ENGINEERING TRIPOS PART IA

Thursday 4th June 2009 9 to 12

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book CUED approved calculator allowed

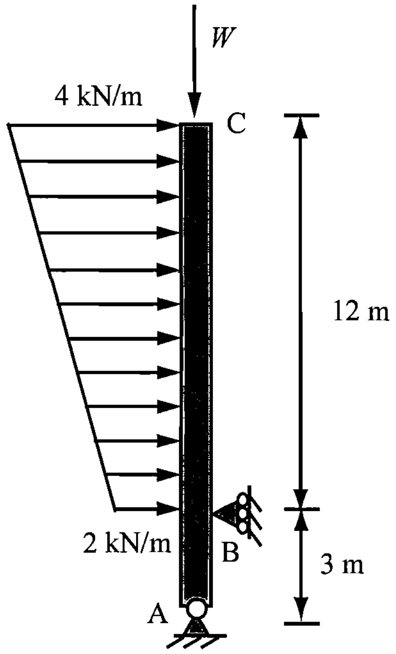
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 **(short)** A vertical tower structure is subjected to a horizontal wind load as shown in Fig. 1 where the wind load is 2 kN/m at B and increases to 4 kN/m at C. A vertical load of $W = 400$ kN is also applied to the structure. The structure is supported on a pin support at its base and on a roller support at B which is a distance 3 m from the base.

(a) Find the support reactions. [4]

(b) The tower has a circular cross-section with a radius of 0.2 m. The details of the support at B are such that the resultant horizontal reaction at B is provided by a uniform bearing pressure around a length which is equivalent to a quarter of the perimeter of the tower and a vertical height of 100 mm. If the maximum allowable bearing pressure is 10 MPa, find the maximum allowable resultant reaction at B. What is the factor of safety for this bearing design? [6]



(not to scale)
Fig. 1

2 (short) A plane pin-jointed truss is shown in Fig. 2. All members have the same cross-sectional area A and are made of a linear-elastic material with Young's modulus E. The self-weight can be neglected. The structure is loaded at joint B, as shown in the figure.

(a) Find the bar forces due to the applied loading. [4]

(b) Find the horizontal displacement at C due to the applied loading. [6]

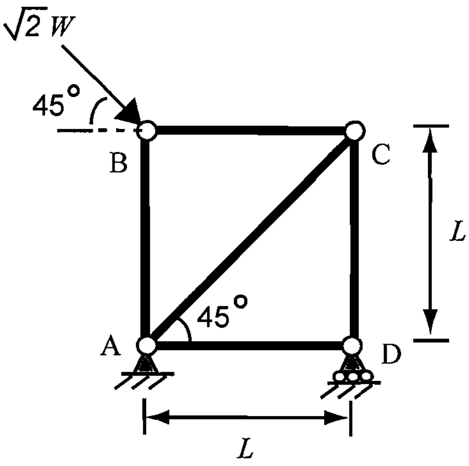


Fig. 2

3 **(short)** A simply supported beam of length 6 m and a uniform bending stiffness EI is shown in Fig. 3. The beam is loaded with a distributed load that varies from 0 kN/m at A to 3 kN/m at C. Linear elastic behaviour can be assumed. When answering the questions that follow, you should clearly state the sign convention used.

(a) Find the bending moment M as a function of x, where x is the distance from the left hand support. [4]

(b) Find the vertical displacement at B, a point which is a distance 2 m from the left hand support. [6]

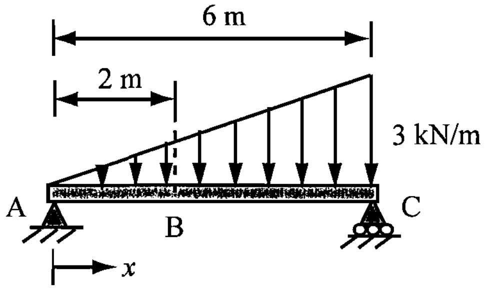


Fig. 3

4 **(short)** Fig. 4(a) shows a side elevation of a simple cantilever truss structure designed as part of the CUED Structural Design Course. The joints can be considered to be pinned. The cross-section of the main compression member consists of two equal angles as indicated in Fig. 4(b). The associated geometric properties for the orientation shown are indicated in Table 1. The member material properties have also been included in the Table.

