SECTION A

1 **(short)**

(a) In the inverting op-amp circuit in Fig. 1, the op-amp may be assumed to be ideal. Derive an expression for the voltage gain in terms of R_1 and R_2**.**

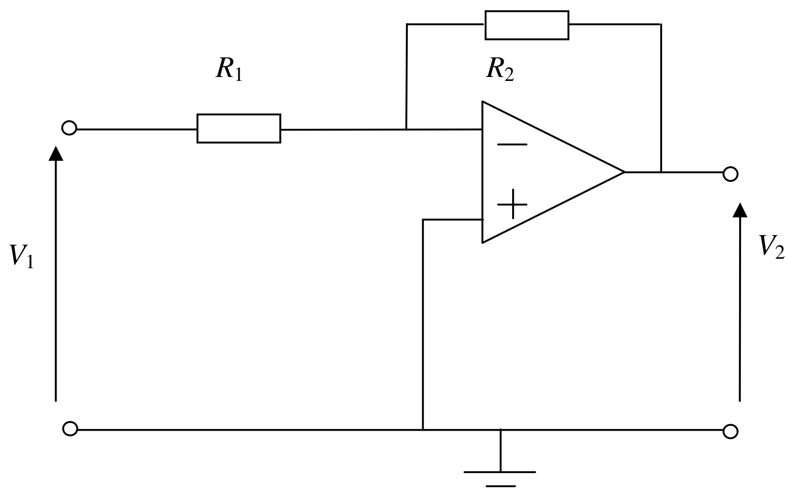


Fig. 1

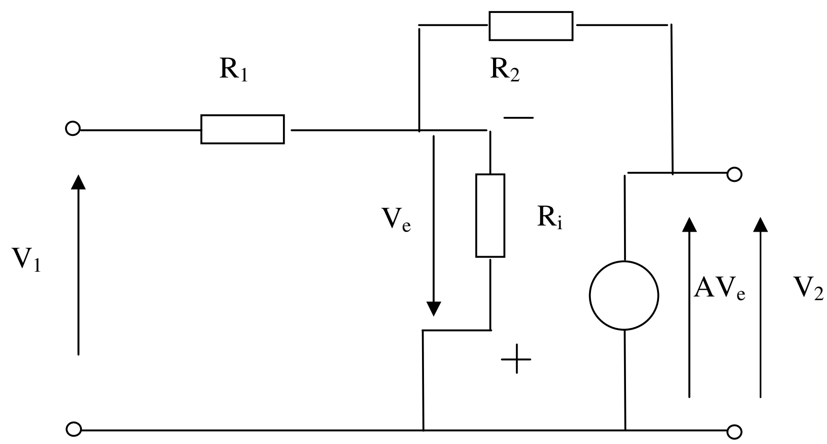
$V_-=0$ (virtual earth principle)

$$\frac{V_1}{R_1}+\frac{V_2}{R_2}=0 \qquad (R_i=\infty)$$

Then

$$\frac{V_2}{V_1}=-\frac{R_2}{R_1}$$

(b) If instead the op-amp has finite input resistance, R_i , and finite gain A, but is otherwise ideal, derive an expression for the voltage gain in terms of A , R_i , R_1 , R_2.



Kirchhoff's current law at the inverting point:

$$\frac{V_1 + V_e}{R_1} + \frac{V_e}{R_i} + \frac{V_2 + V_e}{R_2} = 0$$

$AV_e = V_2$

Then:

$$\frac{V_1 + \frac{V_2}{A}}{R_1} + \frac{V_2}{AR_i} + \frac{V_2 + \frac{V_2}{A}}{R_2} = 0$$

We finally get:

$$\frac{V_2}{V_1} = -\frac{AR_2R_i}{AR_1R_i + R_1R_2 + R_i(R_1 + R_2)}$$

(c) With R_i = 10 kΩ, $A = 10^3$, the op-amp of Fig. 1 is to be used to produce a voltage gain of −20. Using the expression for gain obtained in part (b) above, calculate R_2, if R_1 = 100 Ω.

$$Gain = \frac{V_2}{V_1} = -\frac{AR_2R_i}{AR_1R_i + R_1R_2 + R_i(R_1 + R_2)}$$

$$R_2 = -\frac{\frac{V_2}{V_1}R_1R_i(1+A)}{AR_i + \frac{V_2}{V_1}R_1 + \frac{V_2}{V_1}R_i} = \frac{20\times10^2\times10^4(1+10^3)}{10^3\times10^4 - 20\times10^2 - 20\times10^4}\Omega = \frac{1.001\times10^7}{4899}\Omega = 2.043k\Omega$$

Thus

$$R_2 = 2.043k\Omega$$

2 **(short)**

(a) In the circuit of Fig. 2, R = 200 Ω, L = 70 mH and C = 159 μF. By applying Thevenin's theorem to this circuit, or otherwise, determine the rms magnitude of the current flowing in the capacitor C, its peak value and also its phase with respect to the 100 V voltage source.

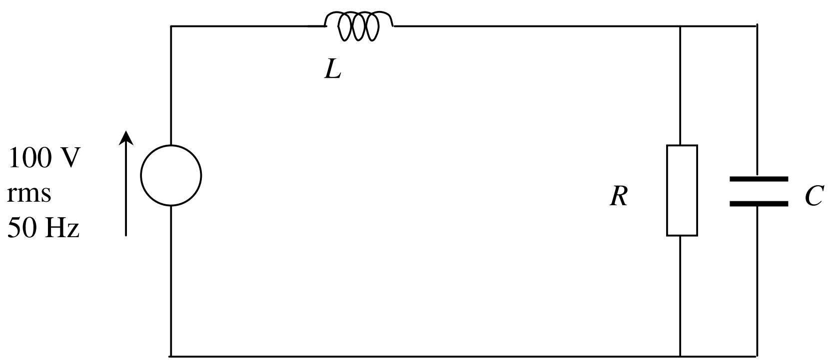


Fig. 2

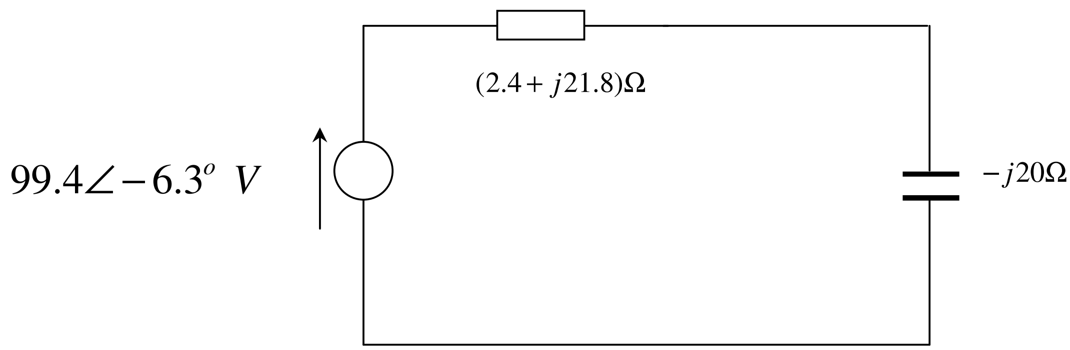
$$\overline{Z_L} = j\omega L = j \times 2\pi \times 50 \times 70 \times 10^{-3}\,\Omega = j22\Omega$$

$$\overline{Z_c} = \frac{1}{j\omega C} = -j20\Omega$$

$$\overline{V_{Th}} = V_{Open\ Circuit} = \frac{200}{200 + j22} 100V = 99.4\angle -6.3^o\ V$$

$$\overline{Z_{Th}} = \frac{V_{Open\ Circuit}}{I_{Short\ Circuit}} = \frac{99.4\angle -6.3^o\ V}{\dfrac{100V}{j22\Omega}} = (2.4 + j21.8)\Omega$$

