ENGINEERING TRIPOS PART IA

Wednesday 2 June 2010 9 to 12

Paper 1

MECHANICAL ENGINEERING

Answer ***all*** *questions.*

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version: mpj4

SECTION A

1 (**short**) A tidal barrage is built to stop seawater flowing inland during high tides and to allow freshwater to flow out to sea during low tides. The tidal barrage is shown in Fig. 1. The barrage is $10\ \text{m}$ wide. Assume that the freshwater depth remains constant at $6\ \text{m}$. The hinge, at point O, is $1\ \text{m}$ above freshwater level. The densities of the freshwater and seawater are $\rho_f = 1000\ \text{kg m}^{-3}$ and $\rho_s = 1030\ \text{kg m}^{-3}$ respectively. The gravitational acceleration is 9.81 m s^{-2}.

(a) Prove that the total hydrostatic force on the seaward side of the barrage acts at an effective point $\frac{2}{3} h_s$ below the seawater surface. [4]

(b) Determine the total fluid force on the gate on the freshwater side, and its effective location. [3]

(c) Show that the gate opens when the seawater depth is $h_s = 5.89\ \text{m}$. [3]

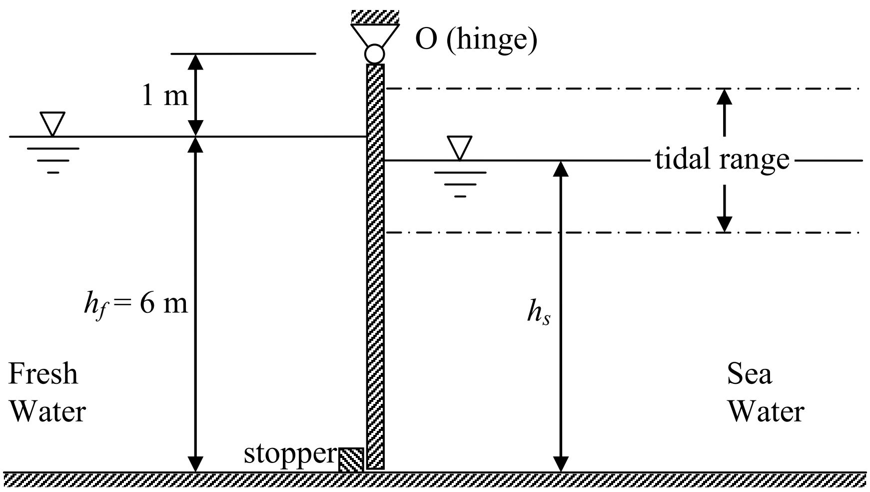

Fig. 1

2 (**short**) Water is discharged from a large container to the atmosphere through a pipeline whose diameter varies, as shown in Fig. 2. The outlet section (section B) is 2 m above the datum. At this section, the pipeline diameter is 25 mm and the flow speed is 5 m s^{-1} . Section A is 6 m above the datum. At this section, the pipeline diameter is 50 mm . Inside the container, the water is 5 m above the datum. The atmospheric pressure is 101×10^{3} Pa , the water density is 1000 kg m^{-3} and the gravitational acceleration is 9.81 m s^{-2}. Neglect any mechanical energy loss in the flow.

(a) Calculate the gauge pressure at section A. [5]

(b) By making appropriate assumptions, calculate the absolute air pressure above the water surface inside the closed container P_c. [5]

