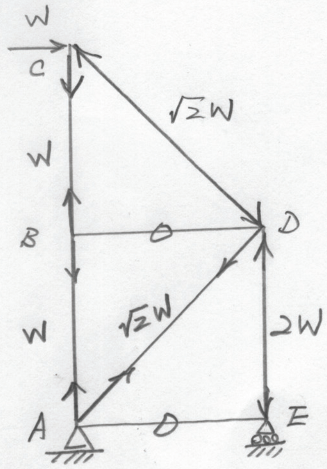
SECTION A:

1 (a)

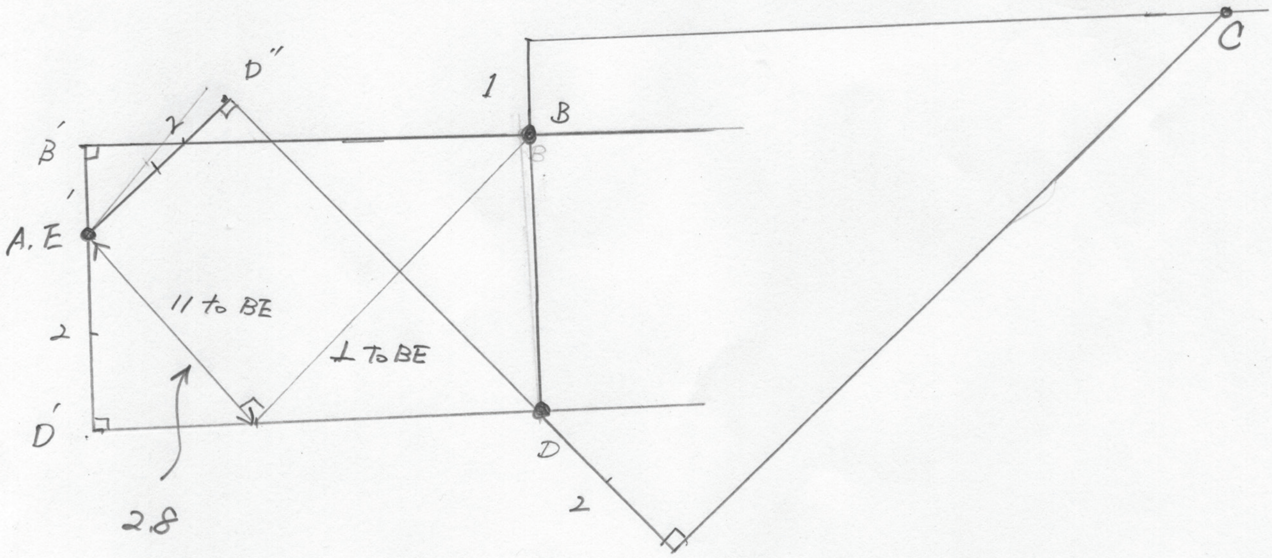


AB	W
BC	W
CD	$-\sqrt{2}W$
BD	0
AD	$\sqrt{2}W$
AE	0
DE	$-2W$

//

(b) Extension

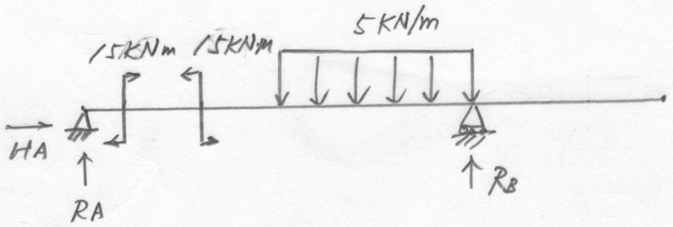
$$e = \frac{WL}{AE}\begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \\ 2 \\ 0 \\ -2 \end{bmatrix}\begin{matrix} AB \\ BC \\ CD \\ BD \\ AD \\ AE \\ DE \end{matrix}$$



$$\delta_{BE} = -\frac{2.8WL}{AE}$$

//

2.



$\Sigma H = 0 \quad HA = 0$

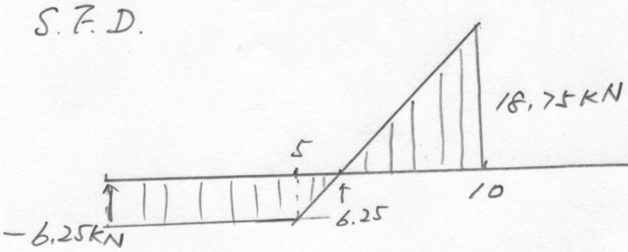
$\Sigma M_A = 0 \quad R_B \times 10 - 5 \times 5 \times 7.5 = 0$

