ENGINEERING TRIPOS PART IA

Thursday 9 June 2011 9 to 12

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There is one attachment: sheet to be handed in with your answer to Question 3.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 **(short)** A plane frame is shown in Fig. 1, which includes frictionless pins at A, B and E. A horizontal load of 50 kN is applied at B and a vertical load of 50 kN is applied at C, as shown in the figure.

(a) Find the vertical and horizontal components of the reactions at the two bottom supports (A and E). [7]

(b) Draw the bending moment diagram on a sketch of the structure with clear indication of the sign convention adopted. [3]

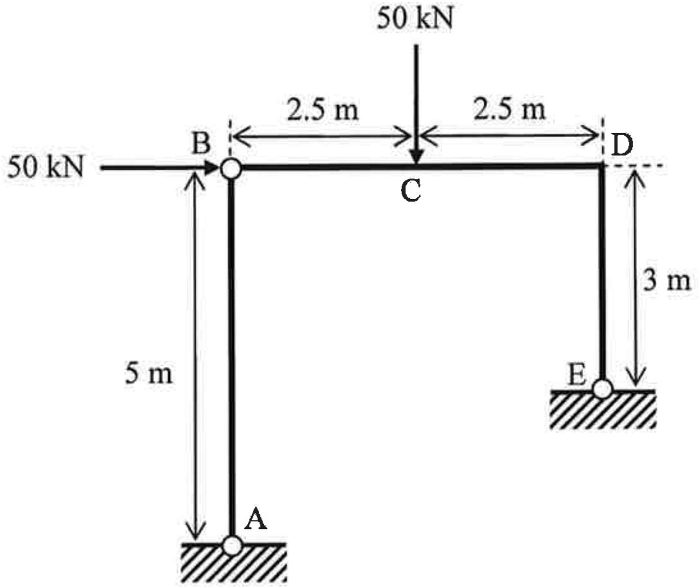


Fig. 1

2 (short) An L-shaped simply supported beam with a uniform flexural stiffness of 10^4 kNm^2 is shown in Fig. 2. The beam is loaded with a distributed load as well as a concentrated load. Linear elastic behaviour is assumed. The self-weight can be neglected. Calculate the vertical displacement at D. [10]

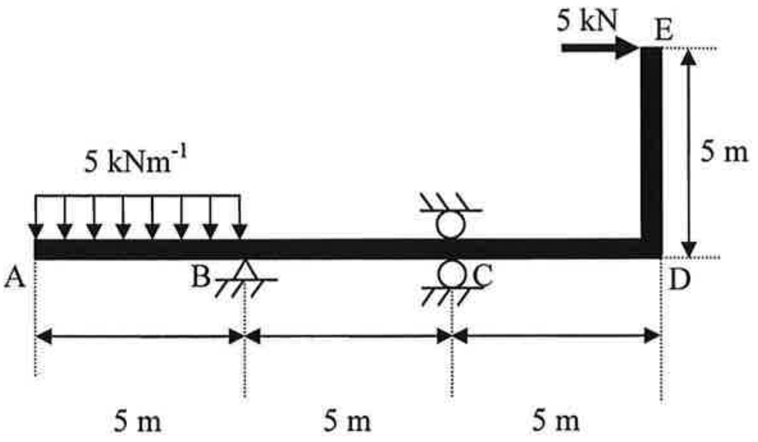


Fig. 2

3 (short) A pin jointed truss is shown in Fig. 3. All members have the same cross-sectional area A and are made of a linear elastic material with Young's modulus E. The self-weight can be neglected. A vertical load of W is applied to the structure at joint B shown in the figure.