Geometric Properties	**Material Properties**
I_{xx} = 6800 mm^4	Young's modulus, E = 70 GPa
I_{yy} = 1500 mm^4	Yield stress, σ_y=260 MPa
A_{total} (the combined area of the two angles) = 120 mm^2	

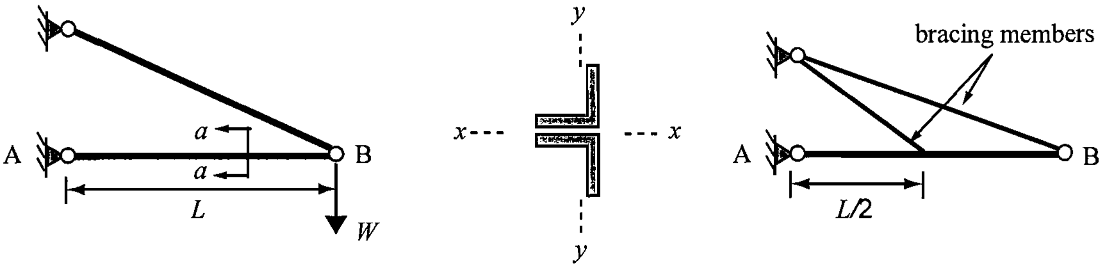
Table 1

The compression member is to carry an axial force of 30 kN and is braced against out-of-plane movement as shown in plan view in Fig. 4(c). The length of the compression member L is 360 mm. It can be assumed that the bracing members, the joints and the tension member do not fail.

(a) By considering the behaviour of the compression member AB in isolation and assuming the compression member is initially straight and has no imperfections, calculate the critical Euler buckling load. [5]

(b) Calculate the axial compressive force associated with failure due to the general yielding of member AB. [2]

(c) Based on your answers to (a) and (b), comment on the failure mode you would expect to see in a practical test and the associated force in the compression member. [3]



(a) side elevation

(b) section a-a

(c) plan view

Fig. 4

5 **(long)** A box section beam is to be made up of a number of component pieces glued together as shown in Fig. 5. The component pieces include four 26 mm × 26 mm × 6 mm thick equal angles made of hard wood and four 18 mm × 6 mm rectangular soft wood sections. The hard wood has a Young's modulus of 12 GPa and can sustain a maximum stress of 40 MPa in tension or compression. The soft wood has a Young's modulus of 8 GPa with a maximum tensile and compressive strength capacity of 16 MPa. The adhesive glue is of negligible thickness and can carry a maximum shear stress of 8 MPa. Self-weight can be neglected and all materials behave elastically. The box beam has a length of 500 mm and is simply supported with a central vertical point load of 2 kN.

(a) Find the maximum longitudinal bending tensile stress at the extreme fibre of the box section:

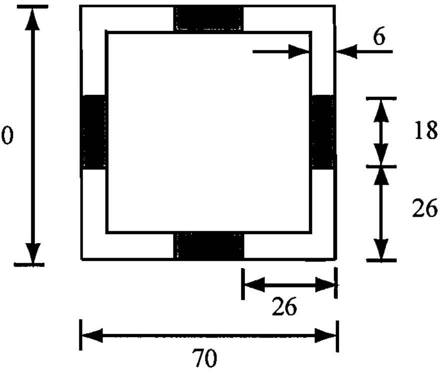
(i) in the hard wood; [6]

(ii) in the soft wood. [6]

(b) Sketch the longitudinal bending stress distribution in one of the vertical 'web' sections of the box. [4]

(c) Determine the maximum shear stress in the adhesive. [8]

(d) Based on your calculations in (a) and (c), what is the maximum applied point load that can be carried by the beam? [6]



all dimensions in mm

Fig. 5

6 (long) A linear elastic beam structure is shown in Fig. 6. The beam members have a uniform bending stiffness *EI*, negligible self-weight and can be assumed to be axially rigid. The structure is supported with a pin support at B and attached to a light flexible cable at D. The applied loading consists of a point load *W* applied at C and a total load *W* which is uniformly distributed between A and B and acts perpendicular to the beam.

