SECTION A

1 (**long**)

An AC generator of RMS voltage V and internal impedance $Z_1\angle\theta$ is connected to a load of impedance $Z_2\angle\varphi$ as shown in Fig 1.

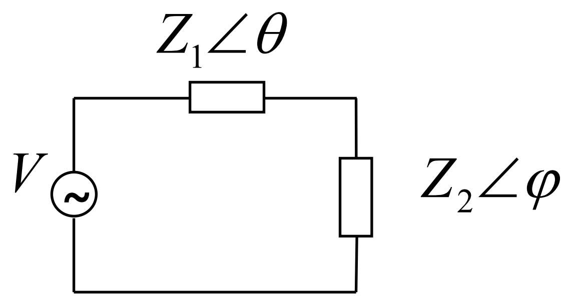


Fig. 1

(a) Show that the average power dissipated in the load is given by:

$$P = \frac{V^2 Z_2 \cos\varphi}{Z_1^2 + Z_2^2 + 2Z_1Z_2\cos(\theta-\varphi)}$$

The current in the circuit is given by:

$$I = \frac{V}{Z_1 + Z_2} = \frac{V}{Z_1\cos\theta + jZ_1\sin\theta + Z_2\cos\varphi + jZ_2\sin\varphi}$$

Hence $|I| = \dfrac{V}{\left[\left(Z_1\cos\theta + Z_2\cos\varphi\right)^2 + \left(Z_1\sin\theta + Z_2\sin\varphi\right)^2\right]^{1/2}}$

The average Power $P = I^2R = I^2\,\mathrm{Re}(Z_2) = I^2 Z_2\cos\varphi$

Hence $P = \dfrac{V^2 Z_2\cos\varphi}{Z_1^2 + Z_2^2 + 2Z_1Z_2\left(\cos\theta\cos\varphi + \sin\theta\sin\varphi\right)} = \dfrac{V^2 Z_2\cos\varphi}{Z_1^2 + Z_2^2 + 2Z_1Z_2\cos(\theta-\varphi)}$

(b) The phase φ of the load is held constant, and the magnitude Z_2 of the load is varied. Show that the condition for maximum power to be transferred to the load is $Z_2 = Z_1$.

To find the maximum power as a function of Z_2, we need to differentiate P with respect to Z_2 and set equal to zero.

Thus

$$\frac{dP}{dZ_2} = V^2 \left\{ \frac{\cos\varphi}{Z_1^2 + Z_2^2 + 2Z_1 Z_2 \cos(\theta - \varphi)} - \frac{Z_2 \cos\varphi}{\left[Z_1^2 + Z_2^2 + 2Z_1 Z_2 \cos(\theta - \varphi)\right]^2} \left[2Z_2 + 2Z_1 \cos(\theta - \varphi)\right] \right\} = 0$$

Hence $\left[Z_1^2 + Z_2^2 + 2Z_1 Z_2 \cos(\theta - \varphi)\right]\cos\varphi - Z_2 \cos\varphi \left[2Z_2 + 2Z_1 \cos(\theta - \varphi)\right] = 0$

Thus: $\left(Z_1^2 - Z_2^2\right)\cos\varphi = 0$ giving $Z_1 = Z_2$

(c) The magnitude of the load is kept constant, but the phase φ is varied. Show that the condition for maximum power to be transferred to the load is now:

$$\sin\varphi = -\frac{2Z_1 Z_2}{Z_1^2 + Z_2^2} \sin\theta$$

Setting

$$\left.\frac{dP}{d\varphi}\right|_{Z_2 = const} = 0$$

Gives:

$$-V^2 Z_2 \sin\varphi \left[Z_1^2 + Z_2^2 + 2Z_1 Z_2 \cos(\theta - \varphi)\right] - V^2 Z_2 \cos\varphi \left[2Z_1 Z_2 \sin(\theta - \varphi)\right] = 0$$

Hence

$$\sin\varphi \left[Z_1^2 + Z_2^2\right] + 2Z_1 Z_2 \left[\sin\varphi \cos(\theta - \varphi) + \cos\varphi \sin(\theta - \varphi)\right] = 0$$

Remembering that: $\sin(A + B) = \sin A \cos B + \cos A \sin B$

We get:

$$\sin\varphi \cos(\theta - \varphi) + \cos\varphi \sin(\theta - \varphi) = \sin\left[\varphi + (\theta - \varphi)\right] = \sin\theta$$