(a) Find the bar forces. A table has been provided on the attached sheet to fill in these values. [5]

(b) Find the vertical displacement of joint B. [5]

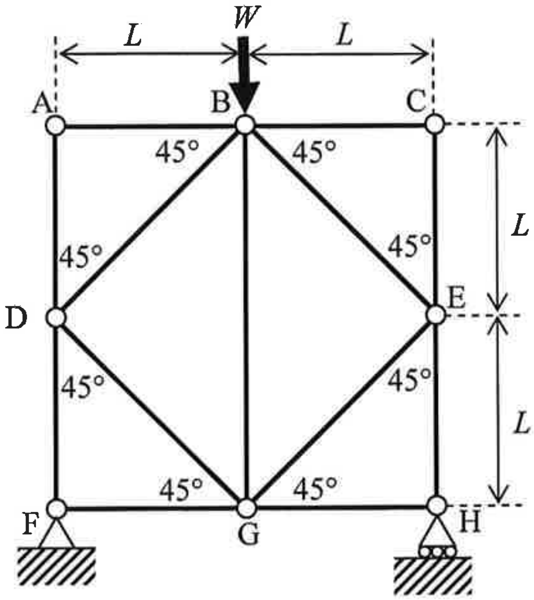


Fig. 3

4 **(short)** A steel beam with T-shaped cross-section is shown in Fig. 4. The beam will be bent about a horizontal axis. A bending moment of 25 kNm is applied about the neutral axis *x*-*x*′ of the beam. The Young's modulus of the steel is 210 GPa.

(a) Determine the location of the neutral axis *x*-*x*′. [3]

(b) Calculate the bending stiffness of the beam. [4]

(c) Find the location and magnitude of the maximum absolute longitudinal stress acting on the cross-section. [3]