$R_B = 18.75\ KN$

$\Sigma V = 0 \quad RA + 18.75 - 5 \times 5 = 0$

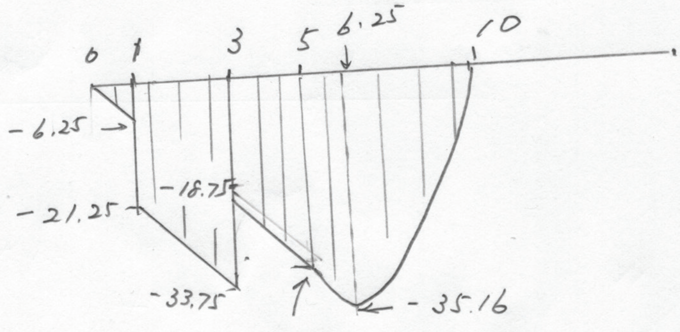
$R_A = 6.25\ KN$

S.F.D.



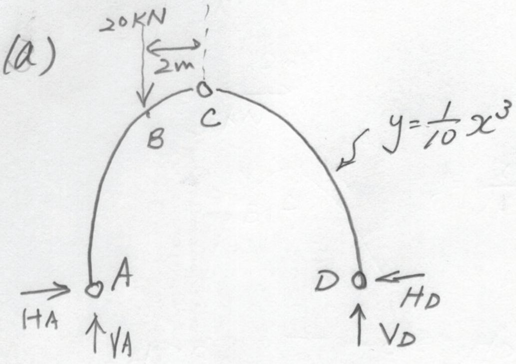
//

B.M.D.



//

3 (a)

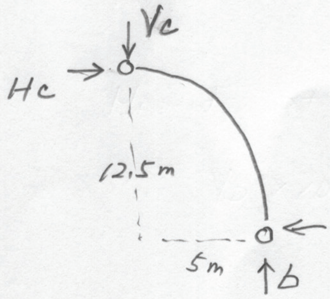


Moment at A

$V_D \times 10 - 20 \times 3 = 0$

$\underline{V_D = 6\ kN}$ //

$\Sigma V = 0 \qquad \underline{V_A = 4\ kN}$ //



Using the right hand side only

Moment at C

$6 \times 5 - H_D \times 12.5 = 0$

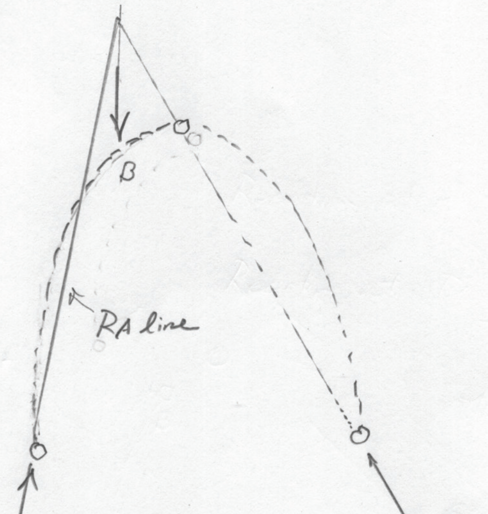
$\underline{H_D = 2.4\ kN}$ //

$\Sigma H = 0 \quad H_A = H_D \qquad \underline{H_A = 2.4\ NN}$ //

Reaction at $A = \sqrt{(14)^2 + (2.4)^2} = \underline{14.20\ kN}$ //

Reaction at $D = \sqrt{(6)^2 + (2.4)^2} = \underline{6.46\ kN}$ //

(b)



Draw the force diagram like the one shown in the left figure using the scaled figure given in the question. By inspection, the largest separation between section AB and the RA-line is at B (2. 0.8).

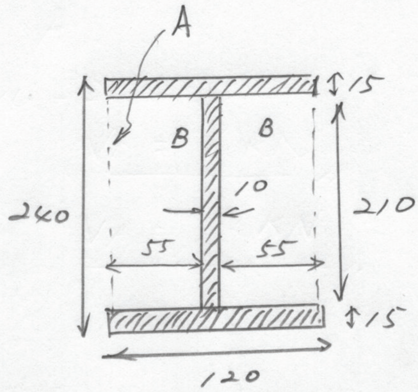
The maximum moment will be at this location.

$M_B + 2.4\,(12.5 - 0.8) - 14 \times (5-2) = 0$

$\underline{M_B = 13.92\ kN.m}$ //

Note that the bending moment will change its sign along A to B. Deriving a general equation for M(x) and solving for $dM(x) = 0$ will not give the correct answer for the location of the maximum moment. It only gives the maximum at the negative side.