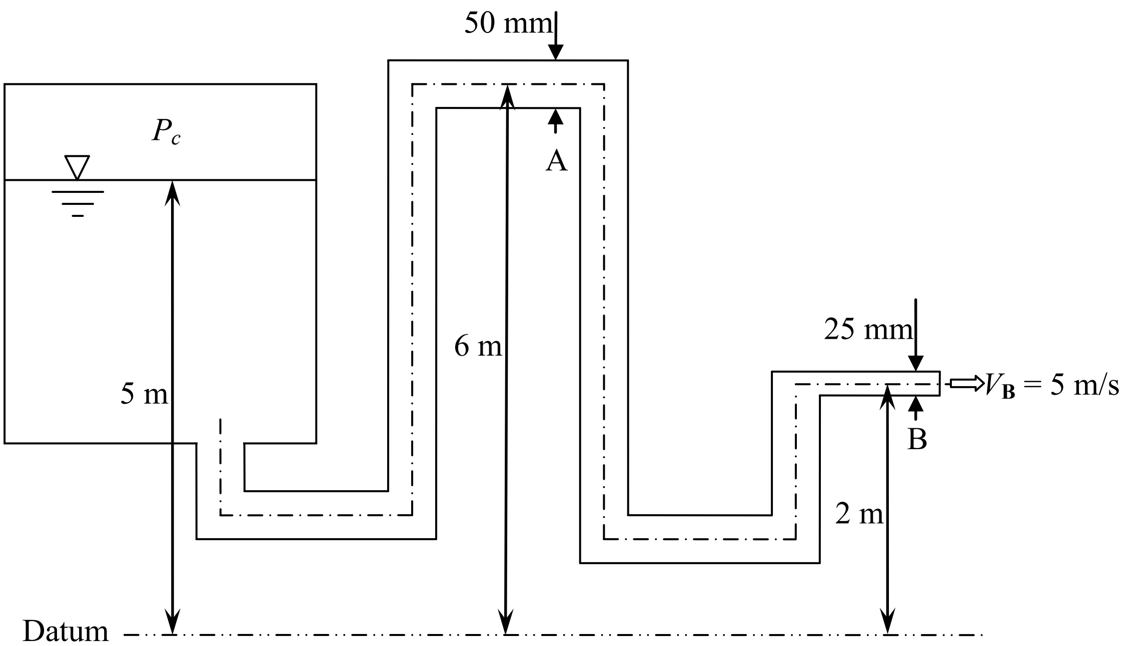

Fig. 2

3 (**short**) A heat engine extracts heat from a river at 10 °C and rejects heat to another river at 4 °C.

(a) Calculate the maximum possible efficiency for the heat engine. [3]

(b) The heat engine is used to drive a heat pump that supplies heat to a house at a temperature of 20 °C . Calculate the maximum COP_P for the heat pump. [3]

(c) Calculate the minimum rate at which heat must be extracted from the warmer river in order to deliver heat at a rate of 5 kW to the house. [4]

4 (**short**) A cylinder contains $0.1\ \mathrm{m}^3$ of CO_2, which is initially at a temperature of 298 K and is maintained at a pressure of 2×10^5 Pa by a weighted piston. You may assume that CO_2 can be treated as a perfect gas with specific gas constant $R = 189\ \mathrm{J\,kg^{-1}\,K^{-1}}$ and with specific heat capacity at constant volume $c_v = 630\ \mathrm{J\,kg^{-1}\,K^{-1}}$.

(a) Calculate the mass of CO_2 in the cylinder. [3]

(b) The CO_2 is heated until its volume doubles. Calculate:

(i) the final temperature of the CO_2 ; [2]

(ii) the work done by the CO_2 against the piston; [2]

(iii) the heat transferred to the CO_2 . [3]

5 (**long**) A hydraulic jump occurs in a horizontal channel with rectangular cross-section. The width of the channel is $W = 15$ m and the mass flowrate is $\dot{m} = 310 \times 10^3$ kg s^{-1}. A concrete block on the channel bed spans the entire channel width, and is submerged under the hydraulic jump, as shown in Fig. 3. At sections 1 and 2, which are upstream and downstream of the jump, the flow is uniform and steady. The water depths at sections 1 and 2 are $h_1 = 1.15$ m and $h_2 = 7.35$ m respectively. The water density is $\rho = 1000$ kg m^{-3} and the gravitational acceleration is $g = 9.81$ m s^{-2}.

(a) Explain why the pressure distribution at sections 1 and 2 may be taken as hydrostatic when these sections are not close to the concrete block. [6]

(b) Assuming that the bed is frictionless, show that the horizontal component of the net fluid force on the concrete block is

$$F = \frac{1}{2}\rho g W\left(h_1^2 - h_2^2\right) + \frac{\dot{m}^2}{\rho}\left(\frac{1}{Wh_1} - \frac{1}{Wh_2}\right).$$ [9]

(c) Calculate the value of F and show its direction. [3]

(d) For the streamline along the bottom of the flow, calculate the values of the Bernoulli constant at sections 1 and 2. Explain whether or not these values should be the same. [6]

(e) Calculate the rate of dissipation of mechanical energy over the jump. [6]