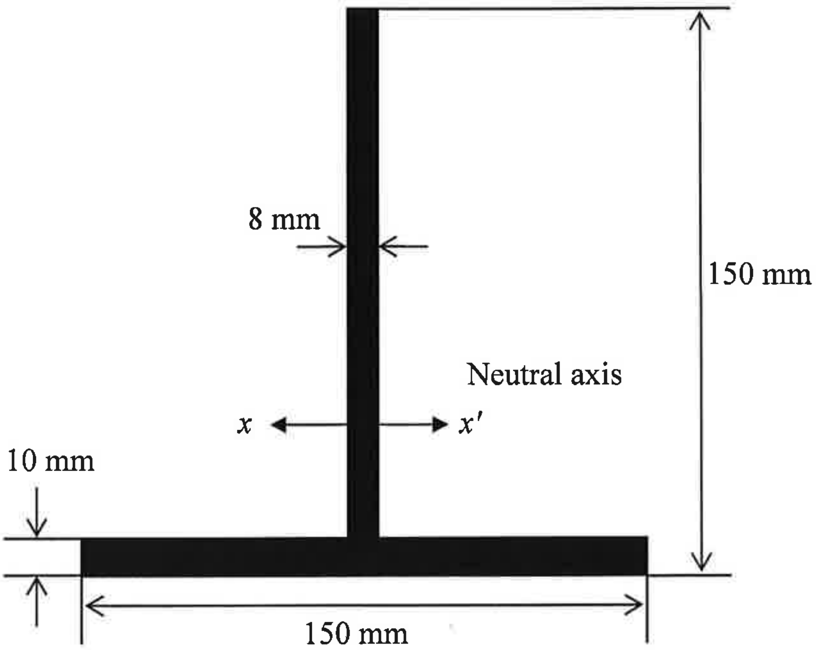


Fig. 4

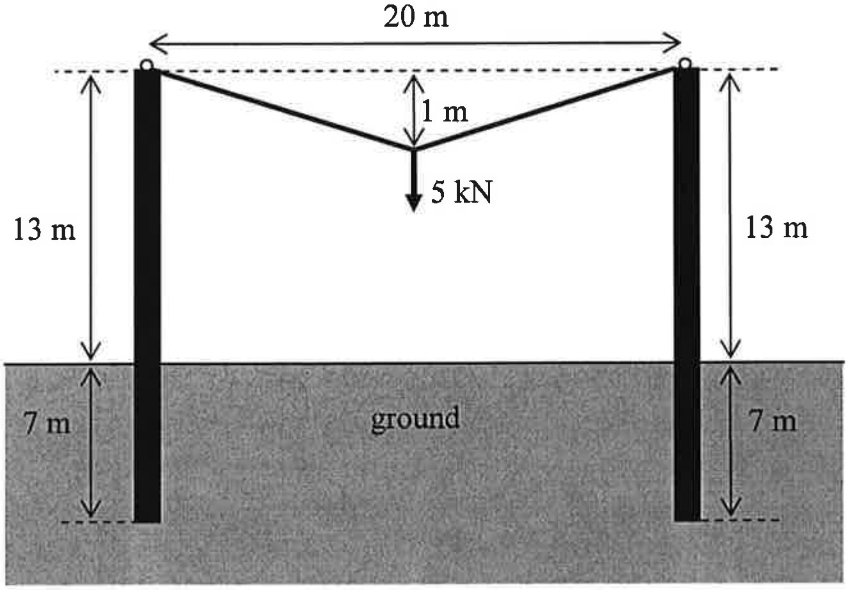
5 (long) Two uniform beams of length 20 m are partially embedded in the ground as shown in Fig. 5(a). The spacing between the two beams is 20 m and the embedment depth of the beams into the ground is 7 m. A flexible cable is attached at the two top ends of the beams as shown in the figure.

(a) A downward point load of 5 kN is applied at the midspan of the cable. The dip of the midspan was found to be 1 m. Assuming that the weight of the cable is negligible compared to the applied load, calculate the vertical and horizontal forces that the cable applies to the beams. [6]

(b) As the cable is loaded, the supporting beams rotate as shown in Fig. 5(b). Soil resistance then develops, counteracting the rotational movement. If the soil resistance is assumed to be a uniformly distributed load of w, show that the depth of the axis of rotation is $x = 3.867$ m and find the value of w. Assume that the angle of rotation is very small. [10]

(c) Draw the shear force diagram for one beam, giving salient values. [6]

(d) The cross-section of the beams is shown in Fig. 5(c). Find the greatest shear stress acting at the interface c-c' shown in the figure. [8]



Not to scale

Fig. 5(a)

(Cont.

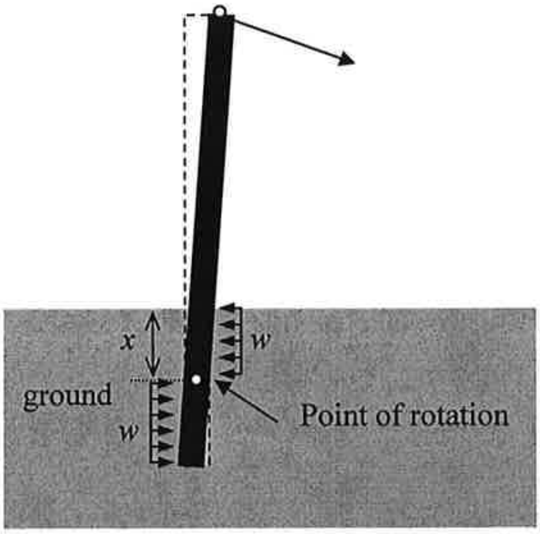

x
w
ground
w
Point of rotation

Fig. 5(b)

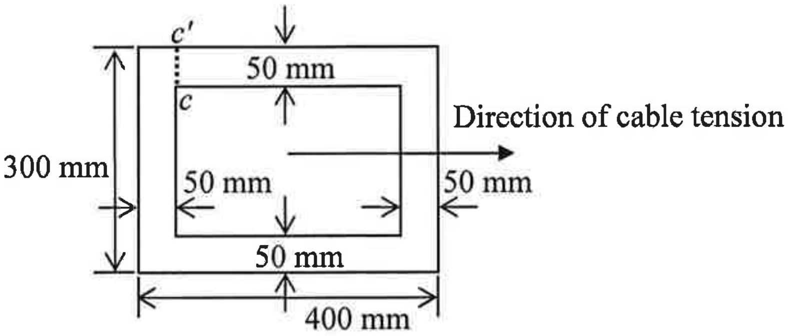

c′
c
50 mm
Direction of cable tension
300 mm
50 mm
50 mm
50 mm
400 mm

Fig. 5(c)

6 (long) The uniform elastic strut with a uniform bending stiffness EI is shown in Fig. 6(a). At A, the strut is fixed in all directions. The support at B is free to move axially and allows in-plane rotation. The strut is stress free when not loaded externally, and is just in contact with the support at B.

