ENGINEERING TRIPOS PART IA

Monday 13 June 2011 9 to 12

Paper 3

ELECTRICAL AND INFORMATION ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 **(long)** An AC generator of RMS voltage V and internal impedance $Z_1\angle\theta$ is connected to a load of impedance $Z_2\angle\varphi$ as shown in Fig. 1.

(a) Show that the average power dissipated in the load is given by

$$P = \frac{V^2 Z_2 \cos\varphi}{{Z_1}^2 + {Z_2}^2 + 2Z_1Z_2\cos(\theta - \varphi)}$$

[8]

(b) The phase φ of the load is held constant, and the magnitude Z_2 of the load is varied. Show that the condition for maximum power to be transferred to the load is $Z_2 = Z_1$. [9]

(c) The magnitude Z_2 of the load is kept constant, but the phase φ is varied. Show that the condition for maximum power to be transferred to the load is now

$$\sin\varphi = -\frac{2Z_1Z_2}{{Z_1}^2 + {Z_2}^2}.\sin\theta$$

[8]

(d) Using the results from (b) or (c), or otherwise, show that, if the magnitude and phase of the load are both varied, the condition for maximum power to be transferred to the load is

$$Z_2\angle\varphi = Z_1\angle - \theta$$

[5]

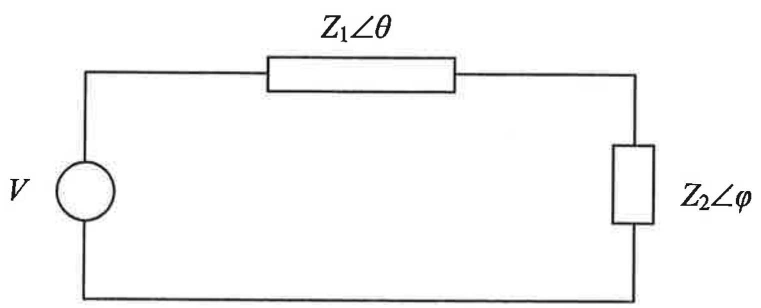

Fig. 1

2 (long) A FET is configured in an amplifier as shown in Fig. 2. The impedances of C_{in}, C_{out} and C_s are negligible at mid-band frequencies. The small signal parameters of the FET are g_m = 10 mA V^{-1} and r_D = 25 kΩ, and its operating point is given by V_{ds} = 15 V, V_{gs} = −2 V and I_{ds} = 1 mA.