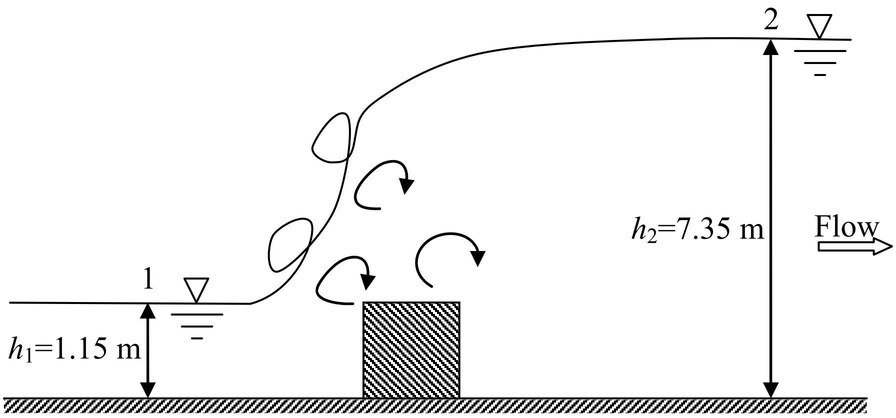

Fig. 3

6 (**long**)

(a) Starting from the first law of Thermodynamics and the definition of entropy

(i) show that, for a reversible process,

$$T\mathrm{d}s = \mathrm{d}u + p\mathrm{d}v\,. \qquad [4]$$

(ii) Furthermore, using the definition of enthalpy show that

$$T\mathrm{d}s = \mathrm{d}h - v\mathrm{d}p\,. \qquad [2]$$

(iii) Hence show that, for a perfect gas changing from thermodynamic state 1 to thermodynamic state 2,

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right). \qquad [6]$$

(iv) Explain why the preceding equation can also be applied to an irreversible process. [2]

(b) A compressor pumps air into a thermally-insulated vessel. Assuming that air can be treated as a perfect gas, that the compressor and pipes are thermally-insulated, and that the process is reversible, use the result from part (a) to show that the pressure, P, and temperature, T, in the vessel are related by

$$\frac{T}{P^{(\gamma-1)/\gamma}} = const\,,$$

where γ is the ratio of the specific heat capacities. [4]

(c) The air in the vessel has a volume of $15\ \mathrm{m}^3$ and is initially at a pressure of 1×10^5 Pa and a temperature of 290 K. Calculate the initial mass of air in the vessel. Air is added by the compressor until the pressure reaches 15×10^5 Pa. Calculate the final temperature and mass of air in the vessel. [4]

(d) The inlet to the compressor is at a pressure of 1×10^5 Pa and a temperature of 290 K. Calculate the total compressor work input during the process. [8]

SECTION B

7 (**short**) The system shown in Fig. 4 includes a straight rod AB of length l and mass m. A particle of mass m is located at point A and a particle of mass $2m$ is located at point B. The bar rotates around the fixed point C which is located at distance x from B.

(a) Show that the mass moment of inertia of the assembly about point C is given by

$$m\left(4x^2 - 3lx + \frac{4l^2}{3}\right)$$ [4]

(b) If x can be varied, determine the minimum possible moment of inertia of the assembly and the value of x that achieves this. [4]

(c) What value should x have in order for C to be at the centre of mass of the assembly? [2]

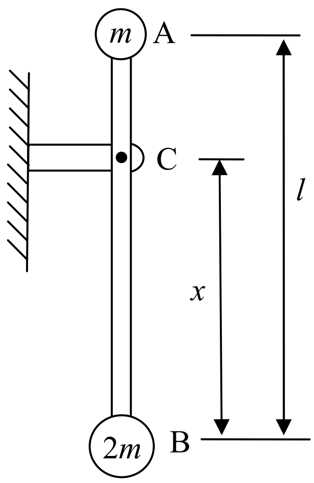

Fig. 4

8 (**short**) Figure 5 shows (in elevation) a system of frictionless pulleys, cables and masses. The two masses A and B are resting on the ground and the combined mass of the pulleys and cables is small compared to m and M.

(a) How large can M be for the system to be in equilibrium as shown? [2]

(b) Show that M would have to be more than $8m$ in order to lift both the resting masses off the ground when the system is released. [4]

(c) If $M = 4m$, find the vertical acceleration of mass A. [4]

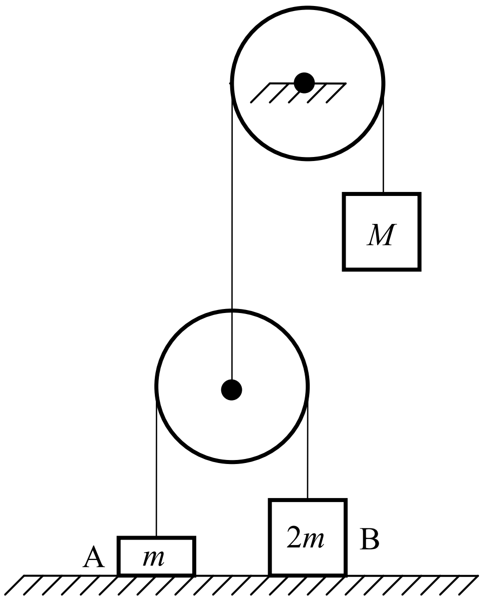

Fig. 5

9 (**short**) A hollow cone with a cone angle of 90° is held with its axis vertical, as shown in Fig. 6. A small particle of mass m slides freely around the frictionless inner surface of the cone, and its height is observed to vary between h and $2h$ above the apex.