(a) Find the vertical support reaction at B and the tension in the cable. [6]

(b) Sketch the shear force and bending moment diagrams for the entire beam structure (ABCDE), marking salient values. [14]

(c) If the cable is extensible with Young's modulus E_c and area A_c, find the vertical displacement at *E*. [10]

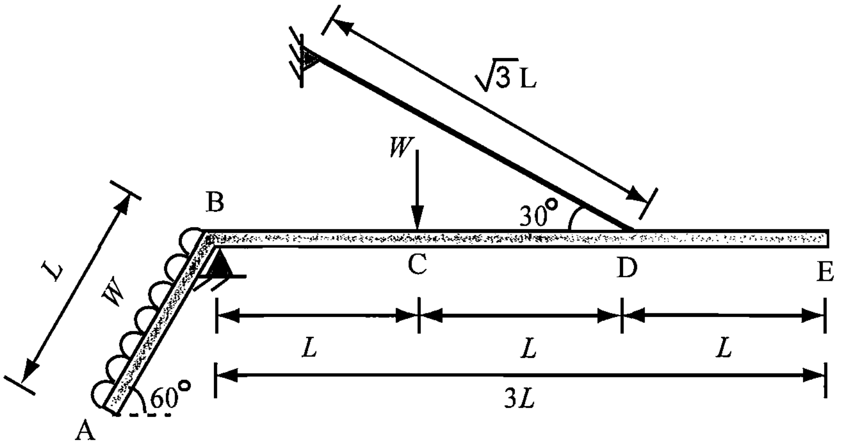


Fig. 6

SECTION B

7 (short)

(a) Soft tissue in the body is predominantly composed of water, which is effectively incompressible at the relevant stresses. Explain, with appropriate sketches and by referring to the definition of Poisson's ratio, why body tissues tend to have a Poisson's ratio close to 0.5. [4]

(b) Fig. 7 shows an idealisation of a cartilage disc in a spine. The square disc has a thickness $t = 1$ mm and a side length $\ell = 40$ mm. Surrounding material constrains the disc so that there is zero strain in the y direction, while in the x direction the disc is free to slide so that the stress σ_x in this direction is equal to zero. Calculate the reduction in thickness of the disc due to normal forces $F = 100$ N applied uniformly to the top and bottom faces of the disc. Assume that the disc behaves linear elastically, with a Young's modulus $E = 3$ MPa and a Poisson's ratio $\nu = 0.5$. [6]

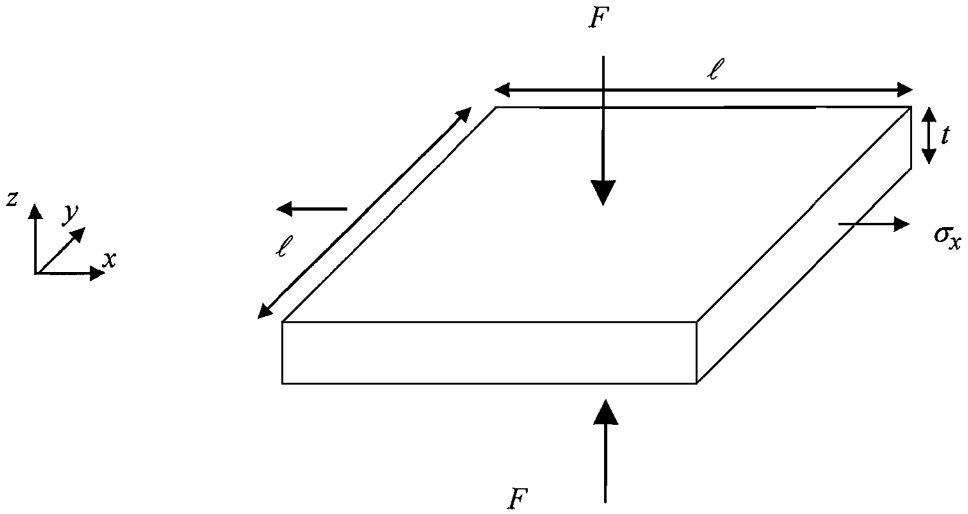


Fig. 7

8 (short)

(a) Fig. 8 shows schematic nominal stress-strain curves I, II and III taken from tensile tests for three aluminium alloys A, B and C described in Table 2. Identify which curve corresponds to which alloy, explaining your reasoning and discussing the reasons for the differences in the curves. [7]

(b) Add to a copy of Fig. 8 a sketch of the nominal stress-strain curve that you would expect after cold rolling alloy A. [3]

A	High purity aluminium, annealed
B	Al-Mg alloy, annealed
C	As alloy B, but after cold rolling