Thus:



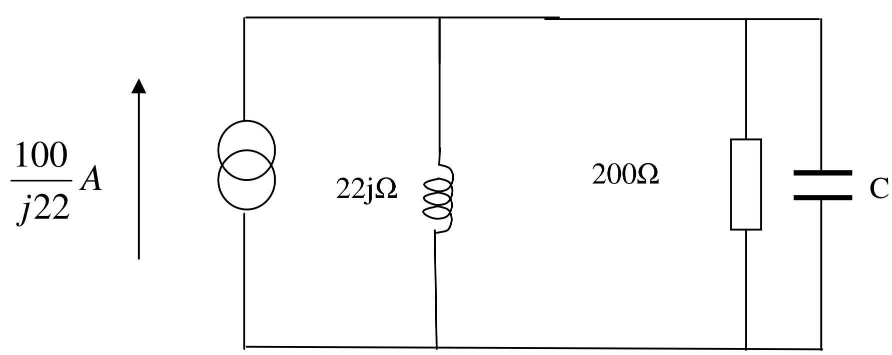
Then

$$\bar{I}_C = \frac{99.4\angle -6.3^o\ V}{(2.4 + j21.8)\Omega - j20\Omega} = \frac{99.4\angle -6.3^o}{2.4 + j1.8} A = 33.13\angle -43.32^o \text{ A}$$

$$\widehat{I_C} = \sqrt{2} I_{C_{RMS}} = \sqrt{2} \times 33.13\text{A} = 46.85\text{A}$$

(b) The capacitor is now altered to give a resonant frequency of the circuit of 50 Hz. Find the new value of capacitor *C* and determine the rms magnitude of the capacitor voltage, and its phase, with respect to the 100 V voltage source.

Appling Norton’s Theorem:



$$\overline{I_{No}} = I_{Short\ Circuit} = \frac{100}{j22} A$$

At resonance load is purely real

Thus:

$$\overline{Z_c} = \frac{1}{j\omega C} = -j22\Omega$$

Thus

C=144.7 μF

$$\overline{V}_C = \frac{100}{j22} \times 200V = 909.1\angle -90^o \text{ V}$$

3 **(short)**

(a) A 240 V, 50 Hz mains transformer is used to drive a load of (20+10j) Ω with 40 V rms. Assuming the transformer to be ideal, calculate the turns ratio of the windings and the impedance of the load when referred across to the high voltage (primary) side.

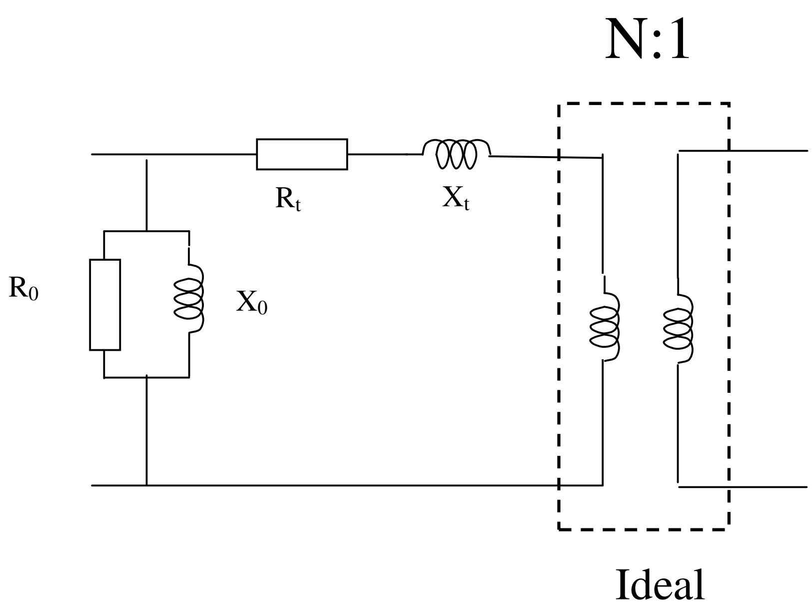
N=Turns ratio=voltage ratio=240:40=6:1

Impedance is transferred across x N^2

Thus

$$\overline{Z_L}' = (20 + j10)\Omega \times 6^2 = (720 + 360j)\Omega$$

(b) Draw the equivalent circuit of a non-ideal power transformer and briefly explain the physical significance of each of the circuit elements.



R_t represents series resistance of windings, i.e. copper loss

X_t represents the leakage flux across the transformer

R_0 represents hysteresis+eddy current losses in the transformer core, i.e. iron loss

X_0 represents inductance of windings on core (finite due to reluctance of flux path)

4 **(long)** Figure 3(a) shows the circuit diagram of a test probe, its cable and the input section of an oscilloscope. The oscilloscope input impedance is equivalent to a 2 MΩ resistor in parallel with a 30 pF capacitor. The cable is represented by a 40 pF capacitance to the ground.

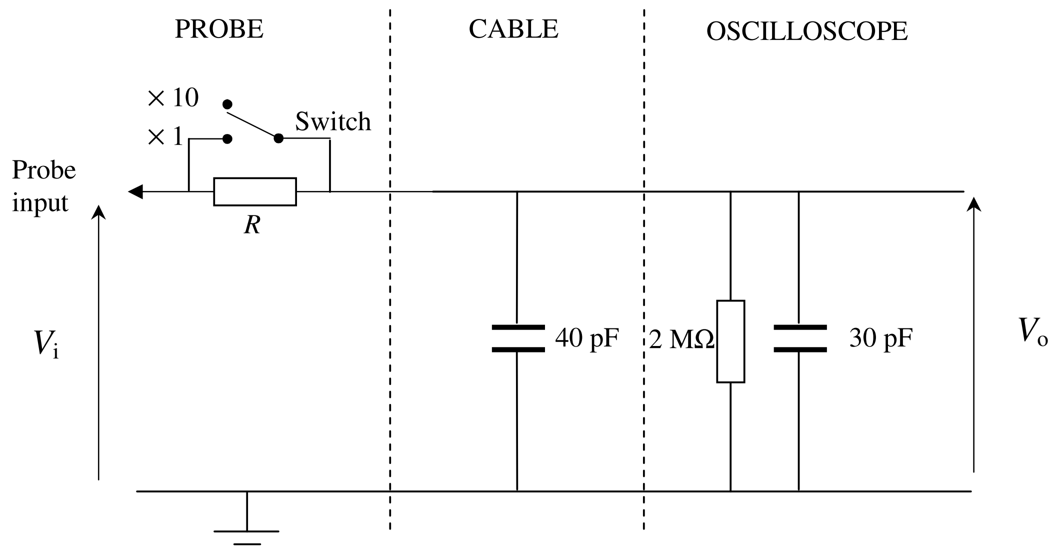
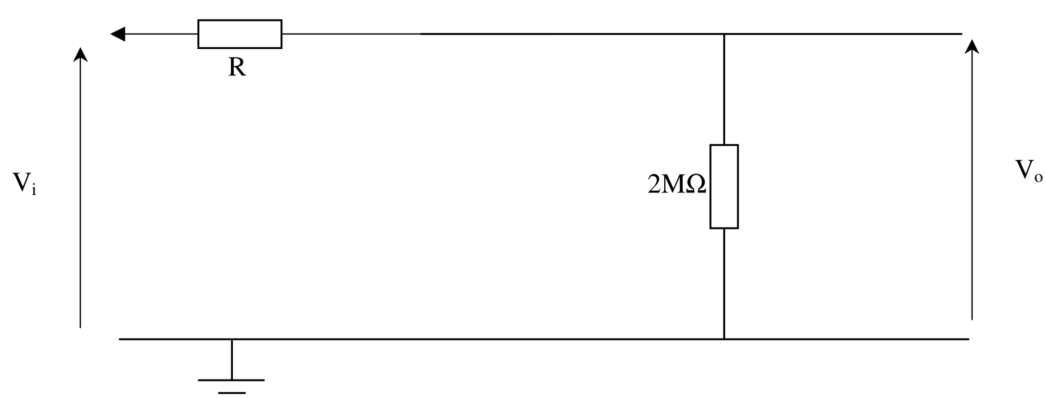