4 (a)



$$I = \underbrace{\frac{bd^3}{12}}_{\text{for}A} - \underbrace{\frac{bd^3}{12}}_{\text{for}B}$$

$$= \frac{120 \times 240^3}{12} - \frac{2 \times 55 \times 210^3}{12}$$

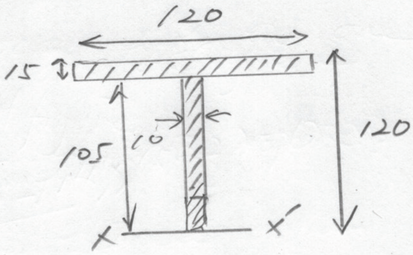
$$= 53.35 \times 10^6 \ mm^4$$

$$\sigma = \frac{My}{I} = \frac{25 \times 10^3 \times 10^3 \times 120}{53.35 \times 10^6}$$

$$= 56.24 \ N/mm^2 = \underline{56.24 \ MPa}$$

$\underline{\tau = 0 \quad \text{free surface}}$

(b) The largest shear stress will be at the centre (x-x axis)



$$\tau \cdot t = \frac{S A c \bar{y}}{I}$$

$$= \frac{10 \times 10^3 \times (120 \times 15 \times 112.5 + 105 \times 10 \times 52.5)}{53.35 \times 10^6}$$

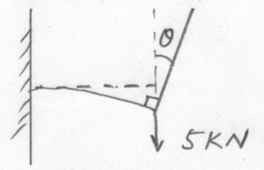
$$= 48.3 \ N/mm$$

$$\tau = \frac{48.3}{10} = 4.83 \ N/mm^2 = \underline{4.83 MPa}$$

5 (a)

(i)

$$I_{wood} = \frac{bd^3}{12} = \frac{0.5 \times (0.5)^3}{12} = 5.2 \times 10^{-3}\ m^4$$



$$EI = 10 \times 10^9 \times 5.2 \times 10^{-3} = 5.2 \times 10^7\ Nm^2$$

From Structures Databook

- vertical displacement at A

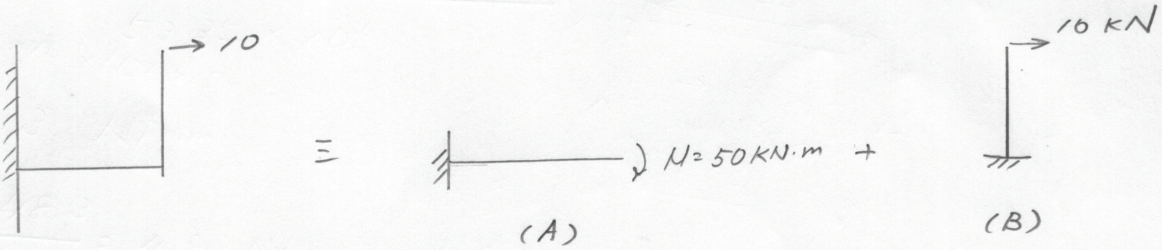
$$\delta V_{(A)} = \frac{WL^3}{3EI} = \frac{5 \times 10^3 \times 5^3}{3 \times 5.2 \times 10^7} = \underline{4 \times 10^{-3}\ (m)}$$ //

- horizontal displacement at B

$$\theta_A\ (\text{rotation}) = \frac{WL^2}{2EI} = \frac{5 \times 10^3 \times 5^2}{2 \times 5.2 \times 10^7} = 1.2 \times 10^{-3}$$

$$\delta H_{(B)} = \theta_a \times 5 = 1.2 \times 10^{-3} \times 5 = \underline{6 \times 10^{-3}\ (m)}$$ //

(ii) Axial deformation is small



- Vertical displacement at A

$$\delta V_{(A)} = \frac{ML^2}{2EI} = \frac{50 \times 10^3 \times 5^2}{2 \times 5.2 \times 10^7} = \underline{1.2 \times 10^{-2}\ (m)}$$ //

- Horizontal displacement at B
  - Due to mechanism (A)

    $$\text{rotation} = \frac{ML}{EI} = \frac{50 \times 10^3 \times 5}{5.2 \times 10^7} = 4.8 \times 10^{-3}$$

    $$\text{Displacement} = 4.8 \times 10^{-3} \times 5 = 2.4 \times 10^{-2}\ (m)$$

  - Due to mechanism (B)

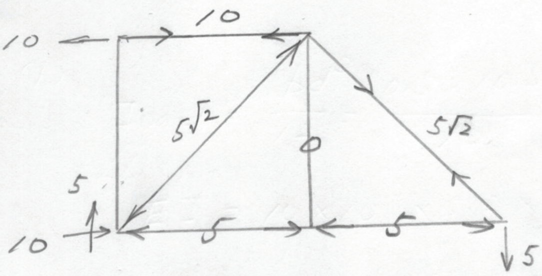
    $$\frac{WL^3}{3EI} = \frac{10 \times 10^3 \times 5^3}{3 \times 5.2 \times 10^7} = 8 \times 10^{-3}\ (m)$$

Total (A) + (B)

$$\delta H_B = 2.4 \times 10^{-2} + 8 \times 10^{-3} = \underline{3.2 \times 10^{-2}\ (m)}$$ //