Table 2

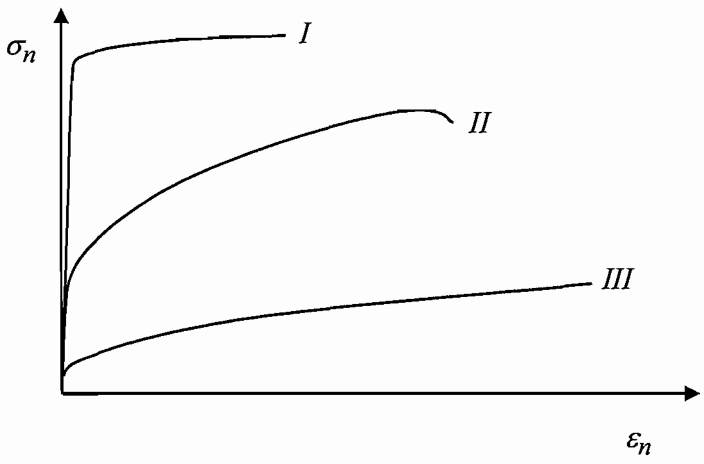


Fig. 8

9 (short)

(a) Discuss design, production and maintenance procedures that you would recommend to prevent fatigue failure of a safety-critical aluminium casing. [5]

(b) A crack in a large steel component grows in length a from 1 to 5 mm during a given number of load cycles with a design load range $\Delta\sigma$. For this range of crack lengths the stress intensity factor range $\Delta K = \Delta\sigma\sqrt{\pi a}$. Experiments show that the crack growth can be modelled using the equation

$$\frac{\mathrm{d}a}{\mathrm{d}N} = A\Delta K^n$$

where N is the number of cycles, and A and n are material constants. Find an expression for the percentage reduction in the magnitude of the load range which will double the number of cycles needed for the same crack growth from 1 to 5 mm? [5]

10 **(short)**

(a) Renewable energy promoters are frequently challenged about the environmental friendliness of energy production systems, over their lifetime. How would you assess the environmental impact of a small hydroelectric power station on the river Cam? [5]

(b) Natural materials commonly have micro-architecture at length scales of 10s or 100s of micrometres. Discuss the benefits of such micro-architecture, illustrating your answer with examples and relevant sketches. [5]

11 **(long)** Hybrid composites using glass and carbon fibres in a polymer matrix are used to give a good combination of strength, stiffness and toughness. Fig. 9 illustrates an idealisation of such a composite, in which the glass, carbon and matrix components are represented by a repeated pattern of layers of the three materials, with thicknesses t_g, t_c and t_m and Young's moduli E_g, E_c and E_m, respectively.

(a) Derive expressions for the Young's moduli E_T and E_L of the composite for loading transverse to and along the plane of the layers, in terms of the thicknesses and Young's moduli of the three layers. [10]

(b) Briefly outline three methods that could be used to measure the moduli of the composite. [6]

(c) Describe, with sketches, likely cracking failure mechanisms in the composite and explain how such mechanisms govern the fracture toughness K_{IC} of the composite. [6]

(d) A large panel of the hybrid composite is modelled as an infinite plate with a crack of length $2a$. The plate is loaded by a bi-axial stress which equals σ_c at fracture of the plate, as illustrated in Fig. 10. The material properties governing fracture of the cracked composite are the composite elastic moduli E_T and E_L transverse to and along the fibres, the uniaxial composite failure stress σ_f and the fracture toughness K_{IC}. The microstructural dimensions of the composite are characterised by the glass fibre diameter d. Use dimensional analysis to find appropriate dimensionless groups which govern the fracture stress σ_c of the cracked composite panel, assuming that only the variables listed in this paragraph are relevant. Comment on the physical significance of the groups that you find. [8]