Fig. 3(a)

(a) The probe may be set to either × 1 or × 10 attenuation by the operation of the switch shown. At low frequencies the effects of capacitance may be ignored. Hence calculate the value of *R* required to achieve the × 10 attenuation at low frequencies.

Ignoring capacitors we get a potential divider:

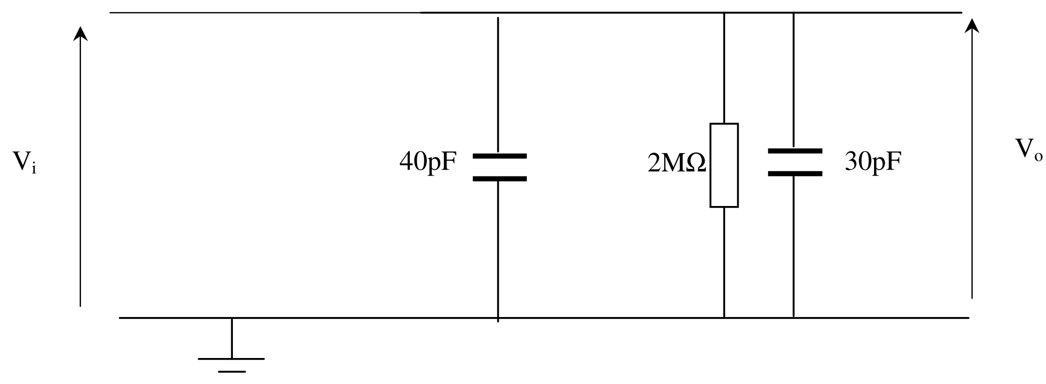


$$V_0 = \frac{2M\Omega}{R + 2M\Omega} V_i = \frac{V_i}{10}$$

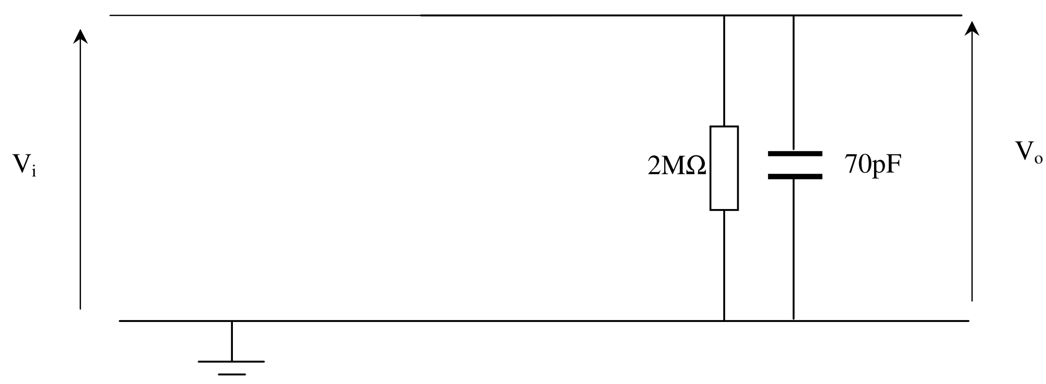
Thus R=18MΩ

(b) For the × 1 switch position, and now also considering the capacitors, derive an expression for the complex input impedance, seen at the input to the probe, as a function of frequency.

In this case the circuit becomes:



Considering that parallel capacitors add, we get:



With R=2MΩ and C=70pF and ω=2πf

$$\overline{Z_{in}} = \frac{V_i}{I} = R \,||\, C = \frac{R}{1 + j\omega CR} = \frac{R(1 - j\omega CR)}{1 + \omega^2 C^2 R^2} = \frac{2\times 10^6 - j560\pi f}{1 + 7.84\times 10^{-8}\pi^2 f^2}\,\Omega$$

(c) An engineer wishes to measure the voltage at a test point within a television which monitors the line frequency at 20 kHz. An equivalent circuit for the test point is given in Fig. 3(b). Determine the voltage measured with the probe set to $\times$ 1 attenuation.

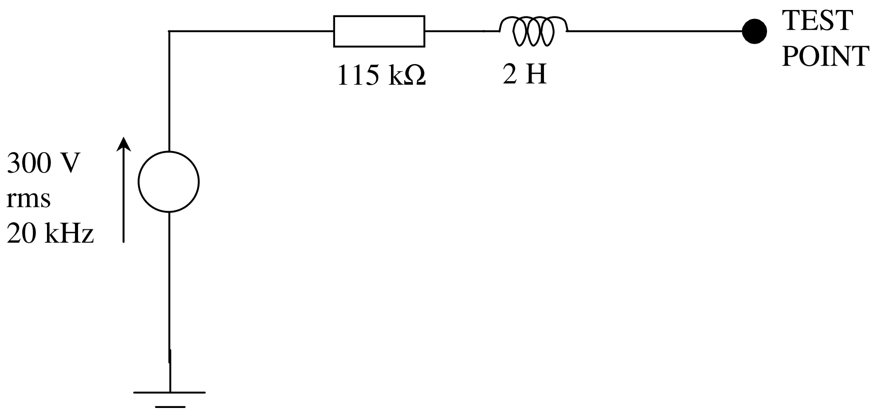
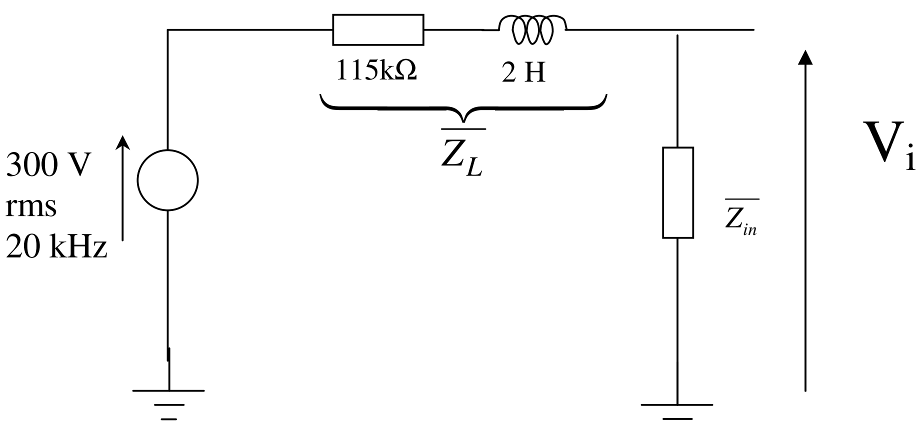