(b) Vertical displacement

Use virtual work.



$$EA = 210 \times 10^9 \times 10^{-4}$$
$$= 2.1 \times 10^7 \ (N)$$

	Force	Length (m)	Extension $(1/EA)e$	Load 1 at E T^*	$T^* e \left(\frac{1}{EA}\right)$
AC	$-5\sqrt{2}$	$5\sqrt{2}$	-50	$-\sqrt{2}$	$50\sqrt{2}$
AD	-5	5	-25	-1	25
BC	10	5	50	2	100
CD	0	5	0	0	0
CE	$5\sqrt{2}$	$5\sqrt{2}$	50	$\sqrt{2}$	$50\sqrt{2}$
DE	-5	5	-25	-1	25
					$\Sigma = 150 + 100\sqrt{2}$

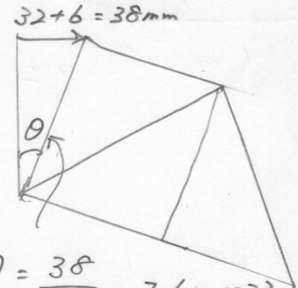
Virtual work

$$1 \times \delta V(E) = (150 + 100\sqrt{2}) / EA$$
$$= \frac{291 \times 10^3}{2.1 \times 10^7} = 1.39 \times 10^{-2} \ (m)$$

Total vertical displacement

$$= 1.39 \times 10^{-2} + 4 \times 10^{-3} + 1.2 \times 10^{-2} + 10 \times 7.6 \times 10^{-3}$$

(from above) (from 5KN at A vertical question (a(i))) (from 10KN at B horizontal question (a(ii))) (rigid body rotation)



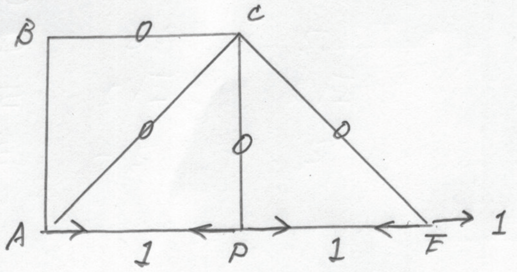
$$\theta = \frac{38}{5000} = 7.6 \times 10^{-3}$$

$$= \underline{10.59 \times 10^{-2} \ (m)}$$ //

Note: the vertical displacement of the truss can also be evaluated using the displacement diagram drawn in Q. 1.

Horizontal displacement.

	Extension e ($1/EA$)	Horizontal Load I at E T^*	T^*e
AC	−50	0	0
AD	−25	1	−25
BC	50	0	0
CD	0	0	0
CE	50	0	0
DE	−25	1	−25
			−50



Virtual work

$$1 \times \delta H_{(E)} = \frac{-50 \times 10^3}{2.1 \times 10^7}$$

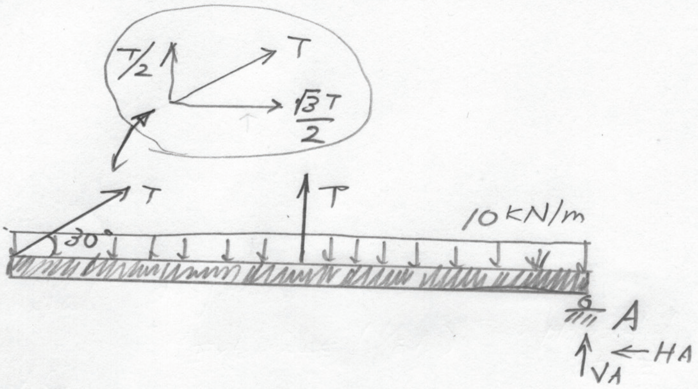
$$= -2.38 \times 10^{-3} \, (m)$$

(Again the displacement diagram in Q1 can be used)

Horizontal displacement

$$= \underline{-2.38 \times 10^{-3} \, (m)} \quad //$$

(the contribution from the beam is neglegible) b.