(a) A uniformly distributed load of w is applied to the strut as shown in the figure.

(i) Find the vertical support reaction at B. [6]

(ii) Draw the shear force and bending moment diagrams for the strut. [6]

(b) A horizontal force P is applied at B on the strut with $w = 0$ as shown in Fig. 6(b) and the strut buckles. A moment of M_0 is generated at A in the buckled configuration.

(i) Draw a free body diagram of the strut and show the reaction forces acting on the strut. [3]

(ii) The vertical displacement of the buckled configuration is defined to be $v(x)$, where x is the distance from A. Set up a differential equation for the bending of the strut as function of M_0 and P. Use the sign convention defined in the Structures Data Book. [6]

(iii) By applying appropriate boundary conditions, derive the equation that could be used to solve for P (there is no need to derive the actual solution for P). [6]

(iv) If w is also applied to the strut, discuss briefly how this would affect the buckling response. There is no need to find the solution. [3]

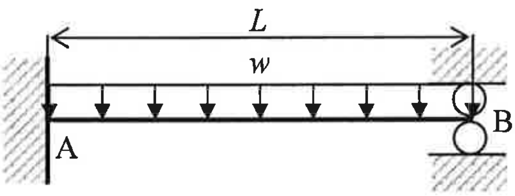


Fig. 6(a)

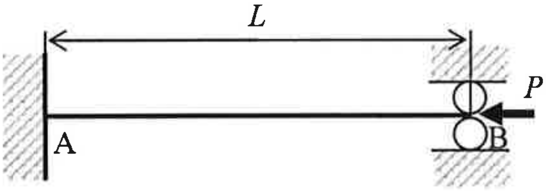


Fig. 6(b)

SECTION B

7 **(short)** An elastic block with Young's modulus E and Poisson's ratio ν has a square cross-section with side length b and height h, as shown in Fig. 7(a).

(a) The block is heated up by a uniform temperature rise of ΔT. If the coefficient of thermal expansion of the block is α, what pressure p must be applied to the x and y faces of the block to maintain a side length of b, as shown in Fig. 7(b)? [4]

(b) A particular block is heated up by $\Delta T = 245°\text{C}$. Again, a pressure is applied to the x and y faces to maintain a side length of b, as shown in Fig. 7(b). If the height of the block after heating is $H = 1.01h$, and the coefficient of thermal expansion of the block is $\alpha = 20 \times 10^{-6}\ °\text{C}^{-1}$, calculate the Poisson's ratio ν of the block material. [6]

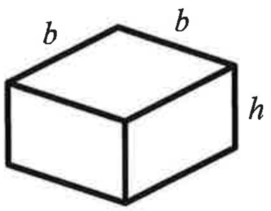


Fig. 7(a)

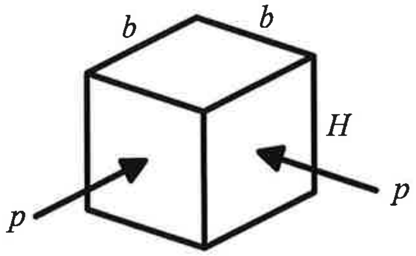
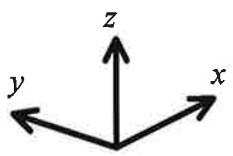


Fig. 7(b)



8 (short)

(a) Using a sketch to illustrate your answer, explain briefly the process of wet corrosion of iron. Indicate on your sketch the location of the anode and cathode, any reactions taking place and the paths of any diffusion or conduction. [6]