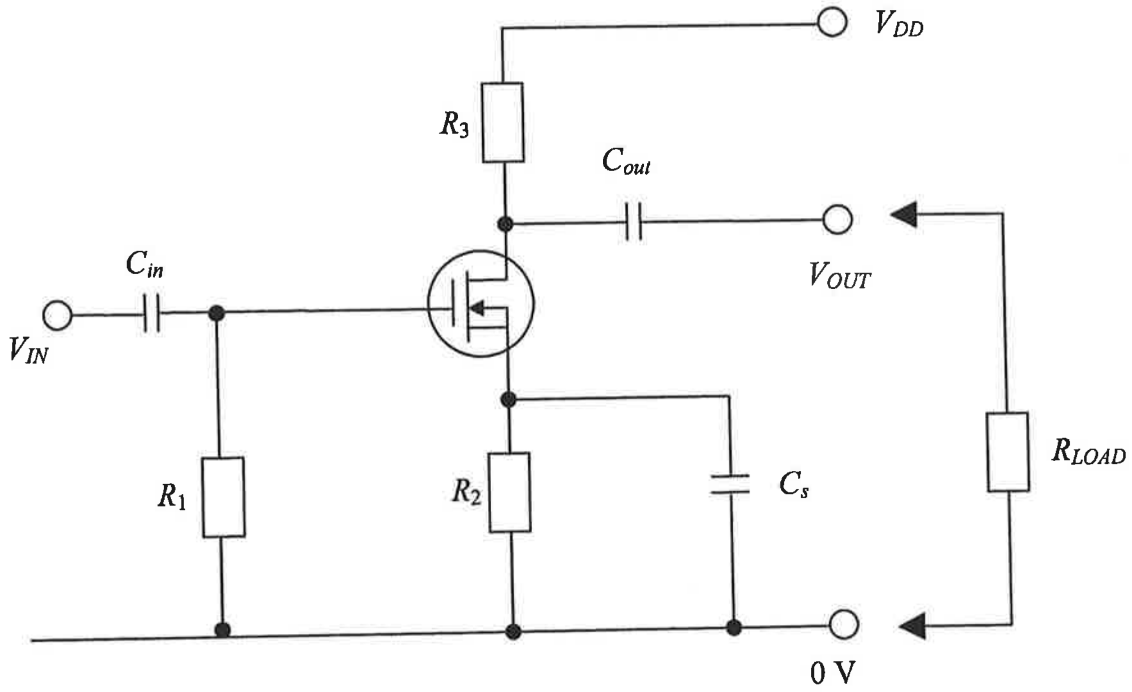

Fig. 2

(a) At the given operating point, calculate the values for R_2 and V_{DD} given that R_3 = 20 kΩ and R_1 = 1 MΩ. [8]

(b) For the circuit in Fig. 2, but without the load connected, draw the small signal equivalent circuit. Hence calculate its voltage gain, and also its input and output impedances, all at mid-band frequencies. [9]

(c) What value of external load resistance, R_{LOAD} , should be connected between the output terminal of the amplifier and ground in order to maximise the signal power in the load? [6]

(d) If the lower 3 dB cut-off frequency of the amplifier is 200 Hz and is dominated by the effect of C_{out} , calculate the value of this capacitor, assuming that R_{LOAD} of part (c) is connected. [7]

(TURN OVER

3 **(short)** A small factory consumes 30 kW power at 230 V with a lagging power factor of 0.85. The line supplying the factory has an impedance $0.1 + j0.1\ \Omega$. The frequency is 50 Hz.

(a) Draw a circuit diagram for the above system. Calculate the power loss in the line, and the voltage at the supply end of the line. [6]

(b) What is the minimum power lost in the line if power factor correction is applied at the factory, and the factory voltage remains 230 V? [4]

4 (short)

(a) State what assumptions have to be made to be able to describe a transformer as ideal. [3]

(b) The characteristics of a non-ideal transformer are determined by performing tests with the low voltage secondary open and short circuited. These give

Open Circuit Test
$V_{PRIMARY} = 260$ V, $I_{PRIMARY} = 0.6$ A, $P = 50$ W, $V_{SECONDARY} = 130$ V

Short Circuit Test
$V_{PRIMARY} = 50$ V, $I_{PRIMARY} = 6$ A, $P = 50$ W

Determine the values of the equivalent circuit parameters (referred to the primary side of the transformer). [7]

(TURN OVER

5 **(short)**

(a) Explain what is meant by a Thevenin equivalent circuit. Draw the Thevenin equivalent circuit for the circuit shown in Fig. 3(a), and derive expressions for the Thevenin voltage and impedance. [3]

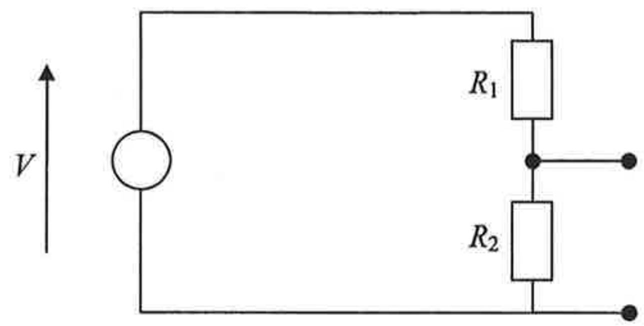

Fig. 3(a)

(b) In the circuit of Fig. 3(b), R = 200 Ω, L = 40 mH and C = 160 μF. By applying Thevenin's theorem, or otherwise, determine the RMS magnitude of the current flowing in capacitor C, its peak value, and its phase with respect to the 150 V voltage source. [7]