(a) Moment around A

$$\frac{T}{2} \times 10 + T \times 5 = 10 \times 10 \times 5$$

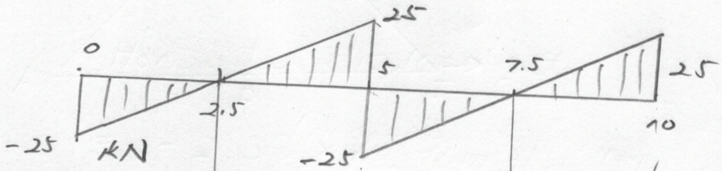
$$T = 50\,kN$$

(b) $\Sigma V = 0$ $\quad \frac{50}{2} + 50 - 10 \times 10 + V_A = 0$

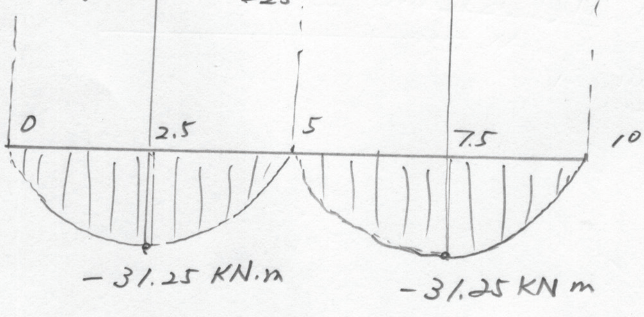
$$V_A = 25\,kN$$

$\Sigma H = 0$ $\quad H_A = \frac{\sqrt{3} \times 50}{2} = 25\sqrt{3}\,kN$

S.F.D.

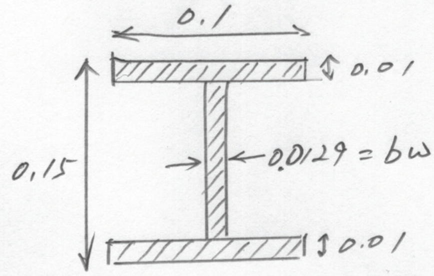


B.M.D.



(C) Shrink wood to Al-Alloy

$E_{Al} = 70\,GPa$ $\quad E_{wood} = 9\,GPa$ $\quad$ Ration $= \frac{E_w}{E_A} = 0.129$



$$b_w = 0.1 \times 0.129 = 0.0129\ (m)$$

$$I_a = \frac{0.1 \times (0.15)^3}{12} - \frac{(0.1 - 0.0129) \times 0.13^3}{12}$$

$$= 12.18 \times 10^{-6}\,m^4$$

$$A_a = 0.1 \times 0.01 \times 2 + 0.0129 \times 0.13$$

$$= 3.68 \times 10^{-3}\,m^2$$

Due to moment

$$\sigma_M = \pm \frac{My}{I} = \pm \frac{-31.25 \times \left(\frac{0.13}{2} + 0.01\right)}{12.18 \times 10^{-6}}$$

$$= \mp 1.92 \times 10^5 \ kN/m^2$$

Due to compression (25√3 kN horizontal force acting on the beam)

$$\sigma_A = \frac{-25\sqrt{3}}{Aa} = \frac{-25\sqrt{3}}{3.68 \times 10^{-3}} = -1.18 \times 10^4 \ kN/m^2$$

$$\text{Top} = -1.92 \times 10^5 - 1.18 \times 10^4 = \underline{-2.04 \times 10^5 \ kN/m^2} \quad //$$

$$\text{bottom} = 1.92 \times 10^5 - 1.18 \times 10^4 = \underline{1.80 \times 10^5 \ kN/m^2} \quad //$$

(d)

$$q = \frac{sA_c \bar{y}}{I}$$

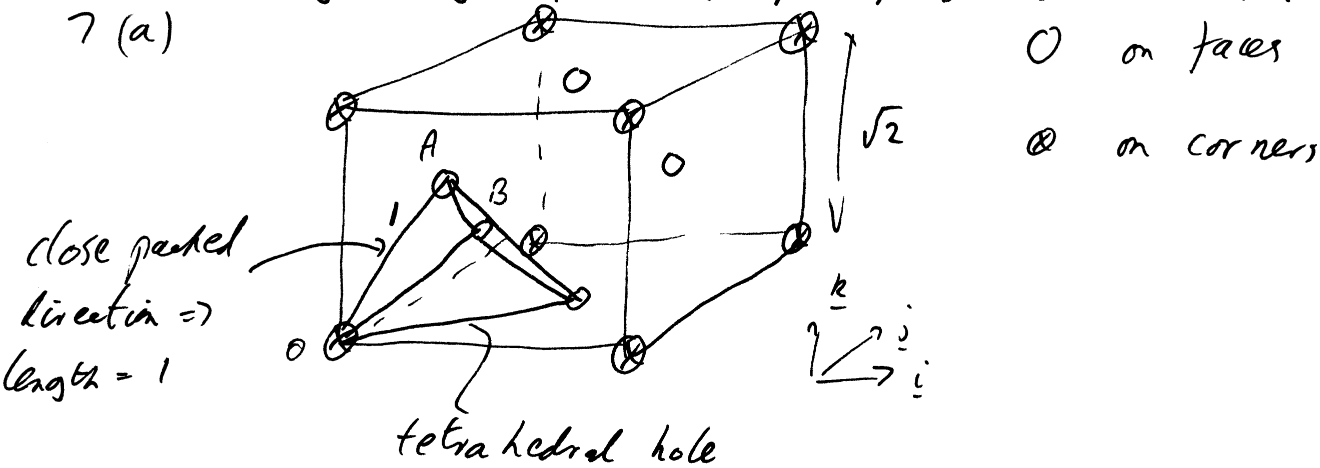
$$= \frac{25 \times (0.01 \times 0.1) \times 0.07}{12.16 \times 10^{-6}}$$

$$= \underline{1.439 \times 10^2 \ kN/m} \quad //$$

Engineering Tripos 1A, Paper 2, Section B MATERIALS. 2010

Dr M Sutcliffe

7 (a)



- Close-packed as consists of stacked hexagonal planes which maximise packing density.
- very ductile when pure allowing forming
- generally tough
- retain ductility and toughness to absolute zero.