Fig. 3(b)

Thus:



From the expression of $\overline{Z_{in}}$ derived above and f=20kHz we get:

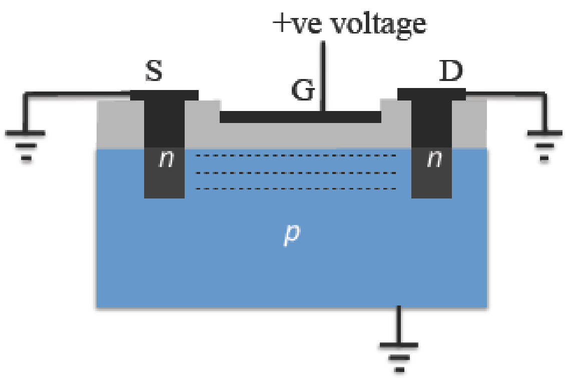
$$\overline{Z_{in}} = \frac{2\times10^6 - j560\pi f}{1+7.84\times10^{-8}\pi^2 f^2}\Omega = (6441\times10^3 - j1.1331\times10^5)\Omega = 1.135\times10^5\angle-86.75^o\ \Omega$$

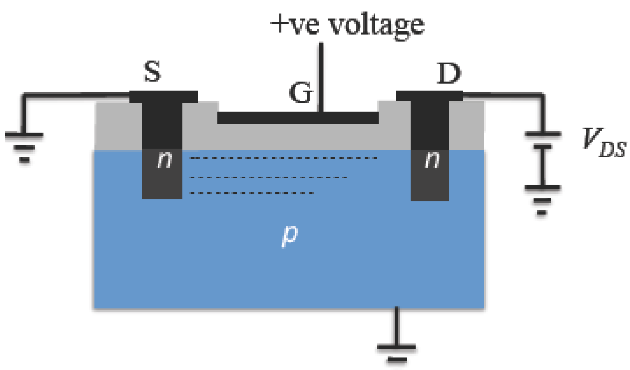
$$\overline{V_i} = \frac{\overline{Z_{in}}}{\overline{Z_L}+\overline{Z_{in}}}\times300V = 183.5\angle-135.41^o\ V$$

5 **(long)**

(a) Briefly describe the electrical characteristics of an enhanced-mode field effect transistor.

The lecture notes discuss the n-channel enhanced mode MOSFET:



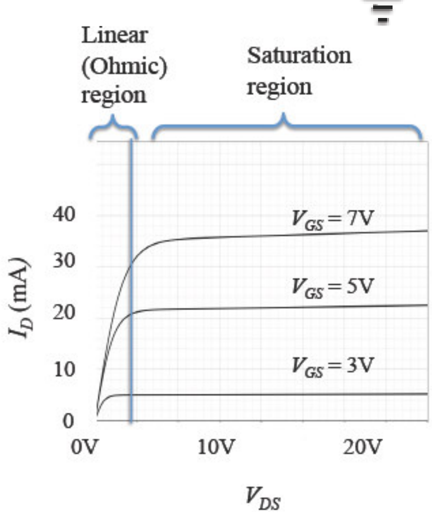


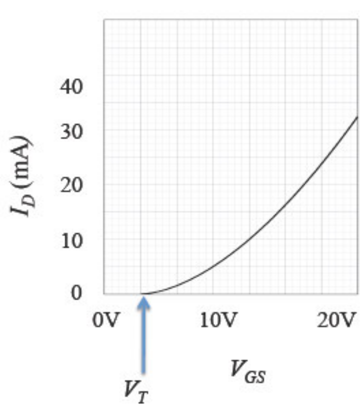
If a voltage is applied between the drain and source when there is no gate voltage, then no drain current will flow, as one of the p-n junctions will always be reverse biased, and there are effectively no charge carriers in the space between the drain and source.

If a positive gate voltage is now applied, there will be an electric field between the gate and the bottom of the device, attracting electrons to the interface between the gate oxide and the p-type substrate. A very small number of these electrons will have come from the p-type substrate but the vast majority of them will have come from the n-type regions at the drain and source.

If V_{GS} is high enough (above the threshold value, V_T), enough electrons will accumulate in the space between the drain and source, resulting in an *inversion layer*, where there is an n-type conducting channel inside the p-type.

If we now apply a voltage between the drain and source as shown on the left, a current, I_D, will flow through this inversion layer. For small values of V_{DS}, I_D increases linearly and rapidly. This is known as the Ohmic region. As V_{DS} is increased, the difference in voltage between the gate and drain decreases, reducing the number of electrons in the inversion layer near the drain, and reducing the conductivity of the channel. This results in a leveling off of the I_D-V_{DS} curve. This is known as the saturation region.





(b) Draw the small signal model for the source-follower circuit shown in Fig. 4 and derive expressions for the:

(i) input impedance;
(ii) gain when no load is connected;
(iii) output impedance.

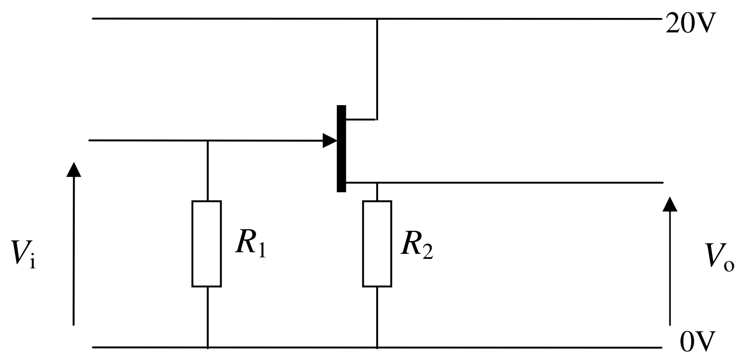
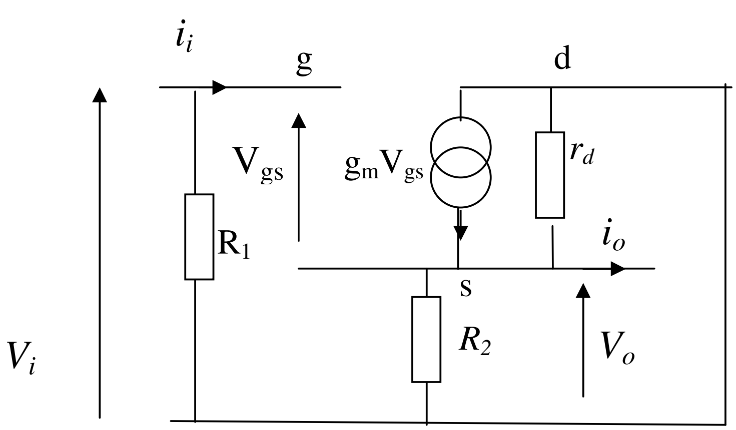


Fig. 4

Evaluate these parameters with $g_m = 3$ mS and $r_d = 15$ kΩ for the transistor, and with $R_1 = 20$ MΩ and $R_2 = 5$ kΩ.