(b) Explain briefly why (i) gold and (ii) aluminium do not corrode in water. [4]

9 (short)

(a) Explain briefly why the survival probability of a ceramic loaded in tension depends on the volume of the specimen. [3]

(b) Ceramic circular cylinders of volume V are tested in tension by applying a stress σ to the ends. Failure data from two series of tests are given in Table 1. Using this data, calculate the Weibull modulus, m. [5]

(c) The same specimens are tested in bending, such that the maximum tensile stress experienced is the same as in the tension test. Explain briefly, and without calculation, whether the survival probability would increase or decrease. [2]

Volume, V (mm^3)	Stress, σ (MPa)	Survival probability, P_s
120	90	0.66
150	95	0.41

Table 1

10 **(short)** A copper wire is tested to failure in a tensile testing machine. The measured nominal stress versus nominal strain curve is shown in Fig. 8.

(a) On a sketch of the stress-strain curve, label (i) the 0.1% proof stress, (ii) the tensile strength and (iii) the permanent strain after fracture. [3]

(b) An identical wire is stretched to a nominal strain $\varepsilon = 0.1$ and unloaded. The same specimen is then re-tested. What would be the yield strength measured during the re-test? [4]

(c) Define the 'ideal strength' of a material. Explain briefly why the yield strength of a metal is often considerably less than the 'ideal strength'? [3]

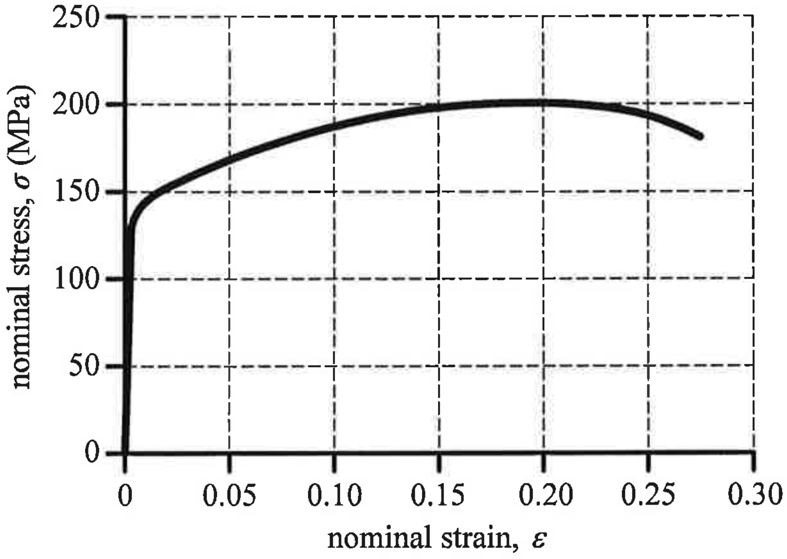


Fig. 8

11 **(long)** A cantilever beam of length L is to be designed to carry a force F at the tip with a specified maximum deflection, while being as light as possible. See Fig. 9(a). It is decided to consider the effect of cross-sectional shape. The 'shape factor' for stiffness-limited design in bending is defined as follows:

$$\Phi_e = \frac{I}{I_{\text{ref}}}$$

where I is the second moment of area, and 'ref' refers to a square cross-section with the same area as the shaped cross-section.

(a) Two candidate cross-sections are considered for the cantilever design: a rectangle and a thin-walled tube. The dimensions are shown in Fig. 9(b). The lengths h and R can be considered free variables, with α and β constants defining the shape ($\beta << 1$).

