ENGINEERING TRIPOS PART IA

Thursday 3 June 2010 9 to 12

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version 2

SECTION A

1 **(short)** A plane pin-jointed truss is shown in Fig. 1. A horizontal load W is applied at joint C as shown in the figure. All members have the same cross-sectional area A and are made of a linear elastic material with Young's modulus E. The self-weight can be neglected.

(a) Find the bar forces due to the applied loading. [4]

(b) Find the extension of the distance of B from E under this loading using a displacement diagram. [6]

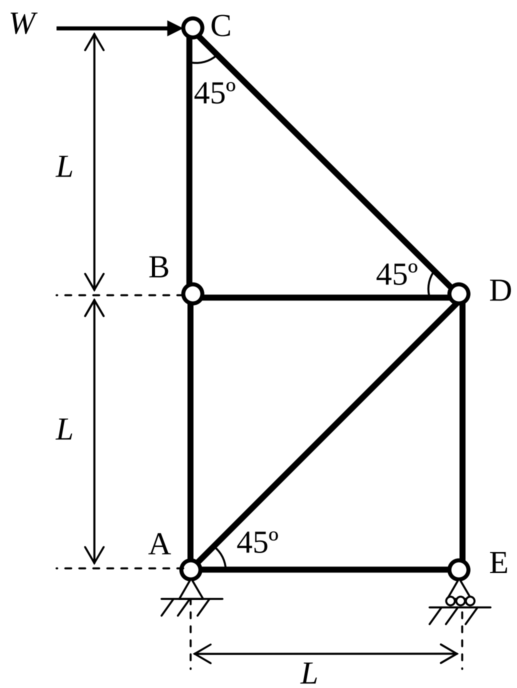


Fig. 1

2 **(short)** A simply supported beam of overall length 15 m and a uniform bending stiffness *EI* is shown in Fig. 2. The beam is loaded with a distributed load as well as two couples. Linear elastic behaviour is assumed. The self-weight can be neglected. Sketch the bending moment diagram and the shear force diagram along the beam, giving salient values. Clearly state the sign convention used. [10]

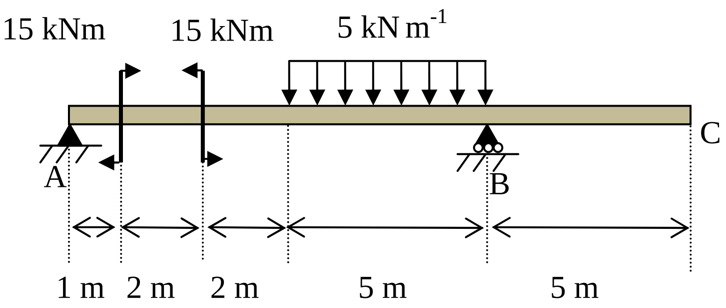


Fig. 2

3 **(short)** A three pin arch structure of height 12.5 m and width 10 m is shown in Fig. 3. The structure is symmetric and the shape of the structure for positive x is given by $y = (1/10)\, x^3$, where the origin is at the middle pin C. A vertical load of 20 kN is applied to the structure at location B shown in the figure.

(a) Find the reaction forces at pin A and D. [5]

(b) Find the location and magnitude of the maximum moment in section AB. [5]

