

Engineering Tripos Part IA 2011

Paper 4: Mathematical Methods

Short Answers

1. $\frac{x^2}{4} + \frac{y^2}{3} = 1$
2. (a) $-1/8$ (b) $1/2$
3. Eigenvalues $1, -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$ Eigenvector $(1,1,1)$ or $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$
4. (a) $\sqrt{35}$
5. (a) Volume = $80 - 4t$ litres, Concentration $\frac{x}{80-4t}$ kg/litre
(b) $20-t - \frac{(20-t)^3}{20^2}$ (c) $\frac{15}{2}$
(d) $\frac{40}{3\sqrt{3}}$
6. (b) Step Response $1 - (t+1)e^{-t}$ Impulse Response te^{-t}
(c) $\frac{e^{-\alpha} - e^{-t}}{(\alpha-1)^2} + \frac{te^{-\alpha}}{\alpha-1}$ (d) $\frac{t^2}{2}e^{-t}$
7. $\sum_{n=1}^{\infty} \frac{2}{n\pi} (1 - \cos nT) \sin nt$
8. (a) .24 (b) .45
9. $\frac{11}{10}e^{-3t} - \frac{1}{10}\cos t + \frac{3}{10}\sin t$
10. Maximum $\left(\frac{1-\sqrt{13}}{3}, 0\right)$, Minimum $\left(\frac{1+\sqrt{13}}{3}, 0\right)$, Saddle Points $(1, \pm\sqrt{3})$
11. (b) Change "while (t2 != x)" to "while (t2 != x && m > 1)"
12. (c) $O(n)$