Thus

$$\sin\varphi = -\frac{2Z_1 Z_2}{Z_1^2 + Z_2^2} \sin\theta$$

(d) Using the results from (b) or (c), or otherwise, show that, if the magnitude and phase of the load are both varied, the condition for maximum power to be transferred to the load is:

$$Z_2\angle\varphi = Z_1\angle-\theta$$

From (b) $Z_2 = Z_1$

$$\Rightarrow\ P = \frac{V^2 Z_2}{2Z_2^2}\frac{\cos\varphi}{1+\cos(\theta-\varphi)}$$

$$\frac{dP}{d\varphi} = 0$$

$$\Rightarrow -\sin\varphi\left[1+\cos(\theta-\varphi)\right] - \cos\varphi\left[\sin(\theta-\varphi)\right] = 0$$

$$-\sin\varphi - \left[\sin\varphi\cos(\theta-\varphi) + \cos\varphi\sin(\theta-\varphi)\right] = 0$$

$$-\sin\varphi - \left[\sin\theta\right] = 0$$

$$-\sin\varphi = \sin\theta$$

$$\varphi = -\theta$$

$$Z_2\angle\varphi = Z_1\angle-\theta$$

Alternatively

From (b) $Z_2 = Z_1$

From (c) $\sin\varphi = -\dfrac{2Z_1Z_2}{Z_1^2+Z_2^2}\sin\theta$

$$\Rightarrow \sin\varphi = -\sin\theta$$

$$\Rightarrow Z_2\angle\varphi = Z_1\angle-\theta$$

2 **(long)** A FET is configured in an amplifier as shown in Fig. 2. The impedances of C_{in}, C_{out} and C_s are negligible at mid-band frequencies. The small signal parameters of the FET are g_m = 10 mA V^{-1} and r_D = 25 kΩ, and its operating point is given by V_{ds} = 15 V, V_{gs} = –2 V and I_{ds} = 1 mA

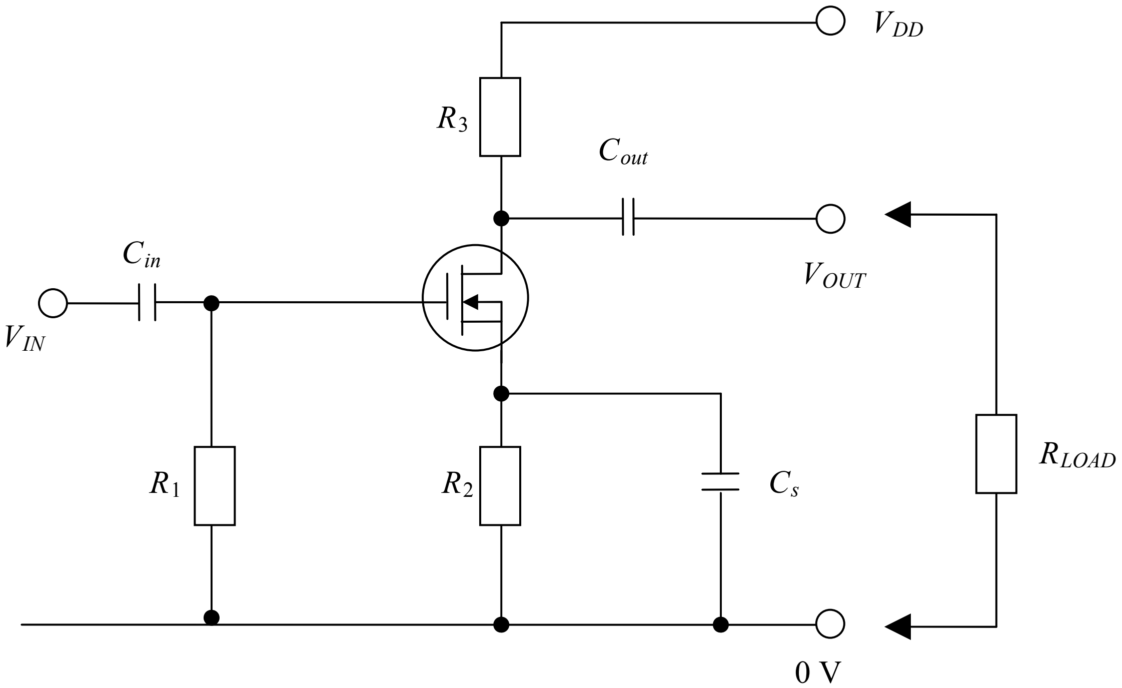


Fig. 2

(a) At the given operating point, calculate the values for R_2 and V_{DD} given that R_3 = 20 kΩ and R_1 = 1 MΩ.