(b) See sketch for tetrahedral hole

$$\underline{r}_B - \underline{r}_A = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} - \begin{pmatrix} 0 \\ +\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$\Rightarrow |\underline{r}_B - \underline{r}_A| = 1$$

So all side lengths are 1.

(a) mostly well done. In (b) the tetrahedral hole was not identified by many.

8 (a) A 'sea' of free electrons is formed from a release of electrons to form positively charged ions. Bonding is by electrostatic interaction between ions and electrons. Because the electrons are in a sea bonding is non-directional. Hence the modulus, determined by the stiffness of the bond, is more-or-less the same in all directions.

Stiffness and strength confused. Dislocations and glide planes irrelevant.

(b) (i) $\varepsilon_3 = \dfrac{-\nu\sigma_1 - \nu\sigma_2 + \sigma_3}{E} = -\dfrac{3}{2}\dfrac{\nu\sigma}{E}$

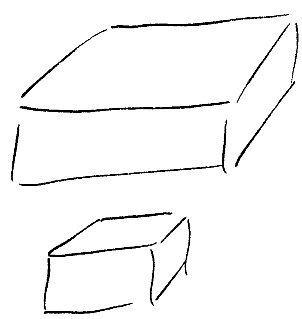
$\varepsilon_1 = \dfrac{\sigma_1}{E} - \dfrac{\nu\sigma_2}{E} = \dfrac{\sigma}{E}\left(1 - \dfrac{\nu}{2}\right)$

(ii) For $\varepsilon_1 = \dfrac{\sigma}{E} \Rightarrow \dfrac{\sigma}{E}\left(1 - \dfrac{\nu}{2}\right) - \dfrac{\nu\sigma_3}{E} = \dfrac{\sigma}{E}$

(b)(i) and ii well answered

$\Rightarrow \sigma_3 = -\dfrac{\sigma}{2}$ – makes sense as Poisson strain for 2 and 3 directions cancel.

(iii)



– Thermal strain causes mismatch, but halves are 'glued' together

Not attempted or well answered. Need to identify compatibility between two halves.

9(a) → First identify objectives and constraints.

- eg minimise weight, cost (objective)
- required stiffness & strength (constraint)
- manufacture, shape, environmental (constraint)

→ Derive performance indices

→ Then construct charts

- use selection stages for eg. σ, cost × density
- limit stage for processing
- include toughness limit stage

Need to address objectives and constraints - omitted by many.

→ Link charts

→ ~~Make selection~~
Move selection criterion to draw up shortlist

→ Identify a few materials for future consideration.

(b) $\sigma_m = \sigma_y = \frac{\alpha p l^2}{t^2} \Rightarrow t = \sqrt{\frac{\alpha p l^2}{\sigma_y}}$

$\text{Cost} = C t l^2 \rho$

Eliminate free variable t $\Rightarrow \text{Cost} = C l^2 \rho \sqrt{\frac{\alpha p l^2}{\sigma_y}}$

Material performance index to maximise is $\sigma_y / C^2\rho^2$

Could have used σ_{TS} or σ_y →

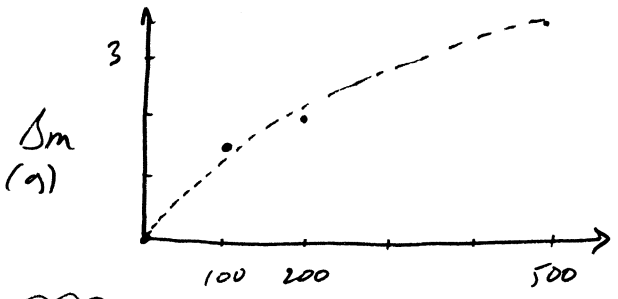
	$\sigma_y = \sigma_{TS}$	C	ρ	$\sigma_y / C^2\rho^2$
Al	304	1	2.7	41.7 ← choose Al
SS	1360	4.4	7.85	1.13

N.B. log scales

consistent units from tables, using mid-values

Commonly mass rather than cost was minimised.