The small signal model is:



(i)

$$R_{in} = \frac{V_i}{i_i} = R_1 = 20M\Omega$$ as no gate current flows

(ii) $V_{gs} = V_i - V_o$

$$V_o = (g_m V_{gs} - i_o) \times R_2 \parallel r_d$$

Calling $R = R_2 \parallel r_d = \dfrac{R_2 r_d}{R_2 + r_d} = 3.75k\Omega$

We get

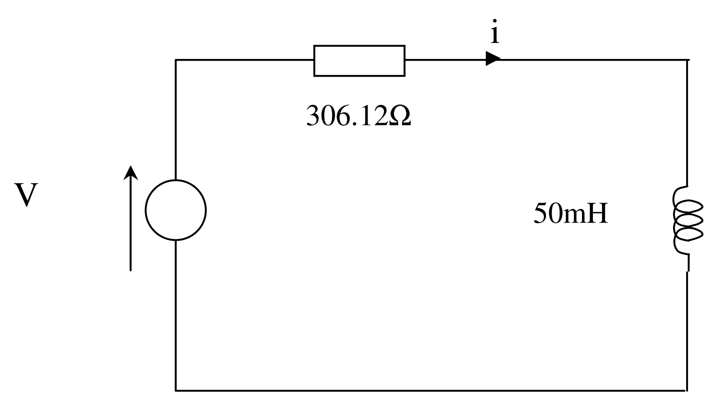
$$V_o = \frac{g_m R}{1 + g_m R} V_i - \frac{R}{1 + g_m R} i_o$$

Thus

Gain= $\dfrac{g_m R}{1 + g_m R} = \dfrac{g_m R_2 r_d}{R_2 + r_d + g_m R_2 r_d} = 0.92$

(iii) Ouput Impedance= $\dfrac{R}{1 + g_m R} = \dfrac{R_2 r_d}{R_2 + r_d + g_m R_2 r_d} = 306.12\Omega$

(c) If the circuit is used to drive an inductive load of 50 mH, calculate the frequency at which the current through the inductive load drops to 70% of its mid-band value.



$$i = \frac{V}{306.12 + j2\pi f 50\times10^{-3}} A$$

70% of mid-bad frequency when the real part of the denominator equals its imaginary part:

$$306.12 = 2\pi f 50\times10^{-3}$$

Thus f=974Hz

Part 1A; Paper 3, Section B (2010) Author: R.V. PENTY

8. (a) $F = A.B.C + B.C.D + A.\bar{C}.D$

$A.\bar{B}.C.D = \bar{A}.\bar{B}.C.D = A.B.\bar{C}.\bar{D} = X$

CD \ AB	00	01	11	10
00			X	
01			1	1
11	X	1	1	X
10			1	

Simplest S.O.P.

$F = A.B + C.D + A.D$

(3 marks)

(b) For NAND gates

$F = A.B + C.D + A.D$

By De Morgan $= \overline{\overline{A.B}\,.\,\overline{C.D}\,.\,\overline{A.D}}$ (3 marks)

(c) For NOR, find simplest S.O.P for $\bar{F}$

CD \ AB	00	01	11	10
00			X	
01			1	1
11	X	1	1	X
10			1	

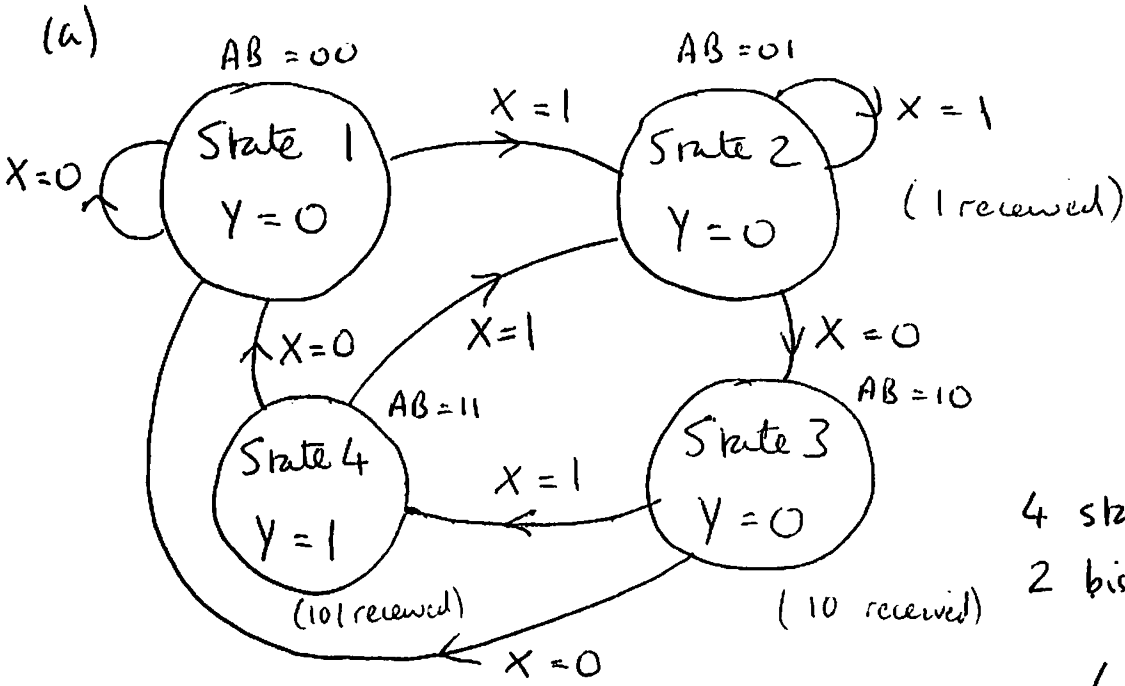
$\bar{F} = \bar{A}.\bar{C} + \bar{A}.\bar{D} + \bar{B}.\bar{D}$

$= \overline{A+C} + \overline{A+D} + \overline{B+D}$

$F = \overline{\overline{A+C} + \overline{A+D} + \overline{B+D}}$

(4 marks)

7. (a)



4 states ⇒
2 bistables needed

(5 marks)

(b) Choose states AB as shown on state diagram

Input X	Current State A	Current State B	Next State A	Next State B	Bistable Inputs J_A	K_A	J_B	K_B
0	0	0	0	0	0	x	0	x
1	0	0	0	1	0	x	1	x
0	0	1	1	0	1	x	x	1
1	0	1	0	1	0	x	x	0
0	1	0	0	0	x	1	0	x
1	1	0	1	1	x	0	1	x
0	1	1	0	0	x	1	x	1
1	1	1	0	1	x	1	x	0

(5 marks)

8. (a) Data lines $d = 16$ Address lines $a = 14$

Total capacity $= d \times 2^a$

$= 16 \times 2^{14}$

$= 262,144$ bits

$= 256$ kbytes (NB 1kbyte = 1024 bits)

(3 marks)

(b)

A_{15}	A_{14}	A_{12} . . . . . . A_0		
0	0	0 0	0000_H	
0	0	1 1	$3FFF_H$	
0	1	0 0	4000_H	
0	1	1 1	$7FFF_H$	
1	0	0 0	8000_H	Required memory map for chip
1	0	1 1	$BFFF_H$	
1	1	0 0	$C000_H$	
1	1	1 1	$FFFF_H$	