V_{gs}= -2V must be the voltage across $R_2 \Rightarrow V_{gs} = R_2 I_{ds}$

$$\Rightarrow R_2 = \frac{2V}{10^{-3}A} = 2k\Omega$$

Voltage across R_2 = 2V

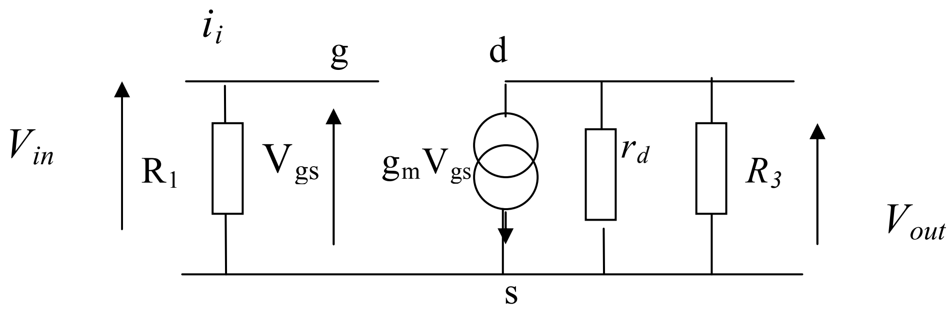
Voltage across FET = 15V

$\Rightarrow R_3 I_{ds}$ = 20kΩ.1mA = 20V

Therefore: $V_{DD} = 37V$

(b) For the circuit in Fig. 2, but without the load connected, draw the small signal equivalent circuit. Hence calculate its voltage gain, and also its input and output impedances, all at mid-band frequencies.

Capacitive impedances are negligible



Input impedance = R_1=1MΩ

Ro =output impedance = $r_d \parallel R_3$

$$\frac{1}{R_o} = \frac{1}{r_d} + \frac{1}{R_3}$$

$$R_o = \frac{r_d R_3}{r_d + R_3} = 11.1k\Omega$$

GAIN = $g_m.R_0 = -111.1$

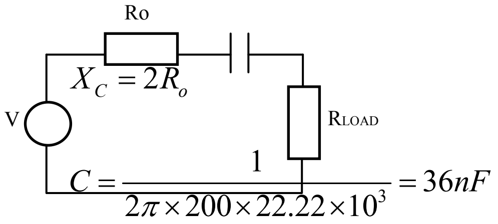
(c) What value of external load resistance, R_{LOAD} , should be connected between the output terminal of the amplifier and ground in order to maximise the signal power in the load?

For max power

$$R_{LOAD} = R_o = 11.1k\Omega$$

(d) If the lower 3 dB cut-off frequency of the amplifier is 200 Hz and is dominated by the effect of C_{out}, calculate the value of this capacitor, assuming that R_{LOAD} of part (c) is connected.

At the 3dB point, the Thevenin equivalent of load circuits is:

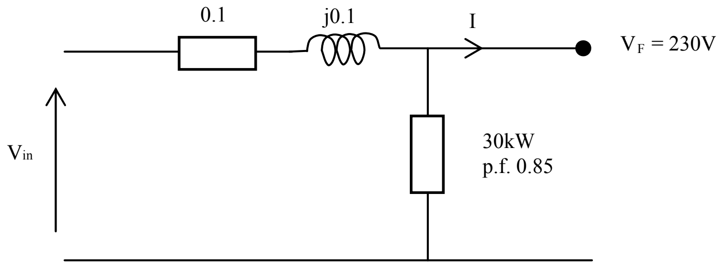


$$X_C = 2R_o$$

$$C = \frac{1}{2\pi \times 200 \times 22.22 \times 10^3} = 36nF$$

3 (**short**) A small factory consumes 30kW power at 230 V, with a lagging factor of 0.85. The line supplying the factory has an impedance 0.1+j0.1Ω. The frequency is 50 Hz

(a) Draw a circuit diagram for the above system. Calculate the power loss in the line, and the voltage at the supply end of the line.



$$230 \times I \times \cos\phi = P$$

$$\Rightarrow I = \frac{30.10^3}{2300.85} = 153.45A$$

$$\frac{Q}{P} = \tan\phi$$

$$\Rightarrow Q_{LOAD} = P_{LOAD}\tan\phi = P_{LOAD}\frac{\sqrt{1-\cos^2\varphi}}{\cos\varphi} = 18692VAR$$

$P_{LINE} = I^2R = 153.45^2 \times 0.1 = 2355W$

$Q_{LINE} = I^2X = 153.55^2 \times 0.1 = 2355VAR$

Input P $= 30000 + 2355 = 32355W$

Input Q $= 18592 + 2355 = 20957VAR$
But: $(VA)^2 = P^2 + Q^2$

$\Rightarrow$ Input VA=38544VA

Input $V = \frac{VA}{I} = 251.2V$

(b) What is the minimum power lost in the line if the power factor correction is applied to the factory, and the factory voltage remains 230 V?