10 (a)



Could be linear or parabolic - this looks parabolic: check

t	Δm	$t/\Delta m^2$
100	1.4	51
200	1.9	55
500	3.3	46

$\rightarrow 51$

Some candidates attempted linear fits

Parabolic growth - $(\Delta m)^2 = kt$ with $k \sim \frac{1}{51} \sim 0.02$

Parabolic growth due to increase in mass as O binds to M. Film stays intact so, as oxide forms, a protective layer builds up slowing growth. Rate governed by diffusion of M / O ions or conduction of electrons through oxide.

(b) Assuming mechanism continues to hold

$$\Delta m_{1000} = \sqrt{\tfrac{1000}{51}} = \sqrt{0.02 \times 1000} = 4.47\text{g} \simeq \underset{\sim}{4.5\text{g}}$$

To get depth of metal lost:

Avagadro's Number

Number O gained $= \dfrac{\Delta m \times N_A}{N_O}$ ~ Atomic weight of O $\quad \dfrac{kg \times {}^1/kmol}{kg\,/kmol}$

Number M lost $= \dfrac{\Delta m \times N_A}{N_O} \times \dfrac{1}{2}$ ~ MO_2 ← This step often omitted

atomic weight of M

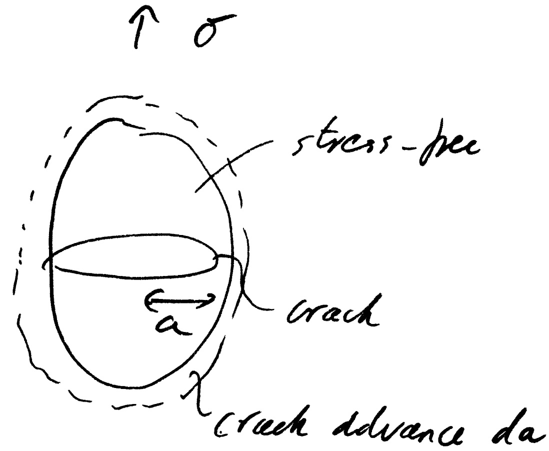
Mass of M lost $= \Delta m \times \dfrac{\cancel{N_A}}{N_O} \times \dfrac{1}{2} \times \dfrac{N_M}{\cancel{N_A}} = \dfrac{\Delta m R}{2} \quad \left(R = \dfrac{N_M}{N_O}\right)$

Vol of M lost $= \dfrac{\Delta m R}{2} \dfrac{1}{\rho} \quad \dfrac{kg}{kg/m^3}$

Depth of M lost $= \underset{\sim}{\dfrac{\Delta m R}{2 \rho A}} \quad \dfrac{m^3}{m^2} = m$

This question omitted by a significant proportion. Generally well done by those attempting the question.

11 (a)



- assume semi-circle is unstressed
- crack advance da gives change in stressed area $2\pi a\,da$
- change in stored energy $= 2\pi a\,da \cdot \frac{1}{2}\frac{\sigma^2}{E}$ $\boxed{\text{per unit depth into page}}$
- this corresponds to energy released $G \times$ new area ($2da$)

$$\Rightarrow G = \pi a \cdot \frac{1}{2}\frac{\sigma^2}{E} = \frac{\sigma^2 \pi a}{2E}$$

But $K = \sigma\sqrt{\pi a}$

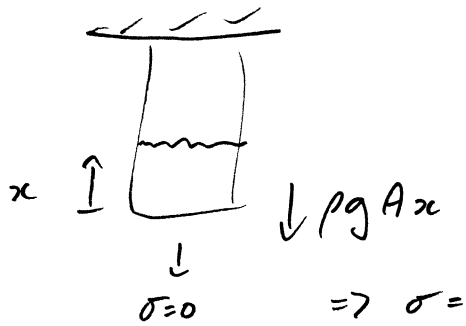
$$\Rightarrow G = \frac{K^2}{2E}$$

$\rightarrow$ result in Databook $\sqrt{\frac{GE}{1-\nu^2}} = K$

except omitting Poisson's effects and a numerical factor.

$\boxed{\text{This section surprisingly poorly done, with few people sketching the relevant model.}}$

11(b)



$$A = \frac{\pi (0.05L)^2}{4}$$

$$\Rightarrow \sigma = \rho g x \leftarrow$$

This free body analysis poorly done, many not identifying the variation of σ with x.

