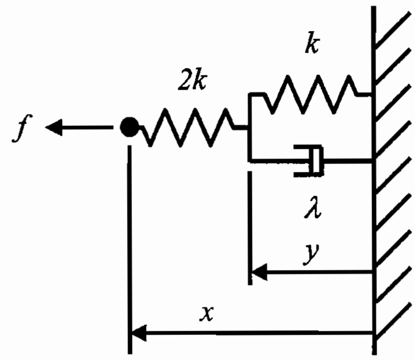

Fig. 7

11 **(long)** Figure 8 shows a side elevation of an opening window. The window ECF is supported by two linkages, one on each side of the window. Each linkage comprises rigid links ABC, BD and DE. There are pins at B, C, D and E, and a pinned slider at A. At the instant shown the window is opening and point A is moving upwards at 0.030 m s^{-1}.

(a) By using a velocity diagram or otherwise, find:

(i) the horizontal component of velocity at F;

(ii) the angular velocity of the window. [15]

(If you draw a velocity diagram, you are advised to let 1 mm represent 1 mm s^{-1}, and to place the origin of the diagram half way down the left hand side of your answer page.)

(b) The window can be considered to be a uniform lamina of weight 200 N. The other moving parts have negligible weight. If inertia can be ignored, use the principle of virtual power to determine the horizontal force required at F to open the window. [5]

(c) Use the principle of virtual power to determine the total friction force required in the sliders at A if the window is to remain open when a wind force of 100 N acts on the window. The wind force acts half way between E and F, normal to the plane of the window and in a direction that acts to close the window. [10]

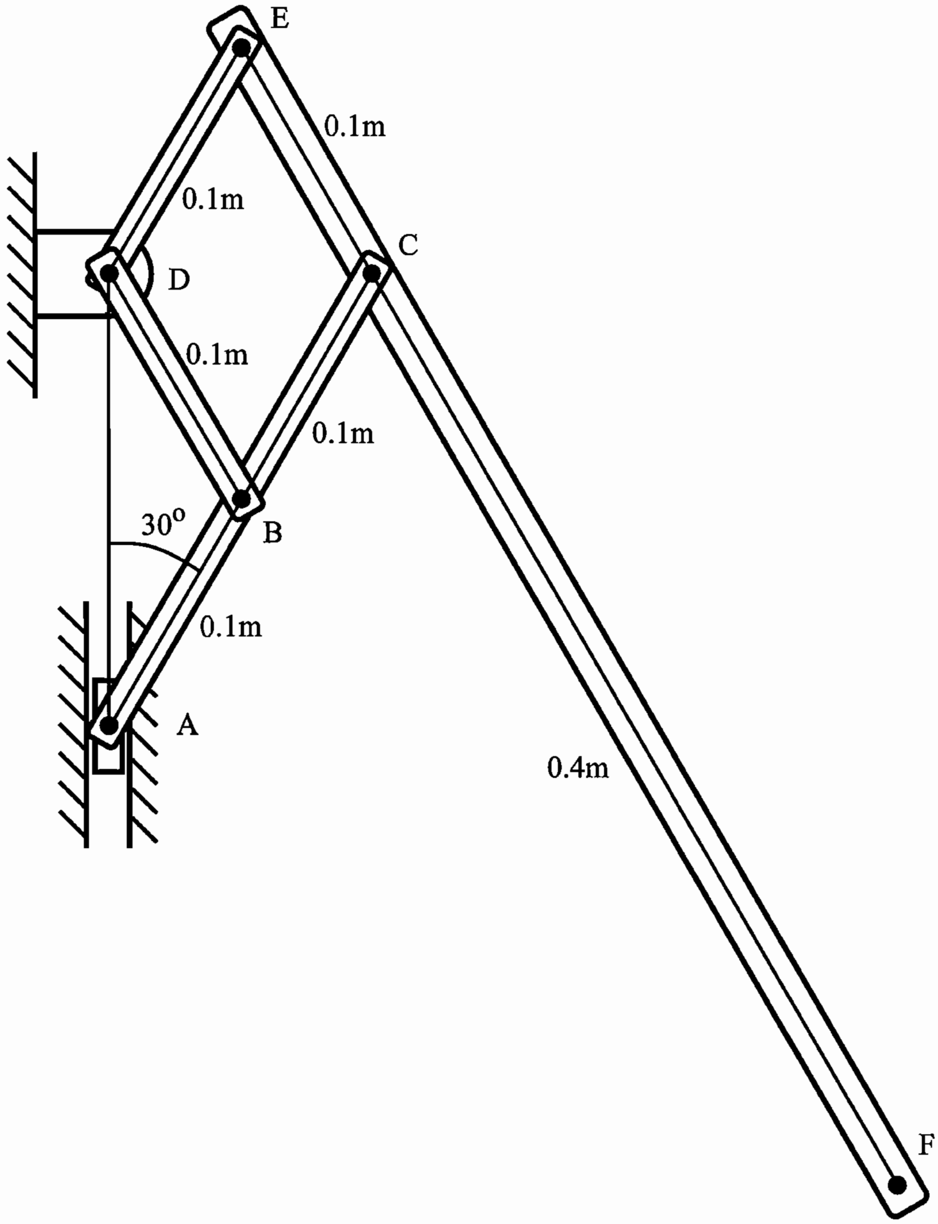

Fig. 8

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12 **(long)** Figure 9 shows two masses, m and $2m$, connected by a spring of stiffness k. The lower mass is connected to ground by a second spring of stiffness k. A force f acts on the upper mass.

(a) Derive the equation of motion in the form

$$\begin{pmatrix} 2m & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{pmatrix} + (\mathrm{K}) \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} f(t) \\ 0 \end{pmatrix}$$

and define the matrix K. [4]

(b) By considering free vibration, $f(t) = 0$, and harmonic displacements, $x_1(t) = X_1 e^{i\omega t}$ and $x_2(t) = X_2 e^{i\omega t}$, show that the natural frequencies squared are [10]

$$\frac{(5 \pm \sqrt{17})}{4} \frac{k}{m}.$$

(c) Find and sketch the mode shape (eigenvector) corresponding to the higher natural frequency. [6]

(d) If harmonic force excitation $f(t) = F e^{i\omega t}$ is applied to the upper mass, derive an expression for $\frac{X_1}{F}$. [10]

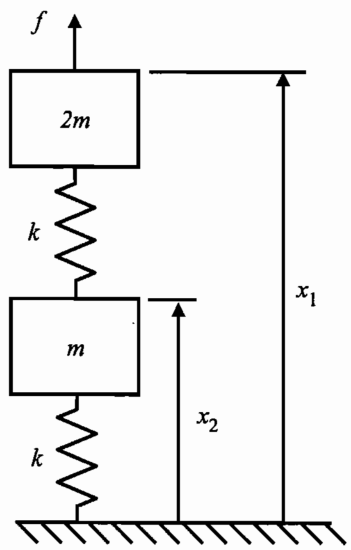

Fig. 9

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