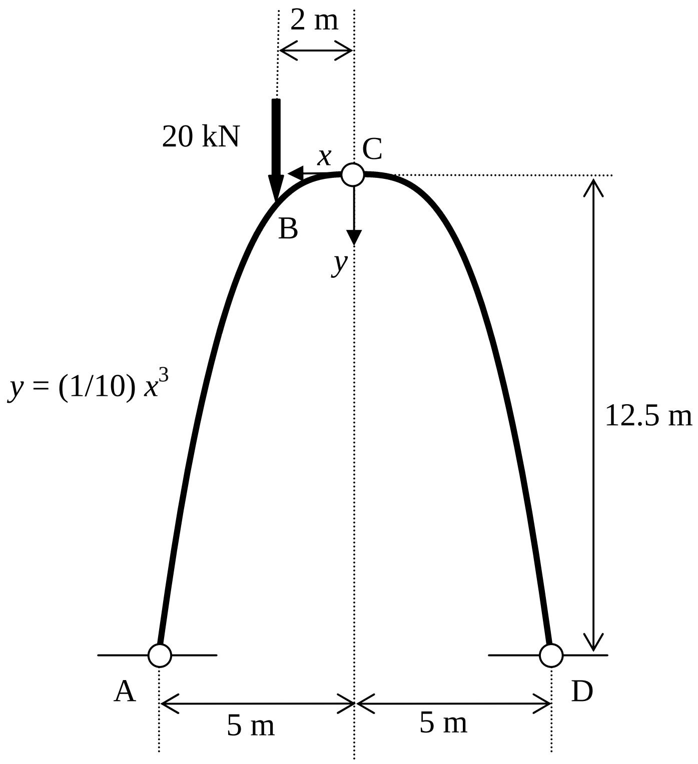


Fig. 3

4 **(short)** The cross-section of a steel I-beam is shown in Fig. 4. A vertical shear force of 10 kN and a bending moment of 25 kN m are applied about the *x*-*x* axis of the beam shown in the figure. Linear-elastic behaviour may be assumed.

(a) Find the longitudinal stress and the shear stress on the cross-section at point A, located on the outer surface of the flange. [6]

(b) Find the location and magnitude of the largest shear stress on the cross-section shown. [4]

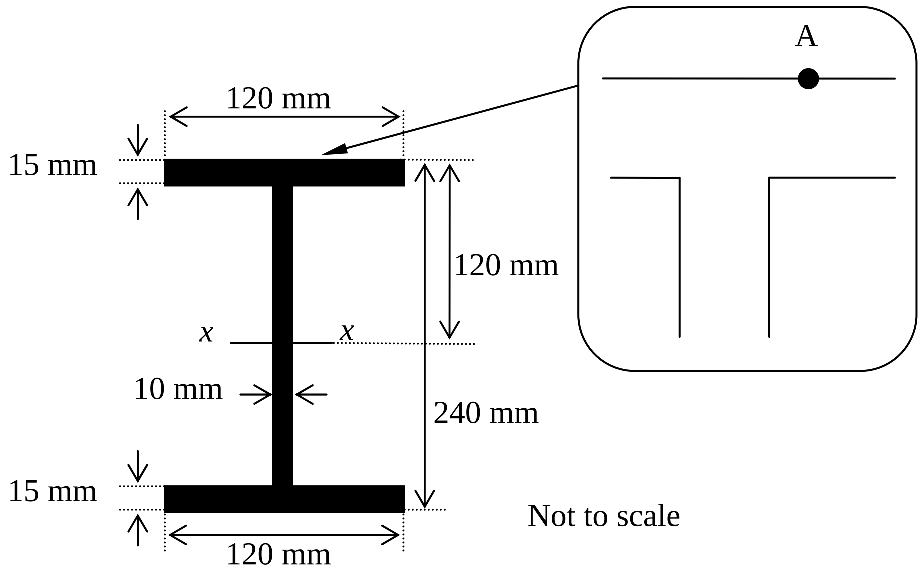


Fig. 4

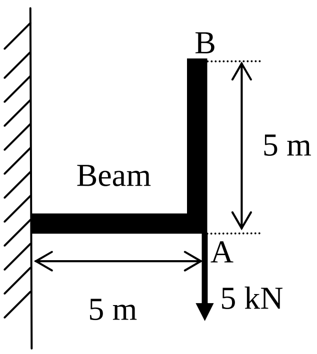
5 **(long)**

(a) An L-shaped beam is shown in Fig. 5(a). The beam is made of wood and its cross section is 0.5 m × 0.5 m. The material behaves elastically and the Young's modulus is 10×10^{9} N m^{-2}.