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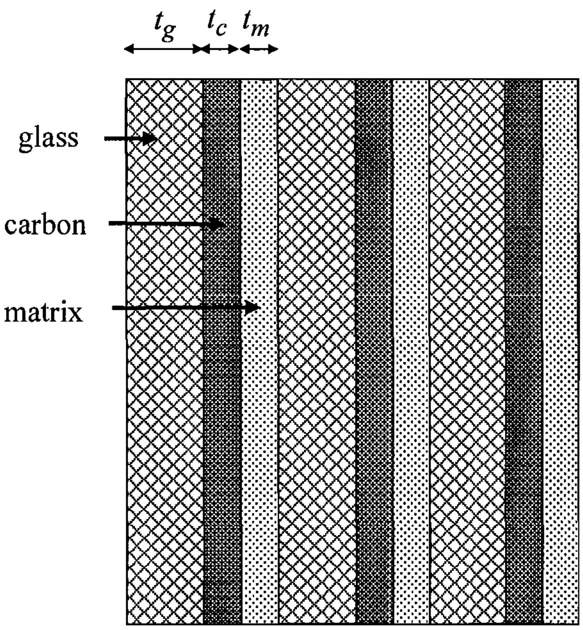
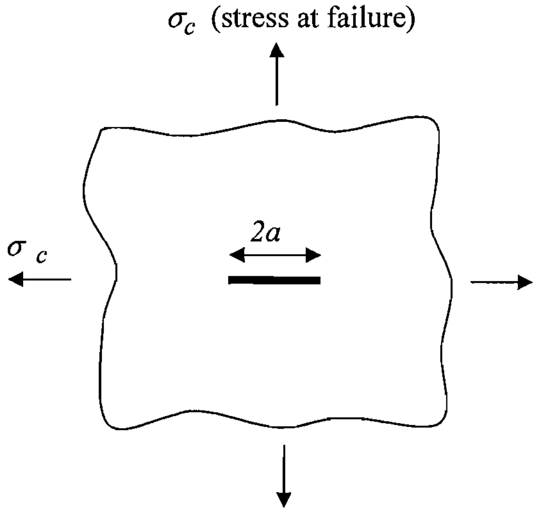


t_g
t_c
t_m
glass
carbon
matrix

Fig. 9



σ_c (stress at failure)
σ_c
$2a$

Fig. 10

12 (**long**)

(a) Discuss the factors likely to have governed the choice of material for the following applications:

(i) glass fibre composite for a cast for a broken limb;
(ii) bamboo for large wind turbine blades;
(iii) steel for motorway crash barriers. [8]

(b) Consider the selection of a material for a model helicopter rotor blade, illustrated in Fig. 11. Aerodynamic considerations have fixed the blade outer geometry, which is uniform along its length with corresponding extreme 'fibre' distance from the neutral axis y_m. The skin thickness t, which is uniform over the blade, can be varied, to give a resulting cross-sectional area αt and second moment of area βt, where α and β are constants. A pressure load W is applied uniformly along the length ℓ of the blade, which is modelled as a cantilever built-in at the hub. Self-weight loading can be neglected. The maximum tip deflection δ of the rotor is fixed.

(i) Derive the following expression for the minimum rotor mass m_δ required to meet the deflection constraint.

$$m_\delta = \frac{\rho \alpha W L^4}{8 E \beta \delta}$$

where E is Young's modulus and ρ is density. Hence use the Materials Data Book to identify a shortlist of eight appropriate materials to minimise the rotor mass while meeting the deflection constraint. [8]

(ii) In addition to the deflection constraint, a fatigue strength constraint is imposed, that the maximum stress range at the root section should not exceed the endurance limit σ_e of the material. Using the candidate material and design data given in Tables 3 and 4, choose a material which minimises the mass of the rotor, considering both design constraints. [11]

(iii) What additional factors would you take into account in making a final material selection? [3]

(cont.

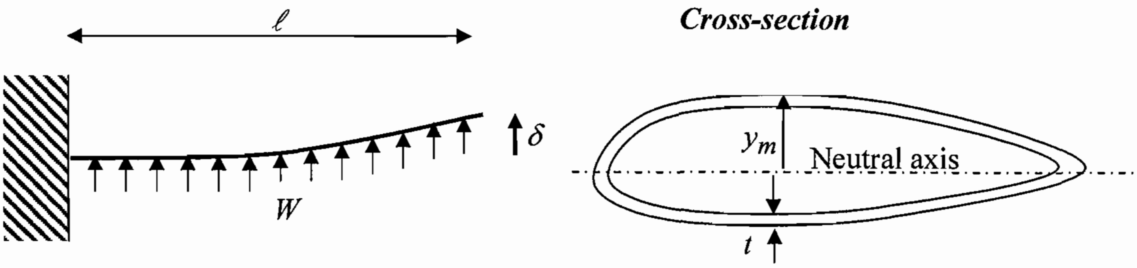


Fig. 11

	Wood	Steel	Aluminium
Young's modulus E (GPa)	20	200	70
Fatigue endurance limit σ_e (MPa)	20	200	200
Density ρ (Mg/m^3)	0.8	7.8	2.7

Table 3. Candidate material data for part (b)(ii)

α (mm^2)	20
β (mm^4)	40
y_m (mm)	2
ℓ (mm)	300
W (N)	5
δ (mm)	15

Table 4. Design parameters

END OF PAPER