(i) For both cross sections, derive the shape factor Φ_e for stiffness-limited design in terms of the constants α and β. [8]

(ii) Table 2 contains data for three candidate materials. Evaluate the minimum values of α and β for each material. Explain why a maximum shape factor is often specified, and why it depends on the material. [6]

(iii) For the thin-walled tube only, derive a suitable shape-dependent performance index for minimum mass, in terms of Φ_e and material properties. The effect of self weight on deflection may be neglected. [8]

(iv) Using the data in Table 2, select the best material for the cantilever. [4]

(b) If the design was strength-limited rather than stiffness-limited, define a shape factor Φ_f for this situation. Why is shape unimportant for the light-weight design of components loaded only in tension? [4]

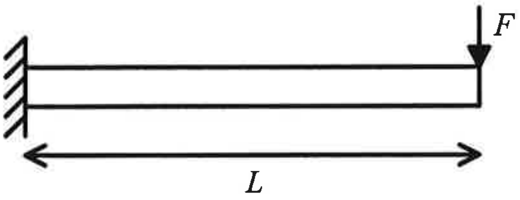


Fig. 9(a)

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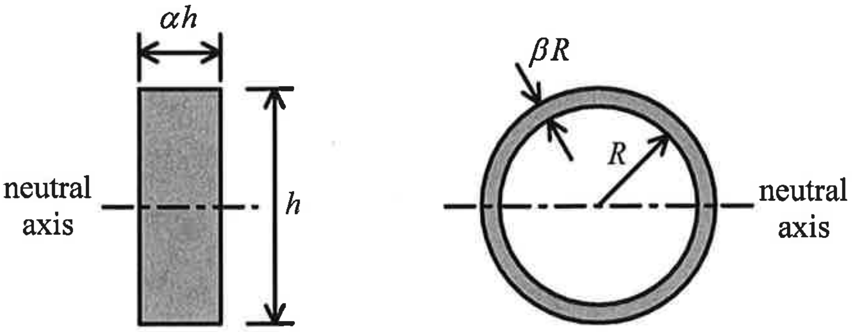

Fig. 9(b)

Material	Young's modulus (GPa)	Density (kg m^{-3})	Maximum shape factor, Φ_e
Steel	210	7600	64
Aluminium alloy	70	2700	49
CFRP	100	1500	36

Table 2

12 **(long)** A structural component consists of a steel rod with a circular cross-section which carries a tensile stress σ. In normal service, the stress σ ramps up to a value $\sigma_m = 150$ MPa about which it oscillates with stress range $\Delta\sigma = 20$ MPa for a large number of cycles.

(a) Most rods are observed to fail by fatigue. Sketch the expected fracture surface, labelling any key features. [3]

(b) In order to assess the fatigue life, laboratory tests are performed on identical rods by applying a cyclic stress with a range $\Delta\sigma_0$ at zero mean stress. The rod fails after 7×10^6 cycles when $\Delta\sigma_0 = 35$ MPa and after 5×10^{10} cycles when $\Delta\sigma_0 = 4$ MPa. Calculate the fatigue life N_f of the rod during normal service conditions. The material has tensile strength $\sigma_{ts} = 550$ MPa. [12]

(c) NDT inspection of a particular rod reveals a ring crack of depth $a = 2$ mm (much less than the rod diameter) which runs around the circumference of the rod. The ring crack results in a stress intensity factor:

$$K = 1.12\sigma\sqrt{\pi a}$$

If subjected to cyclic loading, the crack depth a increases with the number of cycles N following:

$$\frac{da}{dN} = A(\Delta K)^n$$

When stresses are expressed in units of MPa and lengths in mm, $A = 5\times10^{-12}$ and $n = 4$. Calculate the number of cycles required to increase the crack depth to 4 mm during normal service. Explain why it will take fewer cycles to increase the crack depth by a further 2 mm. [10]

(d) Cracked rods are to be proof tested to make sure that the initial crack growth rate will not exceed 0.001 mm per cycle under service conditions. Calculate a suitable tensile stress σ for the proof test. The fracture toughness $K_{IC} = 50\ \text{MPa}\sqrt{\text{m}}$. [5]

END OF PAPER

ENGINEERING TRIPOS PART IA
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Sheet to be handed in with your answer to Question 3.

Bar	Forces				
AB					
AD					
BD					
DF					
DG					
FG					
BG					
BC					
CE					
BE					
EH					
EG					
GH					