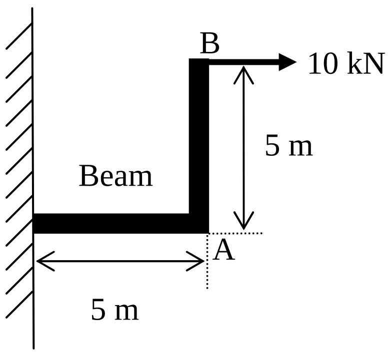
(i) A vertical load of 5 kN is applied at location A as shown in Fig 5(a). Find the vertical displacement at location A and the horizontal displacement at location B. [8]

(ii) A horizontal load of 10 kN is applied at location B as shown in Fig 5(b). Find the vertical displacement at location A and the horizontal displacement at location B. [8]

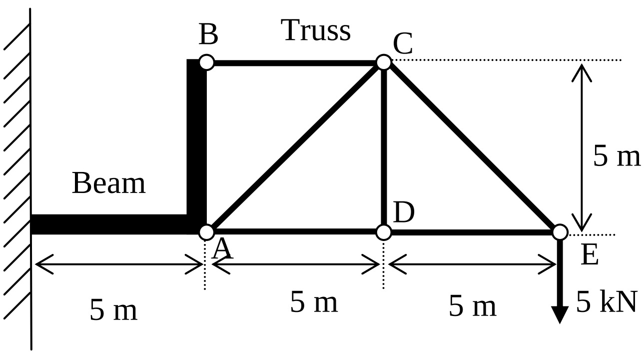
(b) A pin-jointed truss is now attached to the beam as shown in Fig. 5(c). The truss is made of steel and its cross-sectional area is 0.0001 m^{2}. The material behaves elastically and the Young's modulus is 210×10^{9} N m^{-2}. A vertical force of 5 kN is applied at location E of the truss as shown in the figure. Find the vertical and horizontal displacements at location E. [14]



(a)



(b)



(c)

Fig. 5

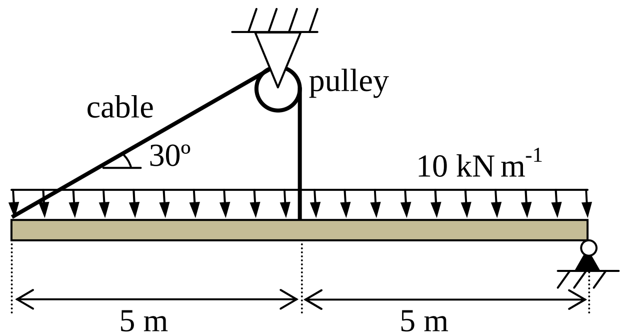
6 **(long)** A uniform beam of length 10 m is supported by a frictionless pin and a cable with a pulley as shown in Fig. 6(a). The pulley is free to rotate. The beam is a structure made of wood and aluminium alloy as shown in Fig. 6(b). The two materials are bonded by adhesive. Values of Young's modulus for the two materials are given in the Structures Data book. A distributed load of 10 kN m^{-1}, including the self weight, is applied along the whole length of the beam.

(a) Show that the tension in the cable is 50 kN. [5]

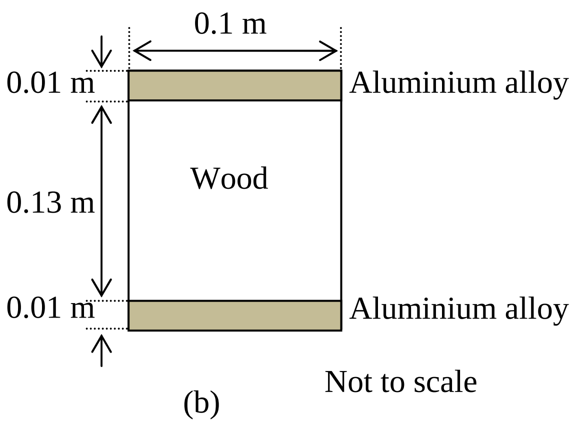
(b) Sketch the shear force diagram and the bending moment diagram along the beam, giving salient values. [10]