Min power in line when p.f. = 1.
$\Rightarrow \cos\varphi = 1$

$P_{Line} = VI\cos\varphi = VI$

$\Rightarrow I = \frac{30kW}{230} = 130.43$ A

Thus

$P_{Line} = I^2 \times 0.1 = 1.701$ kW

4 (**short**)

(a) State what assumptions have to be made to be able to describe a transformer as ideal

1) Reluctance of the iron core = 0
2) All flux in primary links the secondary
3) Winding resistances are Zero
4) No power losses in the iron core

(b) The characteristics of a non-ideal transformer are determined by performing tests with the low voltage secondary open and short circuited. These give

Open Circuit Test

$V_{PRIMARY}$=260 V, $I_{PRIMARY}$=0.6 A, P= 50 W, $V_{SECONDARY}$= 130 V

Short Circuit Test

$V_{PRIMARY}$=50 V, $I_{PRIMARY}$=6 A, P= 50 W

Determine the values of the equivalent circuit parameters (referred to the primary side of the transformer)

$$R_o = \frac{V^2}{P} = 1352\Omega$$

$$X_0 = \frac{V^2}{Q} = \frac{V^2}{\sqrt{(VI)^2 - P^2}} = 457.5\Omega$$

Turns Ratio=2

$$\mathrm{R}_t = \frac{P}{I^2} = 1.39\Omega$$

$$X_t = \frac{Q}{I^2} = \frac{\sqrt{(VI)^2 - P^2}}{I^2} = 8.21\Omega$$

5 (**short**)

(a) Explain what is meant by a Thevenin equivalent circuit. Draw the Thevenin equivalent circuit for the circuit shown in Fig. 3(a), and derive expressions for the Thevenin voltage and impedance.

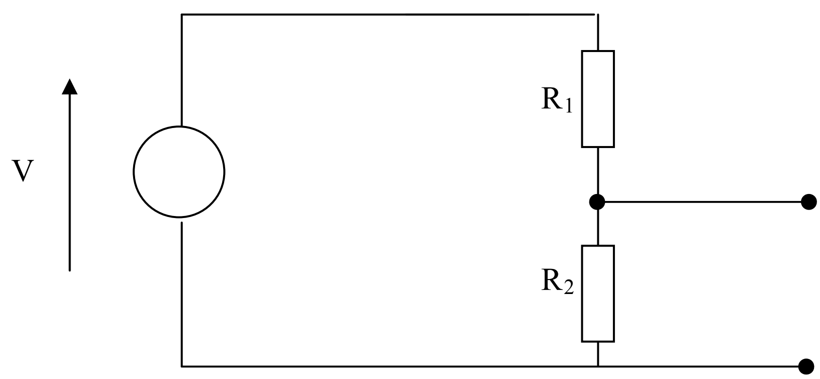
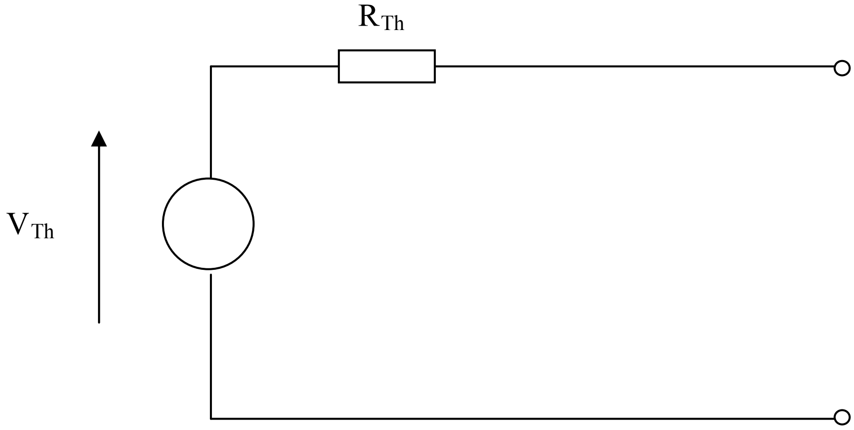


Fig. 3(a)

Thevenin: Any linear circuit may be represented as:



$V_{Th} = V_{Open\ Circuit}$

$R_{Th} = V_{Open\ Circuit} / I_{Short\ Circuit}$

Thus, the Thevenin equivalent for the circuit in Fig. 3(a) has the following parameters:

$$V_{Th} = V_{OC} = \frac{R_2}{R_1 + R_2} V$$

$$I_{SC} = \frac{V}{R_1}$$

$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2}$$

(b) In the circuit of Fig. 3(b), R= 200Ω, L=40 mH and C= 160μF. By applying Thevenin's theorem, or otherwise, determine the RMS magnitude of the current flowing in the capacitor C, its peak value, and its phase with respect to the 150 V voltage source.