(i) Weibull: $P_s(V) = \exp\left[\int_V -\left(\frac{\sigma}{\sigma_0}\right)^m \frac{dV}{V_0}\right]$

well answered

$$= \exp\left[\int_{x=0}^{L} -\left(\frac{\rho g x}{\sigma_0}\right)^m \frac{A\,dx}{V_0}\right] = \exp\left[-\left(\frac{\rho g}{\sigma_0}\right)^m \frac{\pi (0.05L)^2}{4V_0} \frac{L^{m+1}}{m+1}\right]$$

(ii) Uniform tension $\Rightarrow P_s = \exp\left(-\left(\frac{\sigma}{\sigma_0}\right)^m \frac{V}{V_0}\right)$

To find m take ratios $\frac{\ln[P_s(0.0848)]}{\ln[P_s(0.1)]} = \left(\frac{0.0848}{0.1}\right)^m$ $\Rightarrow 0.192 = (0.848)^m$ $\quad \frac{\ln 0.98}{\ln 0.9}$

key step to avoid arithmetic errors

$\Rightarrow m = 10.0$

Taking ratio of feature and test sample

$$\frac{\ln(0.99)}{\ln(0.90)} = \frac{\left(\frac{\rho g}{\cancel{\sigma_0}}\right)^m \frac{\pi\, 0.05^2 L^2}{4\cancel{V_0}} \frac{L^{m+1}}{m+1}}{\left(\frac{0.1\text{ MPa}}{\cancel{\sigma_0}}\right)^m \frac{0.2\text{ m}^3}{\cancel{V_0}}}$$

$\rho g = 2000 \times 9.81$
$= 0.0196\ \text{MN/m}^3$

Working in MN and m

$$\Rightarrow L^{13.0} = \frac{\ln 0.99}{\ln 0.9}\left(\frac{0.1}{0.0196}\right)^{10.0} \frac{0.2.11.4}{0.05^2\pi} = 1.28 \times 10^9\ \text{m}^{13}$$

$$\Rightarrow \underline{L < 5.0\text{m}}$$

(c) Gripping will be a problem, causing local stress raisers, as well as defects on the edges of the sample

$\Rightarrow$ careful preparation and use of a bend test.

more than 'bend test' needed for 4 marks

12 (a) Hardness testing: find a smooth flat part and push in an indentor (eg diamond) and measure the projected area. $H = \frac{\text{Force}}{\text{Projected area}}$

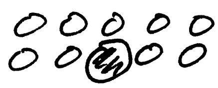
{Not an impact test as stated by some}

Apply different forces for different materials.
Useful as it provides an easy non-destructive test for hardness, which can be used to estimate yield stress $\simeq \frac{H}{3}$

Difficulties: - need a smooth surface
- mounting specimen
- surface / edge effects
- flaw / indent may be unacceptable.

{All parts of question need to be addressed for full marks}

(b) - Solid solution hardening

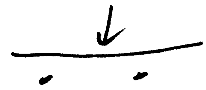


- solute atoms roughen slip plane
- increases dislocation resistance

Precipitation hardening

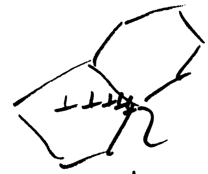
- hard particles pin dislocations
- depends on size / strength / density of ppts

Work hardening



- dislocations tangle impeding progress
- work hardening increases dislocation density

Grain boundary hardening



change in slip orientation

- dislocations pile up at boundaries
- gives a relatively small increase in yield stress as grain size falls.

{sketches required for full marks}

12(c) (i) $\varepsilon_n = \frac{l}{l_0} - 1 \qquad \varepsilon_T = \ln \frac{l}{l_0}$

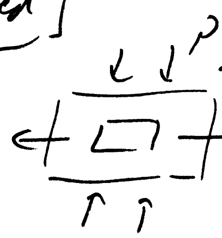
	ε_n	ε_T
$a \to b$	$\frac{7.5}{10} - 1 = -0.25$	$\ln \frac{7.5}{10} = -0.288$
$b \to c$	$\frac{5}{7.5} - 1 = -\frac{1}{3}$	$\ln \frac{2}{3} = -0.405$
$a \to c$	$\frac{5}{10} - 1 = -0.5$	$\ln \frac{5}{10} = -0.693$

True strains add, eg $-0.288 - 0.405 = -0.693$

Nominal strains have no simple relationship.

{This section well answered}

(ii) For specimen (b)



— yield when p = yield stress

$\varepsilon_n = \frac{4}{3} - 1 = 33\%$

In tensile test nominal stress at a nominal tensile strain of 33% equals 350 MPa.

But in this test the area will have been reduced, so true stress to cause yield $= 350 \times \frac{A_0}{A} = 350 \times \frac{l}{l_0} = 350 \times 1.33 = 466$ MPa

This is the yield stress that needs to be applied to compress (b).

{Not such an easy question, not well done, requiring relating the compression and tensile tests.}

(iii) Assume volume conservation.

(a) → (b) area is unchanged so no change in length

(b) → (c) $5 \times 18 \times x = 100 \Rightarrow x = 1.11 \Rightarrow$ elongation of 11%.

Some students appealled to symmetry, reasonably, though (a)→(b) is not symmetrical in the out-of-plane direction. Poisson's ratio effects are not relevant.

MPFS 22/6/10