(c) At the location of the maximum bending moment, find the longitudinal stresses at the top and bottom surfaces of the beam. [10]

(d) Determine the longitudinal shear force per unit length of beam transmitted across the interface between the wood and the aluminium alloy at the location of the maximum shear force. [5]



(a)



(b)

Fig. 6

SECTION B

7 (**short**)

(a) Sketch the face-centred cubic (fcc) lattice structure. Why is this termed close-packed? What mechanical characteristics do fcc metals normally exhibit? [5]

(b) On your sketch of the fcc structure, identify a tetrahedral interstitial hole. Why are such holes important? Calculate the lengths of the edges of this tetrahedron, taking the diameter of the host spheres to be unity. [5]

8 (**short**)

(a) What determines the Young's modulus in metals? Why do metals generally have effectively the same Young's modulus for all loading directions? [3]

(b) Figure 7 illustrates a cube of linear elastic material with Young's modulus E and Poisson's ratio ν subject to stresses σ and $\sigma/2$ in the 1 and 2 directions, respectively.

(i) Write down expressions for the strains in the 1 and 3 directions, in terms of σ, E and ν. [2]

(ii) What stress would need additionally to be applied in the 3 direction to give a strain equal to σ/E in the 1 direction? [2]

(iii) Discuss qualitatively how the stresses in the cube in the 1 direction would vary with position if the top half of the cube was subjected to a uniform increase in temperature, with no external stresses now being applied to the cube. [3]

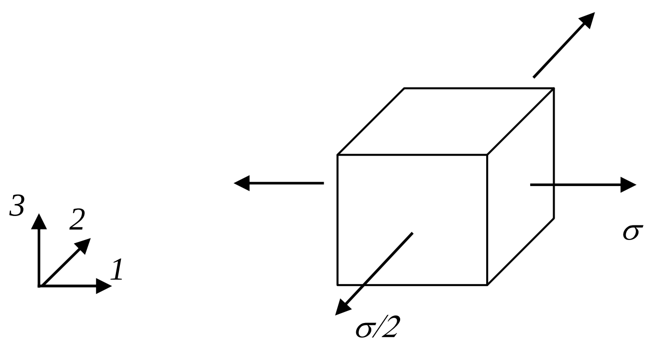


Fig. 7

9 (**short**) Figure 8 shows an idealisation of part of a boat hull. The panel, which is supported along its edges, has thickness t and side lengths L and carries a normal pressure load p.

(a) Describe how you would use the Cambridge Engineering Selector (CES) software to identify a shortlist of appropriate materials for the hull. [5]

(b) The maximum stress σ_m in the panel is given by

$$\sigma_m = \alpha \frac{pL^2}{t^2}$$

where α is a numerical constant. The side length L of the panel is fixed, but the thickness t can be varied. Derive an appropriate material performance index to minimise the cost of the hull, based on a strength criterion. Use the Materials Data Book to choose between aluminium alloy and stainless steel, based on this performance index. [5]

