

ENGINEERING TRIPOS PART IA

Tuesday 9th June 2009 9 to 12

Paper 4

MATHEMATICAL METHODS

Answer all questions.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS

Engineering Data Book

CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 **(short)** Express

$$\frac{\tan x}{(1+4x^2)^{1/2}}$$

as a power series in x , up to and including the term in x^3 . [10]

2 **(short)** Find the solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 10y = 6$$

with the initial conditions that $y(0) = \frac{dy}{dx}(0) = 1$. [10]

3 **(short)** Solve the difference equation

$$y_{n+2} = 3y_{n+1} - 2y_n$$

with initial values $y_0 = 1$ and $y_1 = 2(\sqrt{3} + 1)$. [10]

4 (long)

(a) Find

$$(i) \lim_{x \rightarrow 2} \frac{x^3 - 8}{2x^2 - x - 6} \quad [4]$$

$$(ii) \lim_{x \rightarrow 0} \frac{\ln(1+x) - x}{\sqrt{1+x^2} - 1} \quad [4]$$

$$(iii) \lim_{x \rightarrow 0} \frac{\tan x - x}{x^2 \sin x} \quad [4]$$

(b) Find the solutions of the following equations and plot them on an Argand diagram

$$(i) z^3 + 8 = 0 \quad [8]$$

$$(ii) z^4 - z^2 + 1 = 0 \quad [10]$$

5 (long)

(a) Find the equation of the plane containing the line

$$\underline{r} = \begin{bmatrix} 1 \\ -5 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 6 \\ 3 \\ 5 \end{bmatrix}$$

and the point (3,1,1). Express your answer in the form $ax + by + cz = d$ [10]

(b) Find the shortest distance between the two lines

$$\underline{r} = \begin{bmatrix} 1 \\ -5 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 6 \\ 3 \\ 5 \end{bmatrix} \quad \text{and} \quad \underline{r} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 6 \\ -3 \\ 5 \end{bmatrix} \quad [10]$$

(c) Find the 3×3 matrix \mathbf{R} which represents a rotation of 90° about the x -axis followed by a reflection in the plane $x = y$. The sense of the rotation is anti-clockwise when looking back towards the origin along the positive x -axis. This matrix has only one real eigenvalue. Calculate this eigenvalue and its corresponding eigenvector. [10]

(TURN OVER)

SECTION B

6 (short) Two functions P and Q are defined according to

$$P(x, y) = \frac{a}{y} + \frac{1}{3x+2}$$

$$Q(x, y) = \frac{y}{y^2+9} - 2xy^b$$

where a and b are constants. For a certain function $f(x, y)$ it is known that $df = Pdx + Qdy$ is a perfect differential.

- (a) Determine the values of the constants a and b . [4]
- (b) Find a form for the function $f(x, y)$. [6]

7 (short) Solve using Laplace transforms (and no other method) the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = \sin 3t$$

subject to the initial conditions $x(0) = 0$, $\dot{x}(0) = 0$. [10]

8 (short) An engineering company needs to select a project team with seven members. The available personnel consists of 12 men and 4 women. How many ways are there of picking the project team, if

- (a) the gender balance of the team does not matter; [5]
- (b) the team must contain at least two women? [5]

9 (long) A linear system is described by the differential equation

$$\frac{d^2 y}{dt^2} + 5 \frac{dy}{dt} + 4y = f(t)$$

- (a) Determine the step response and the impulse response of the system. [10]
- (b) Determine by convolution the response of the system to the input

$$f(t) = \begin{cases} 0 & t < 0 \\ e^{-\alpha t} & t \geq 0 \end{cases}$$

where α is a positive constant. What is the significance of the limit $\alpha \rightarrow 0$? [20]

- 10 (long) (a) Express the function $f(x) = x^2$ as an even function in the range $0 < x \leq 2$ and find its Fourier series. [20]
- (b) Hence, or otherwise, show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \quad [10]$$

(TURN OVER

SECTION C

11 (**short**) Convert the decimal number -1.1825×10^2 into its IEEE standard single precision floating point representation. What is the percentage error in this decimal value introduced by changing the least significant bit of the mantissa? [10]

12 (**short**) The following declarations form part of a C++ program to analyse pin-jointed frameworks:

```
struct pinjoint {
    float x_coord;
    float y_coord;
    float z_coord;
};
struct member {
    pinjoint joints[2];
};
struct framework {
    member members[80];
};
framework building;
```

Determine the number of bytes of memory required to store the variable `building`. Write a C++ statement to set a variable `y_local` equal to the value of the y-coordinate of the 2nd joint of the 17th member of the variable `building`. [10]

END OF PAPER