(a) What are the maximum and minimum speeds of the particle? [5]

(b) The orbiting particle collides with a second, stationary particle when it is at the lowest point of its orbit. The two particles fuse together and the subsequent motion is a circular orbit at a constant height of h . What is the mass of the second particle? [5]

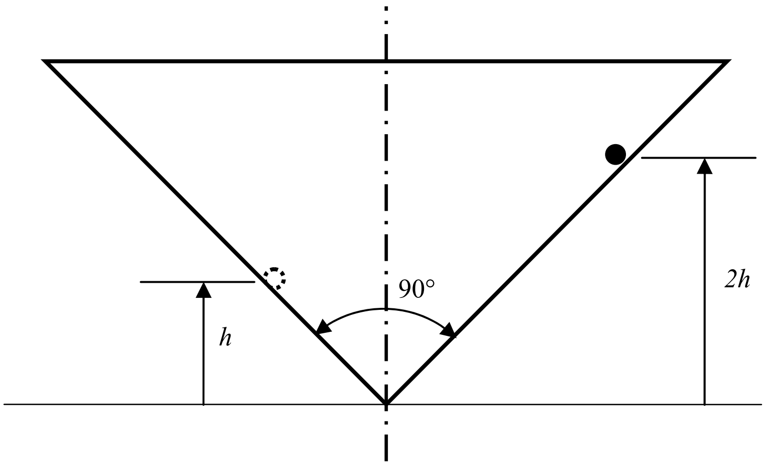

Fig. 6

(TURN OVER

10 (**short**) Figure 7 shows three rotors rigidly mounted on two shafts, each of torsional stiffness k, which are freely supported by two bearings.

(a) Write down the equations of angular motion for this system, in the form

$$[\mathbf{M}]\underline{\ddot{\theta}} + [\mathbf{K}]\underline{\theta} = 0$$

[5]

(b) By inspection, determine all the natural frequencies of this system. [5]

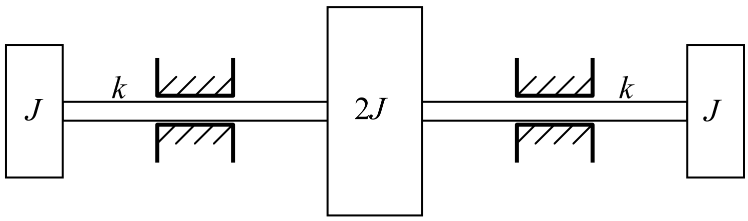

Fig. 7

11 (**long**) The planar mechanism in Fig. 8 is arranged in the vertical plane and is composed of two straight light rigid rods. Rod AC is of length $3l$ and rotates about the stationary pivot A . A point mass m is attached to this rod at C . Rod BE is of length $4l$ and rotates about its midpoint at the stationary pivot D , which is at the same height as pivot A . A slider on rod AC is attached by a pivot to rod BE at B . At the instant shown, both rods make an angle of 60º with the horizontal and rod BE is rotating clockwise with angular velocity ω. The inertia of the system may be neglected.

(a) By constructing a velocity diagram of the system,

(i) Determine the velocity of point C. [12]

(ii) Determine the angular velocity of rod AC. [4]

(b) The mechanism is being driven by a force F , which acts horizontally at point E , as indicated.