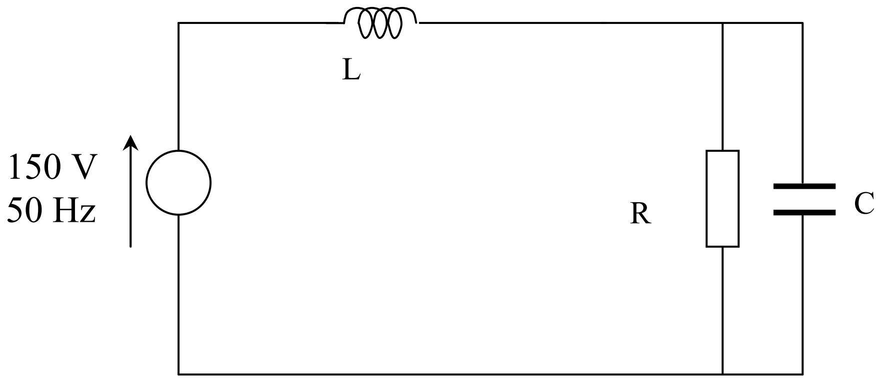


Fig. 3(b)

$$\overline{Z}_L = j\omega L = j12.6\Omega$$

$$\overline{Z}_C = \frac{1}{j\omega C} = -j19.9\Omega$$

Regarding $\overline{Z}_C$ as load

$$\overline{V}_{Th} = \frac{200}{200 + j12.6} \times 150V = (149.41 - j9.41)V = 149.71\angle -3.61V$$

$$\overline{Z}_{Th} = \frac{200 \times j12.6}{200 + j12.6} = (0.79 + j12.55)\Omega = 12.57\angle 86.3\Omega$$

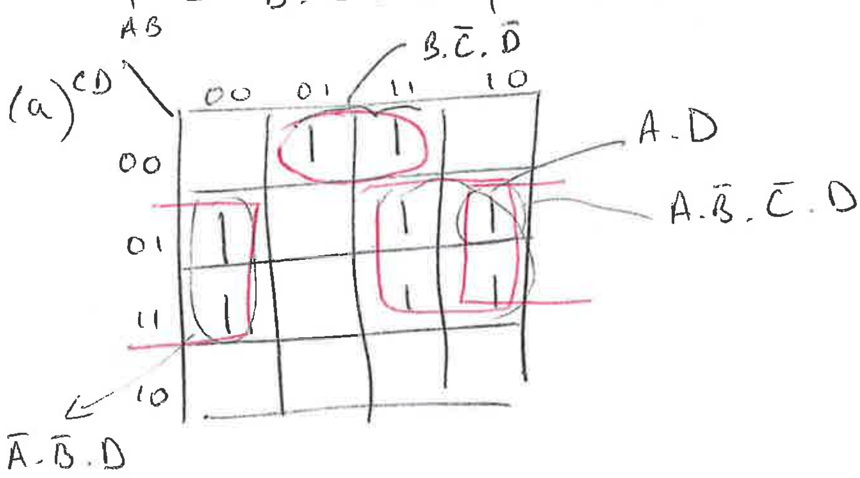
$$\overline{I} = \frac{149.71\angle -3.61}{(0.79 - j7.35)} = 20.26\angle 80.25A$$

$$\hat{I}_C = \sqrt{2} I_{C_{RMS}} = 28.65A$$

Section B.

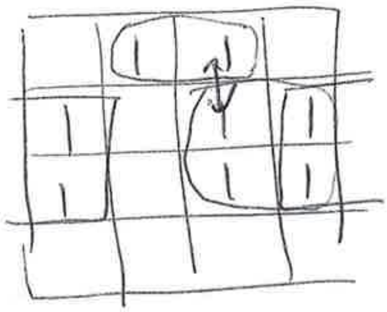
6. $F = B.\bar{C}.\bar{D} + A.D + A.\bar{B}.\bar{C}.\bar{D} + \bar{A}.\bar{B}.D$

(a)



Simplest SOP $F = B.\bar{C}.\bar{D} + A.D + \bar{B}.D$

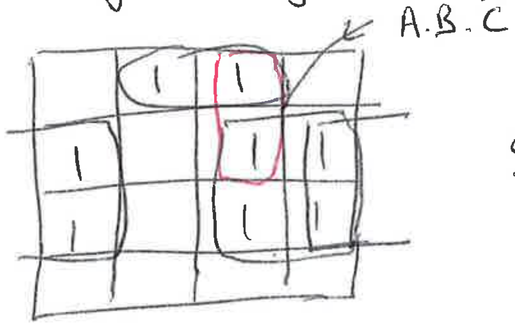
(b)



If $A = B = 1$ and $C = 0$, changing D from $0 \to 1$ or $1 \to 0$ should not have an effect according to the Boolean expression (ie F should be 1). However races in the circuit may mean that F temporarily drops to 0 before returning to 1

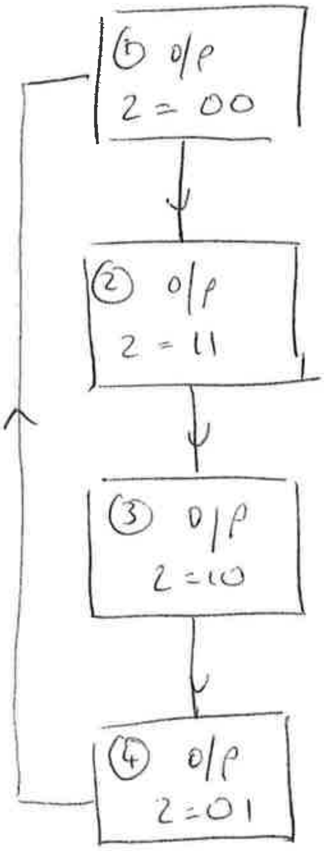
This is known as a static 1-hazard.