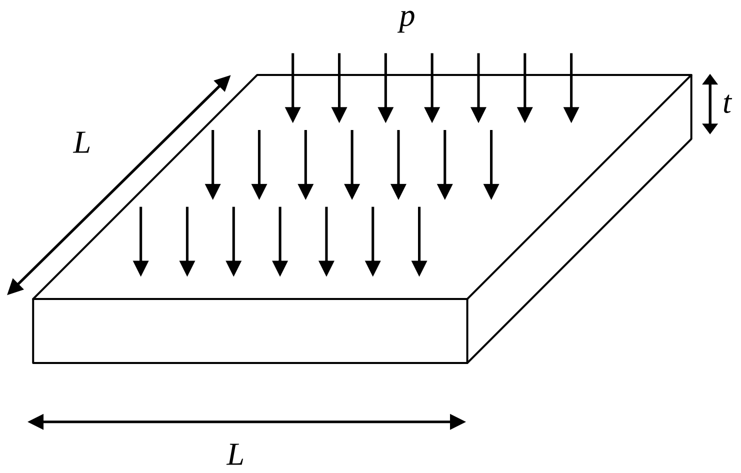


Fig. 8

(TURN OVER

10 (**short**) The variation of mass gain Δm with time t of a component made of metal M due to hot oxidation to form MO_2 is given in Table 1.

(a) Use this data to fit an appropriate expression for Δm as a function of t. Account for the observed form of dependence. [5]

(b) Hence estimate the mass gain Δm_{1000} after 1000 hours and the corresponding depth of metal lost to oxidation, in terms of Δm_{1000}, the ratio R of the metal to oxygen atomic weights, the surface area A of the component and any other relevant material properties. [5]

Time t (hours)	Mass gain Δm (g)
0	0
100	1.4
200	1.9
500	3.3

Table 1

11 (**long**)

(a) Outline a derivation of the formula given on page 6 of the Materials Data Book relating the stress intensity factor K and the strain energy release rate G. There is no need to account for the effect of Poisson's ratio ν. [6]

(b) Figure 9 illustrates an architectural feature, idealised as a cylindrical column of density ρ, length L and diameter $0.05\,L$ hanging vertically downwards under self-weight loading due to a gravitational acceleration g. Failure of the material can be modelled using Weibull statistics, with relevant material properties V_0, σ_0 and m, as defined in the Materials Data Book.

(i) Derive the following expression for the probability of survival P_s of the column:

$$P_s = \exp\left[-\left(\frac{\rho g}{\sigma_0}\right)^m \frac{\pi(0.05L)^2}{4V_0}\frac{L^{m+1}}{m+1}\right] \qquad [10]$$

(ii) Samples of the same material of volume 0.2 m^3, subjected to uniform tensile stresses of 0.0848 or 0.1 MPa, were found to have survival probabilities of 0.98 and 0.9, respectively. Use this data to calculate the column length required to have a survival probability greater than 0.99. The material density is 2 Mg m^{-3}. [10]

(iii) What practical problems might you expect with the uniform tensile tests described in part (b)(ii) used to characterise the material behaviour? Suggest an alternative testing approach. [4]

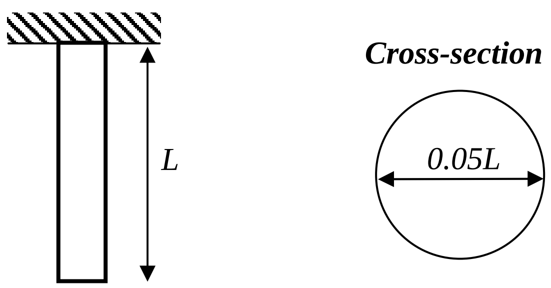


Fig. 9

12 (**long**)

(a) Describe hardness testing, explaining why it is widely used and identifying any potential practical difficulties. How can a hardness test be used to estimate yield stress? [5]

(b) List four potential hardening mechanisms in crystalline metals. Describe, with the help of sketches, how each of the mechanisms works. [8]

(c) Figure 10 (a) illustrates a two-stage process, where an initially unstrained sample of annealed 70/30 brass (a) is compressed to a deformed state (b) and then undergoes hardness testing from (b) to (c). A small square element A (enlarged in the figure for clarity), labelled Aa in (a), is deformed in the process to the corresponding elements Ab and Ac in subsequent steps. The side lengths of the element at each stage, in arbitrary units, are given in the figure, which also includes x and y axes in the horizontal and vertical directions, respectively.