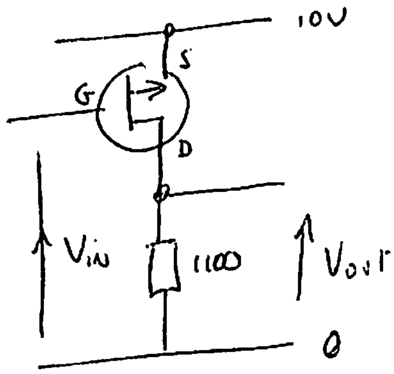
$CS = \bar{A}_{14} \cdot A_{15}$

$\overline{CS} = \overline{\bar{A}_{14} \cdot A_{15}}$

$= A_{14} + \bar{A}_{15}$ Either OK

(7 marks)

4 (a)



For $V_{DS} = 0V$

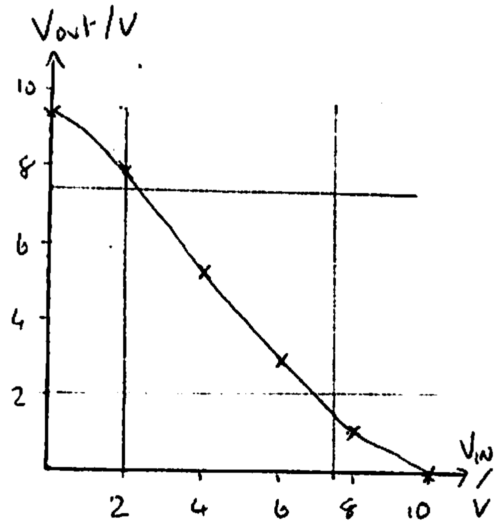
Current through R

$I = \frac{10}{1100} = 9.09\,mA$

For $V_{DS} = 10V$, $I = 0mA$

(See attached sheet for load line).

V_{IN}	V_{GS} ($V_{IN}-10$)	V_{DS}	$V_{out}(=10+V_{DS})$
0	−10	−0.6	9.4V
2	−8	−2.1	7.9V
4	−6	−4.8	5.2V
6	−4	−7.1	2.9V
8	−2	−8.9	1.1V
10	0	−10	0V



Function is inverter ($Y = \bar{X}$) though not a very good transfer characteristic (6 marks)

(b) We could say $> \cancel{8}\,7.5V$ is logic 1

$< 2V$ is logic 0

From transfer function in part (a) above

For $V_{in} > 7.5\,V$, $V_{out} < 1.5V$

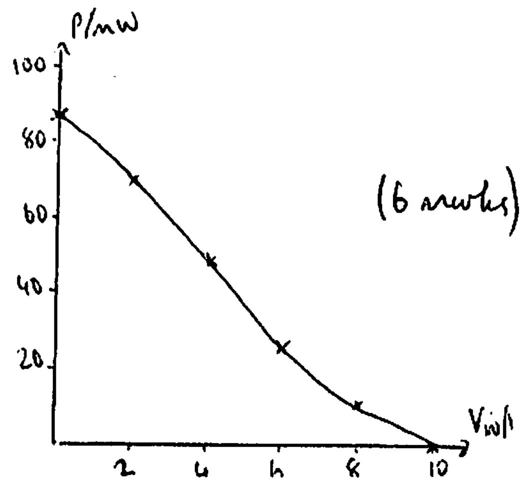
$V_{in} < 2\,V$, $V_{out} > 7.9V$ (4 marks)

(c) Power $= V_{FET}\,I_{FET} + V_R\,I_R$

But $I_{FET} = I_R$ + $V_{FET} + V_R = 10V$

$\Rightarrow$ Power $= 10 \cdot I_{DS}$

V_{IN}	I_{DS}/mA (from load line)	P/mW
0	8.7	87
2	7.1	71
4	4.9	49
6	2.6	26
8	1.1	11
10	0	0

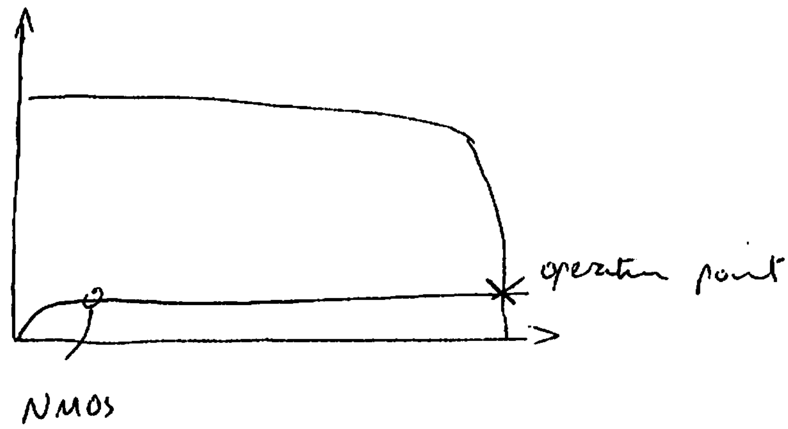


(6 marks)

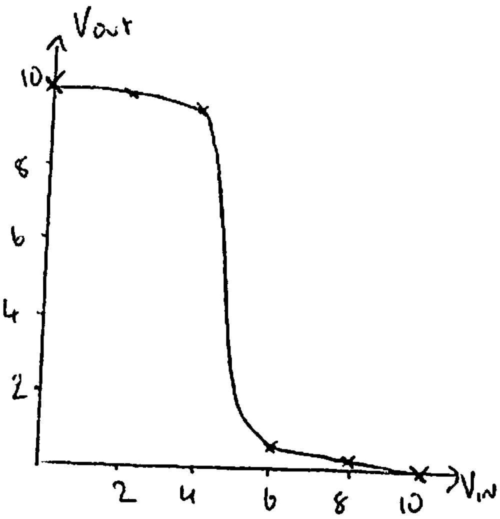
(d) Operating points found by intersecting NMOS characteristic with relevant reflected PMOS characteristic.

So for $V_{IN} = 2V$ (for example) $V_{GS(NMOS)} = 2V$

$V_{GS(PMOS)} = -8V$



V_{IN}	$V_{DS} = V_o$ (NMOS)	I_{DS}	$P (= 10 \times I_{DS})$ /mW
0	10	0	0
2	9.8	1.0	10
4	9.5	2.6	26
6	0.5	2.6	26
8	0.2	1.0	10
10	0	0	0



Much more nonlinear characteristic

Power only dissipated when switching

(10 marks)

(e) Fig (e) Note top transistor is self biased NMOS ⇒ looks like a resistor

(fig(e))

Input A	Input B	T_A	T_B	Y
LOW	LOW	OFF	OFF	HIGH
LOW	HIGH	OFF	ON	LOW
HIGH	LOW	ON	OFF	LOW
HIGH	HIGH	ON	ON	LOW

$$\Rightarrow Y = \overline{A}.\overline{B} = \overline{A+B}$$

(NOR gate)