(c) Fix by adding and extra term to the SOP expression



So $F = B.\bar{C}.\bar{D} + A.D + \bar{B}.D + A.B.\bar{C}$

7(a) Sequence 00, 11, 10, 01



Note no inputs (apart from clock) since sequence generator

4 states ⇒ 2 bistables needed.

(b) Assign JK(A) to MSB & JK(B) to LSB

Current State	Next State	Bistable Inputs J_A	K_A	J_B	K_B
00	11	1	X	1	X
11	10	X	0	X	1
10	01	X	1	1	X
01	00	0	X	X	1

(c)

J_A

$Q_B \backslash Q_A$	0	1
0	1	X
1	0	X

$J_A = \bar{Q}_B$

K_A

$Q_B \backslash Q_A$	0	1
0	X	1
1	X	0

$K_A = \bar{Q}_B$

J_B

$Q_B \backslash Q_A$	0	1
0	1	1
1	X	X

$J_B = 1$

K_B

$Q_B \backslash Q_A$		
	X	X
	1	1

$K_B = 1$

8(a)

Cycles		
1	movlw 127;	puts 127 (decimal) into W
1	movwf 0x30;	puts content of W(127) in loc^n 0x30
1	subwf 0x30,W;	subtracts contents of W (127) from contents of 0x30 (also 127) ⇒ ans = 0 places ans in W, leaving 0x30 contents (127) untouched.
1	movwf ~~0x30~~ 255;	places 255 (decimal) into W
1	andwf 0x30;	bitwise ANDs content of W and 0x30 for ans = 127 (0x7F) answer into 0x30 (by default) decimal
1 (since skip doesn't happen)	btfsc 0x30,2;	test bit 2 of 0x30 and skips if bit 2 is clear (which it isn't) → so doesn't skip
2	goto end ;	goes to end label.
(would both be 1 but not used)	nop ; nop ;	} no operations – not executed.
1	end sleep;	sleep.

a) i) For subwf 0x30, W, need to turn contents of W into −W (ie invert all bits & add 1)

so W	0 1 1 1 1 1 1 1
→ (−W)	1 0 0 0 0 0 0 1
and add 127	+ 0 1 1 1 1 1 1 1
(1)	0 0 0 0 0 0 0 0

note carry into carry bit ; note carry from low nibble to high nibble

So contents of W = 00 (hex)
" " 0x30 = 7F "

C, DC & Z flags all set (=1)

ii)

W	1 1 1 1 1 1 1 1
0x30	0 1 1 1 1 1 1 1
bitwise AND	0 1 1 1 1 1 1 1

Note no carries for (C) & between nibbles ; into

} after instruction
contents of W = FF (hex)
" " 0x30 = 7F "
C, DC, & Z flags all clear (=0)

(b) Code takes 9 clock cycles to execute (see table above). 1 clock cycle = 100 ns
⇒ Execute time = 900 ns.

9 Map of outputs $Z = 2X + Y$

$Y \backslash X$	00	01	11	10
00	0 (0000)	2 (0010)	6 (0110)	4 (0100)
01	1 (0001)	3 (0011)	7 (0111)	5 (0101)
11	3 (0011)	5 (0101)	9 (1001)	7 (0111)
10	2 (0010)	4 (0100)	8 (1000)	6 (0110)

(a) So for each output $Z_0 - Z_3$

Z_0

$Y_1Y_0 \backslash X_1X_0$	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

$Z_0 = Y_0$

Z_1

0	1	1	0
0	1	1	0
1	0	0	1
1	0	0	1

$Z_1 = X_0 \bar{Y}_1 + \bar{X}_0 Y_1$

Z_2

0	0	1	1
0	0	1	1
0	1	0	1
0	1	0	1

$Z_2 = X_1 \bar{Y}_1 + X_1 \bar{X}_0 + \bar{X}_1 X_0 Y_1$

Z_3

0	0	0	0
0	0	0	0
0	0	1	0
0	0	1	0

$Z_3 = X_1 X_0 Y_1$

(b) Write down expression for $\bar{Z}_2$ & use De Morgan

0	0	1	1
0	0	1	1
0	1	0	1
0	1	0	1

$$\bar{Z}_2 = \bar{X}_1 \bar{X}_0 + \bar{X}_1 \bar{Y}_1 + X_1 X_0 Y_1$$

$$= \overline{X_1 + X_0} + \overline{X_1 + Y_1} + \overline{\bar{X}_1 + \bar{X}_0 + \bar{Y}_1} \quad \text{(De Morgan)}$$

$$\bar{Z}_2 = \overline{(X_1 + X_0) \cdot (X_1 + Y_1) \cdot (\bar{X}_1 + \bar{X}_0 + \bar{Y}_1)}$$

$$Z_2 = (X_1 + X_0) \cdot (X_1 + Y_1) \cdot (\bar{X}_1 + \bar{X}_0 + \bar{Y}_1)$$