(i) Calculate the nominal and true strains of element A in the vertical y direction caused by each of the deformation stages, i.e. for the deformation from (a) to (b) and for the hardness test from (b) to (c). Calculate also the nominal and true strains for the overall deformation from (a) to (c). Are there any simple relationships that hold between any of these answers? [7]

(ii) Figure 2.1 in the Materials Data Book gives the appropriate nominal stress versus nominal strain data for the material. Use this to estimate the uniform compressive vertical stresses on the top and bottom faces of specimen (b) which would be required to deform it further. [7]

(iii) How would you expect the element length to change in the direction out of the plane of the figure during these two processes? [3]

(cont.

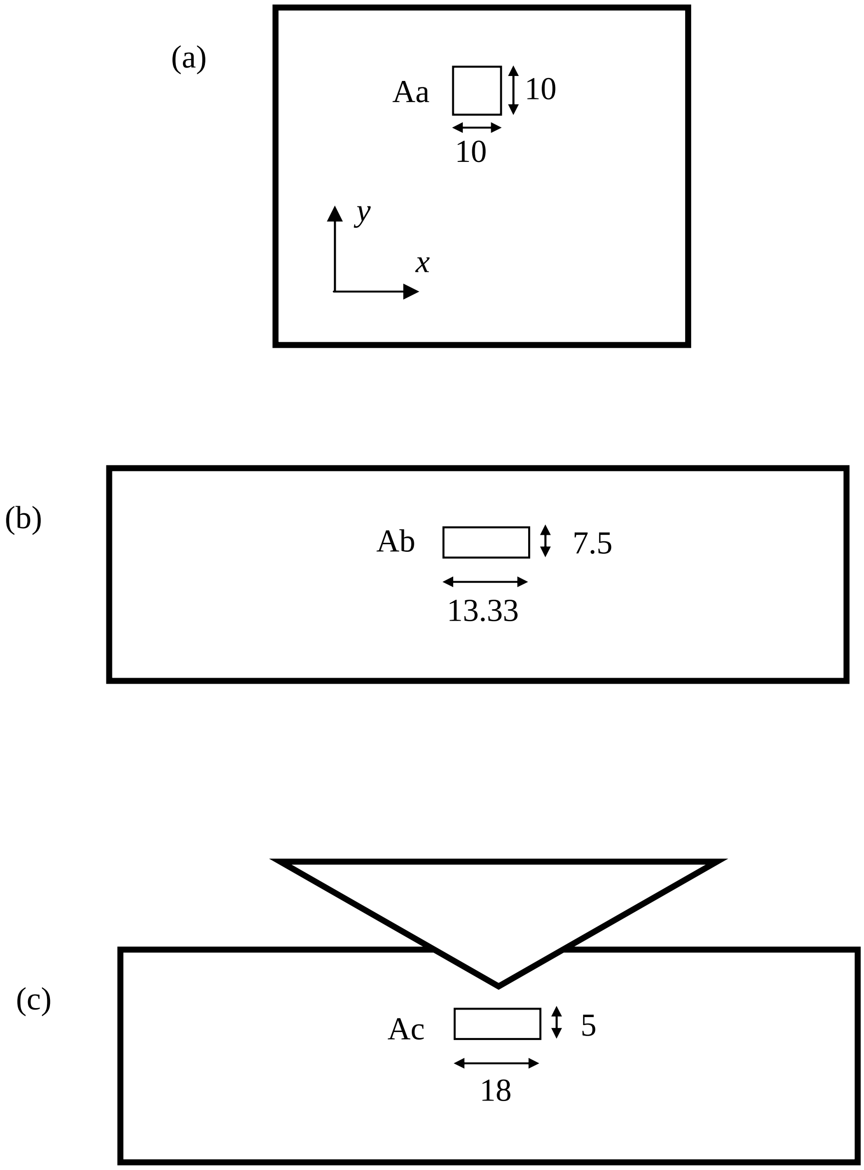


Fig. 10

END OF PAPER