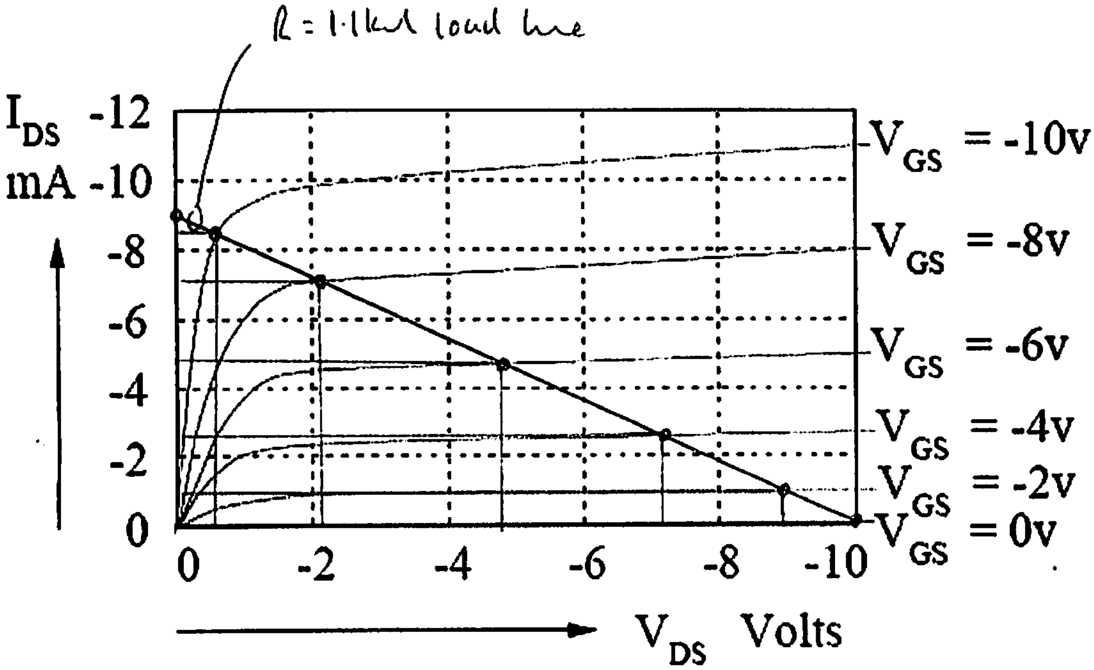
(fig(f))

input A	input B	T_{AN}	T_{AP}	T_{BN}	T_{BP}	Y
LOW	LOW	OFF	ON	OFF	ON	HIGH
LOW	HIGH	OFF	ON	ON	OFF	HIGH
HIGH	LOW	ON	OFF	OFF	ON	HIGH
HIGH	HIGH	ON	OFF	ON	OFF	LOW

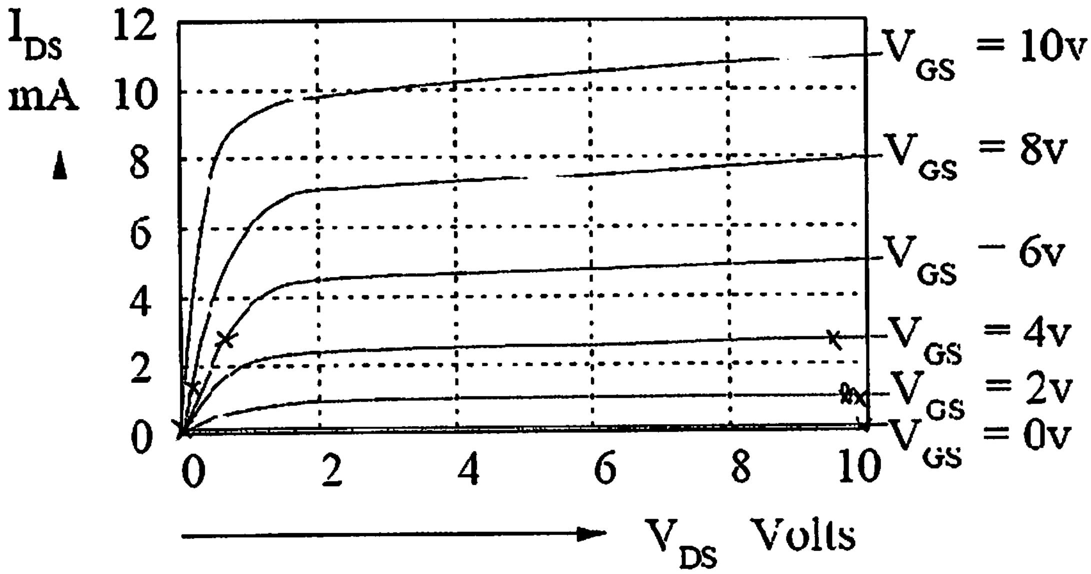
$$\Rightarrow Y = \overline{A.B}$$

(NAND gate).

(4 marks)


R = 1.1kΩ load line
IDS
mA
-12
-10
-8
-6
-4
-2
0
0
-2
-4
-6
-8
-10
VGS = -10v
VGS = -8v
VGS = -6v
VGS = -4v
VGS = -2v
VGS = 0v
VDS Volts

PMOS

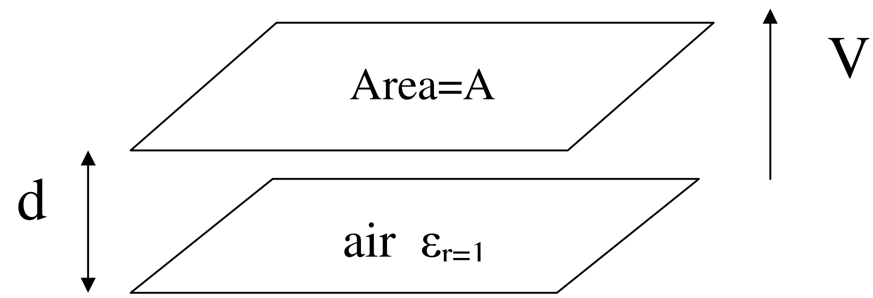

IDS
mA
12
10
8
6
4
2
0
0
2
4
6
8
10
VGS = 10v
VGS = 8v
VGS = 6v
VGS = 4v
VGS = 2v
VGS = 0v
VDS Volts

NMOS

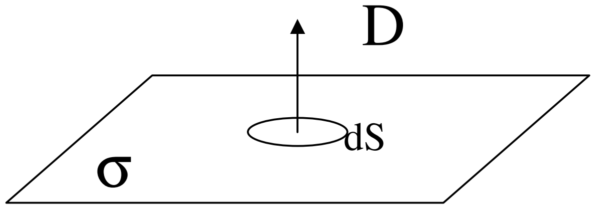
SECTION C

10 **(short)**

(a) Using Gauss's law, derive an expression for a parallel plate, air filled capacitor with plate separation *d*. State any assumptions made.



Gauss' law, flux of D= charge enclosed



D is perpendicular to the plane due to the geometry. We assume infinite plane and ignore edge effects

Calling σ [Cm^{-2}] the charge density:

$$DdS = \sigma dS$$

Thus

$D=\sigma$

But $D=\varepsilon_0\varepsilon_r E$

Then

$E=\sigma/\varepsilon_0$

But

$$V = -\int_{-d}^{0}(-E)dy$$

Then

$$V = Ed = \frac{\sigma d}{\varepsilon_0}$$

Total charge $Q=\sigma A$

$Q=CV$

So:

$$C = \frac{\varepsilon_0 A}{d}$$

(b) Show that the electrostatic force on a capacitor plate is $F = 0.5\ QE$, where Q is the total charge and E is the electric field.