(ii). For Z_0, Z_1

$$Z_0 = Y_0 \implies Z_0 = \overline{\overline{Y_0}}$$

$$Z_1 = Y_0\overline{Y_1} + \overline{X_0}Y_1$$

$$= \overline{\overline{Y_0\overline{Y_1}} \cdot \overline{\overline{X_0}Y_1}} \quad \text{(De Morgan)}$$

For Z_2, Z_3

$$\overline{Z_2} = \overline{X_1}\,\overline{X_0} + \overline{X_1}\,\overline{Y_1} + X_1X_0Y_1$$

$$= \overline{X_1 + X_0} + \overline{X_1 + Y_1} + \overline{\overline{X_1} + \overline{X_0} + \overline{Y_1}} \quad \text{(De Morgan)}$$

$$Z_2 = \overline{\overline{X_1 + X_0} + \overline{X_1 + Y_1} + \overline{\overline{X_1} + \overline{X_0} + \overline{Y_1}}} \quad \text{(De Morgan)}$$

$$\overline{Z_3} = \overline{X_1} + \overline{X_0} + \overline{Y_0}$$

$$Z_3 = \overline{\overline{X_1} + \overline{X_0} + \overline{Y_0}}$$

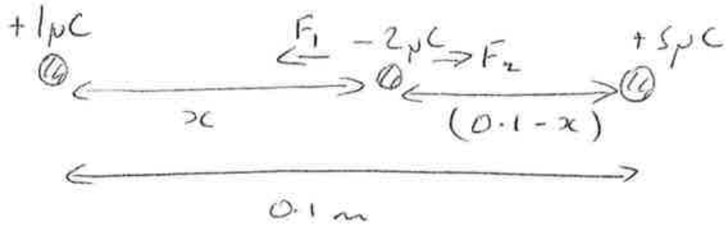
Section C

Q10 (a)

$$F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r^2}$$

Usual definitions

(b)



Zero net force when attractive force due to +1μC (F_1) is the same magnitude as the force due to +5μC (F_2)

$$\cancel{\frac{1}{4\pi\varepsilon_0}} \frac{1\times\cancel{10^{-6}} \times \cancel{2\times10^{-6}}}{x^2} = \cancel{\frac{1}{4\pi\varepsilon_0}} \frac{5\times\cancel{10^{-6}} \times \cancel{2.0\times10^{-6}}}{(0.1-x)^2}$$

$$5x^2 = (0.1 - x)^2$$

$$5x^2 = 0.01 - 0.2x + x^2$$

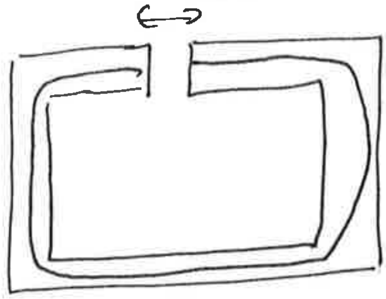
$$4x^2 + 0.2x - 0.01 = 0$$

$$x = \frac{-0.2 \pm \sqrt{0.2^2 + 4\times4\times0.01}}{8}$$

$$= \frac{-0.2 \pm \sqrt{0.2}}{8} = \frac{-0.2 \pm 0.447}{8}$$

$= 0.031\,m$ (or $-0.081\,m$ ignore)

11 (a)



gap length

$l_g = 0.1\,mm$

$l_m = 100\,mm$

magnetic material length

Flux conservation says $B_m A = B_g A$ ①

$\Rightarrow B_m = B_g$ (assuming no fringing)

Ampere's Law $H_m l_m + H_g l_g = NI$ ②

$B_g = \mu_0 H_g$, $B_m = \mu_0 \mu_m H_m$ ③

①, ③ in ② in

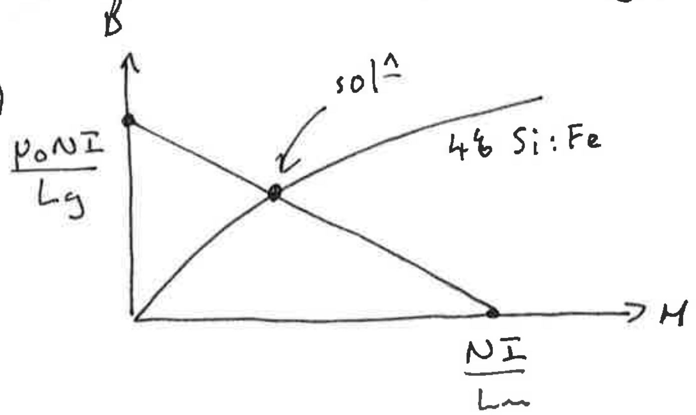
$$\frac{B_m}{\mu_0}\left(l_g + \frac{l_m}{\mu_r}\right) = NI$$

$$B_m = \frac{\mu_0 NI}{\left(l_g + \frac{l_m}{\mu_r}\right)} = \frac{4\pi \times 10^{-7} \times 500 \times 1}{\left(0.1 \times 10^{-3} + \frac{100 \times 10^{-3}}{1000}\right)}$$