(i) If there is no friction in the mechanism, express F in terms of m and g . [6]

(ii) If there is a frictional force R resisting relative motion between the slider and the rod AC and a frictional couple T resisting rotation at each of the mechanism's three joints, express F in terms of m, g, R, l and T . [8]

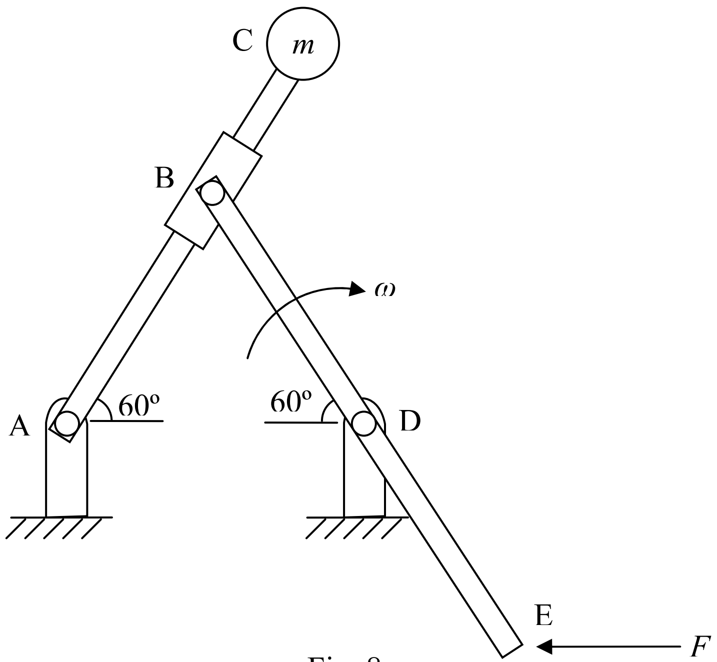

Fig. 8

12 (**long**) The circuit shown in Fig. 9 consists of a capacitor, resistor and inductor with values C, R and L respectively. An input voltage e is applied by means of a voltage generator and an output voltage v is measured across the inductor, as shown.

(a) Show that the differential equation relating the output voltage to the input voltage is:

$$LC\ddot{v} + RC\dot{v} + v = LC\ddot{e}$$

Use the electrical analogy set out in the lecture course to write down expressions for the undamped natural frequency ω_n and damping factor ζ of the circuit. [10]

(b) An input voltage of $e = E\cos\omega t$ is now applied to the circuit. If the value of ζ for the circuit is 0.2, estimate the two frequencies at which the amplitude of the output voltage is $2E$. Find also the phase angle between the input and output voltages at these two frequencies, stating clearly whether the output voltage is leading, or lagging, the input. [10]

(c) Explain the difference between the undamped natural frequency ω_n and the resonant frequency ω_r for such a circuit and calculate ω_r/ω_n when $\zeta = 0.2$. If the values of the other components remained unchanged, by what factor would R need to be increased in order to raise ω_r/ω_n to 1.5? [10]

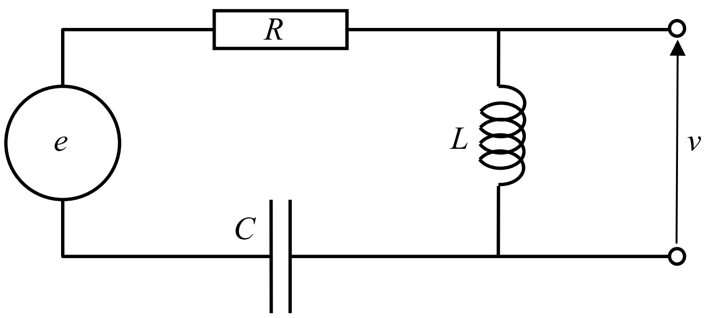

Fig. 9

END OF PAPER

Answers

1. (a) 2/3 *h*; (b) 5 m below hinge O; (c) 5.89 m.
2. (a) -27.5 kPa; (b) 84.07 kPa.
3. (a) 2.12%; (b) 29.3; (c) 12.88 kW.
4. (a) 0.355 kg; (b) 596 K, 20 kJ, 86.7 kJ.
5. (a) - (b) - (c) 822 kN; (d) 76.05 kPa; (e) 29.97 MW.
6. (a) - (b) - (c) 18.02 kg, 628.7 K, 124.7 kg; (d) 21.44 MJ.
7. (a) - (b) 37/48 ml^2; (c) 3/8 *l*.
8. (a) - (b) - (c) *g*/2.
9. (a) $\sqrt{\frac{2gh}{3}}$ $\sqrt{\frac{8gh}{3}}$ (b) 0.633 *m*.
10. (a) - (b) $\omega = 0$, $\omega = \sqrt{\frac{k}{J}}$, $\omega = \sqrt{\frac{2k}{J}}$.
11. (a) 3/2 $l\omega$, $\omega/2$; (b) $-\frac{\sqrt{3}}{4}mg$.
12. (a) - (b) $\frac{\omega}{\omega_n}$ = 0.90, 1.28; 62° and 142° leading (c) 2.64.

(TURN OVER