The force can be derived from the energy balance at equilibrium

$$F = \frac{1}{2}V^2\frac{\partial V}{\partial C}$$

$$C(y) = \frac{\varepsilon_0 A}{y}$$

$$\frac{\partial C}{\partial y} = -\frac{\varepsilon_0 A}{y^2}$$

Then

$$F = -\frac{1}{2}V^2\frac{\varepsilon_0 A}{y^2} = -\frac{1}{2}V^2\frac{C}{y}$$

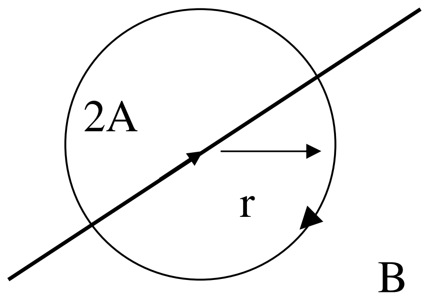
Since E=V/y

Q=CV

$$|F| = \frac{1}{2}Q|E|$$

11 **(short)**

(a) A straight wire carries a current of 2 A. Find the direction and strength of the magnetic flux density *B* at a distance of 30 mm perpendicular to the wire, showing your answer on a diagram.



From Maxwell-Ampere's Law

$B2\pi r=\mu_0 I$

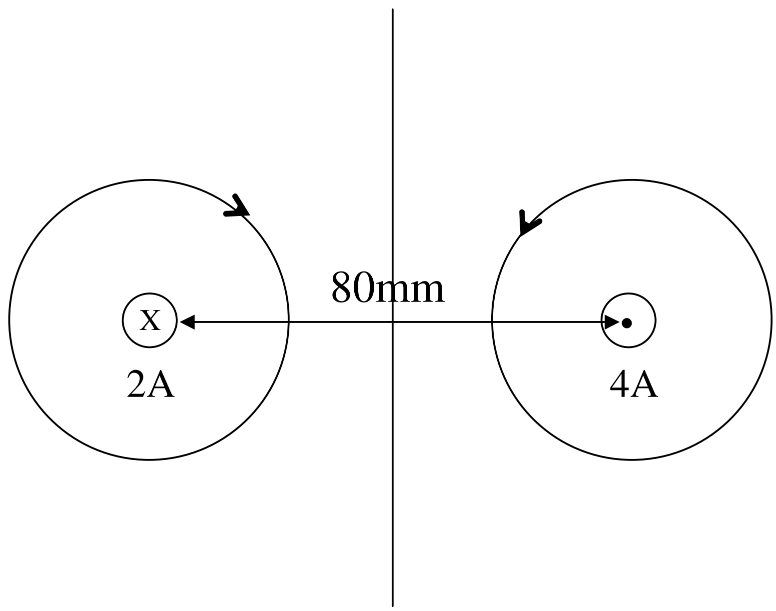
Thus

$$B = \frac{\mu_0 I}{2\pi r}$$

Then, for r=30mm and I=2A

$B=1.33\text{x}10^{-5}$ T

(b) A second wire is placed 80 mm away from and parallel to the first one and contains a current of 4 A in the opposite direction to the first wire. Calculate *B* at the mid-point between the two wires.



At mid point the B from each wire will add:

$$B = \frac{\mu_0 2A}{2\pi 40mm} + \frac{\mu_0 4A}{2\pi 40mm} = \frac{\mu_0 6A}{2\pi 40mm} = 3\times10^{-5}\ \text{T}$$

12 **(long)** Figure 6 shows a semi-circular permanent magnet of radius *R*, thickness δR (small) and depth *d*, which is constructed of COLUMAX. The magnet has a soft iron keeper, which is of very high permeability ($\mu_r \sim 10^4$), but which is prevented from touching the pole pieces by a plastic sheet ($\mu_r = 1$) of thickness *t*. The weight of the plastic and the keeper may be ignored.

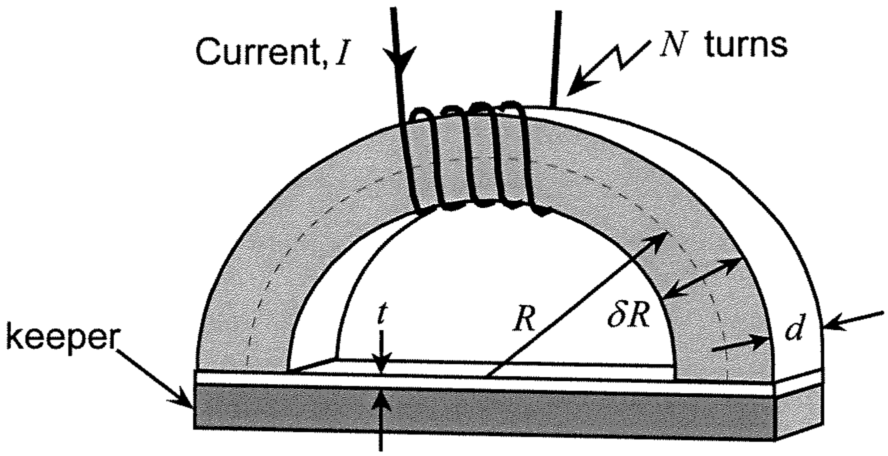


Fig. 6

(a) If $I = 0$ A, $R = 200$ mm, $\delta R = 10$ mm, $d = 20$ mm, and $t = 0.1$ mm, what is the flux density, B_c , in the magnet? (Use Fig. 2b, page 7, Electrical and Information Data Book)

Circulation path length = πR (we can ignore the keeper since it has very high μ_r

From Maxwell-Ampere's Law:

$$\oint Hdl = NI$$

Since I=0

$H_m l_m + 2H_p t = 0$

Where m denotes the magnet and p the plastic

The flux conservation at the magnet/plastic interface gives

$B_m = B_p = \mu_0 H_P$

Thus

$$H_m l_m + \frac{B_m 2t}{\mu_0} = 0$$

Then

$$B_m = -\frac{\mu_0 H_m l_m}{2t} = -3.95 \times 10^{-3}\ H_m$$

Plotting this relation on the Columax graph in the data-book, it crosses the Columax line at $B_m \approx 1.35$T

(b) What force is necessary to pull the keeper from the magnet, with $I = 0$ A?

Using the virtual work principle

$$\delta W = F\delta x$$

Energy per unit volume in the air gap= $\frac{B_m^2}{2\mu_0}$

Then

$$2\frac{B_m^2}{2\mu_0} A\delta x = F\delta x$$

$$F = \frac{B_m^2}{\mu_0} A = \frac{1.35^2 (0.01 \times 0.02)}{4\pi \times 10^{-7}} N = 290N$$

(c) If $N = 10^5$ turns, what current I is necessary for the force between magnet and keeper to be zero?

When the keeper is released $B_m=0$, $B_P=0$, $H_P=0$

Then

$H_m l_m = NI$

$$I_{release} = \frac{H_m l_m}{N}$$

From the Columax graph in the data-book, at $B_m=0$ we have $H_m \approx 5.9x10^4 Am^{-1}$

Thus

$I_{release}=0.37A$

(c) Does the direction of the current matter? What happens if *I* is increased beyond the value calculated in part (c)?

Yes the direction of I matters, otherwise the B from the coil would be on the same direction as the B from the Columax. These would add and saturate the Columax.

If I is in the correct direction to oppose the B from the Columax, then going above $I_{release}$ will turn the magnet in an electro-magnet and this will attract the keeper again.