NB NOT negligible

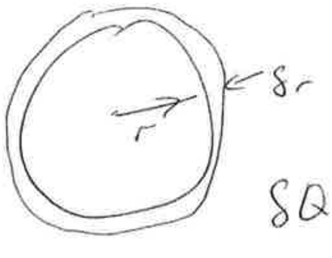
$= 3.14\,T$

(b)



Note from curves in data book, Si:Fe material is well into saturation so magnetic flux density is saturation value ie ~1.3 T

Q12 a) i) $\rho(r) = \rho_0 \left(1 - 4r/_{3R}\right)$



Amount of charge in spherical shell

$$\delta Q = \rho(r)\, 4\pi r^2 \delta r$$

$$= 4\pi\rho_0 \left(1 - \frac{2r}{3R}\right) r^2 \delta r$$

$$Q = 4\pi\rho_0 \int_0^R \left(1 - \frac{2r}{3R}\right) r^2 \delta r$$

$$= 4\pi\rho_0 \int_0^R \left(r^2 - \frac{2r^3}{3R}\right) \delta r$$

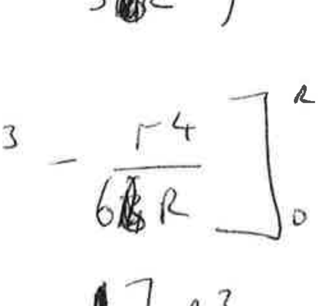
$$= 4\pi\rho_0 \left[\frac{1}{3} r^3 - \frac{r^4}{6R}\right]_0^R$$

$$= 4\pi\rho_0 \left[\frac{1}{3} - \frac{1}{6}\right] R^3$$

$$= \frac{2}{3}\pi\rho_0 R^3$$

ii) for $r > R$,

Gauss's law



$$\oint_s \underline{D} \cdot \underline{ds} = Q \Rightarrow D\, 4\pi r^2 = \frac{2}{3}\pi\rho_0 R^3$$

$$\underline{D} = \frac{1}{6\cancel{\epsilon_0} r^2}\rho_0 R^3 \qquad \underline{D} = \epsilon_0\epsilon_r\underline{E}$$

$$\underline{E} = \frac{1}{6\epsilon_0 r^2}\rho_0 R^3 \qquad (\epsilon_r = 1 \text{ outside sphere})$$

for $r < R$, need to calculate Q within r

from i)

$$Q(r) = 4\pi\rho_0\int_0^r\left(1 - \frac{4r}{6R}\right)r^2\,\delta r$$

$$= 4\pi\rho_0\left[\frac{1}{3}r^3 - \frac{\cancel{4}}{\cancel{6}}\frac{r^4}{R}\right]$$

$$= 4\pi\rho_0\left[\frac{1}{3} - \frac{r}{6R}\right]r^3$$

$$\Rightarrow \quad D 4\pi \cancel{r^2} = 4\pi\rho_0\left[\frac{1}{3} - \frac{r}{6R}\right]r^{\cancel{3}}$$

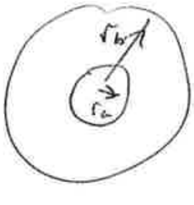
$$\underline{D} = \rho_0\left[\frac{1}{3} - \frac{r}{6R}\right]r \quad \hat{\underline{r}}$$

$$\underline{E} = \frac{\rho_0}{\epsilon_0\epsilon_r}\left[\frac{1}{3} - \frac{r}{6R}\right]r \quad \hat{\underline{r}}$$

(b) i) Let inner shell have charge $+Q$ & outer $-Q$

By Gauss, between shells

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \quad \text{and} \quad E = 0 \text{ elsewhere}$$



$$V_a - V_b = \int_{r_b}^{r_a} \frac{Q}{4\pi\varepsilon_0 r_a} - \frac{Q}{4\pi\varepsilon_0 r_b}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$C = \frac{Q}{V_a - V_b} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}$$

ii) $r_a = 1\text{cm}$, $r_b = 5\text{cm}$

$$\Rightarrow C = \frac{4\pi\varepsilon_0}{\left(\frac{1}{0.01} - \frac{1}{0.05}\right)} = 1.4\,pF$$

ii) $Q = CV = 1.4 \times 10^{-12} \times 220V$

$$= 0.31\ nC$$