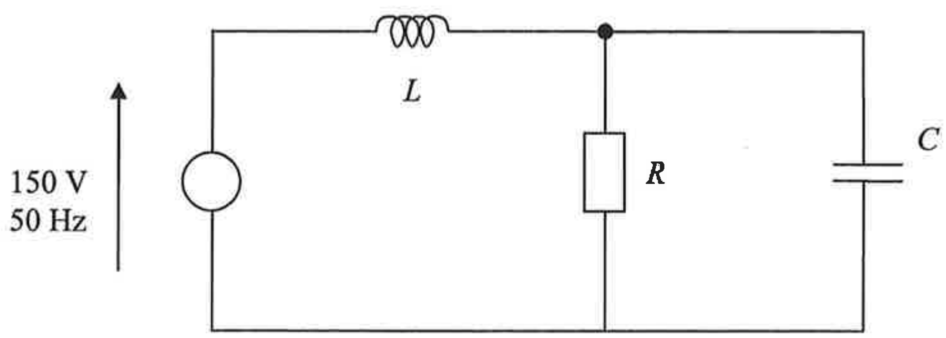

Fig. 3(b)

SECTION B

6 **(short)** A four variable Boolean function is defined as

$$F = B.\bar{C}.\bar{D} + A.D + A.\bar{B}.\bar{C}.D + \bar{A}.\bar{B}.D$$

Using a Karnaugh map, or otherwise:

(a) Find the simplest Sum of Products expression for *F*. [4]

(b) Identify under what circumstances a change to a single input variable may cause a momentary unexpected change in *F*. Give the technical term for this problem. [3]

(c) Modify the Sum of Products expression found in part (a) to remove this possible problem. [3]

7 **(short)** A sequential circuit is to be designed using J-K bistables to generate the sequence 00, 11, 10, 01, which is to repeat indefinitely.

(a) Draw the state diagram and state how many bistables are required. [3]

(b) Write down the state transition table for the circuit. [4]

(c) Derive expressions for the inputs to the J-K bistables used in the circuit. [3]

(TURN OVER

8 (short) The following PIC12F629 code is executed.

```
    movlw 127;
    movwf 0x30;
    subwf 0x30, W;
    movlw 255;
    andwf 0x30;
    btfsc 0x30, 2;
    goto end;
    nop;
    nop;
end sleep;
```

(a) What will be the contents of W, 0x30 and of the C, DC and Z flags after

(i) the **subwf 0x30, W** instruction?

(ii) the **andwf 0x30** instruction? [6]

(b) Determine the time taken to execute the code assuming a 10 MHz clock. [4]

9 (long) A logic circuit has four inputs and four outputs. The inputs X_1, X_0 and Y_1, Y_0 represent two 2-bit unsigned binary numbers. The circuit calculates the output Z according to the equation

$$Z = 2X + Y$$

(a) Using a Karnaugh map or otherwise, derive Sum of Products expressions for each of the outputs Z_0 (the LSB) to Z_3 (the MSB). [14]

(b) Express Z_2 as a Product of Sums. [4]

(c) Design the logic circuit using only NAND and inverter gates for Z_0, Z_1 and only NOR and inverter gates for Z_2, Z_3. The NAND and NOR gates may have any number of inputs. [12]

SECTION C

10 **(short)**

(a) Write down an expression for the electrostatic force between two charges, Q_1 and Q_2, which are separated by a distance r. [2]

(b) Two point charges, with values of +1 μC and +5 μC, are separated by a distance of 0.1 m. A third point charge, with a value of -2 μC, is free to move along the straight line joining the two charges. Determine the position where it experiences zero net force. [8]

11 **(short)** In the magnetic circuit shown in Fig. 4, the electromagnet consists of an ideal magnetic material with relative permeability of 1000 and a coil of 500 turns carrying a current of 1 A. The cross-sectional area of the electromagnet is 10 mm^2. For simplicity, you may assume that all flux lines have a nearly equal length of 100 mm around the magnetic circuit.

(a) Calculate the magnetic flux density in the air gap. You may assume that fringing effects are negligible. [5]

(b) The ideal magnetic material is replaced by 4% Silicon-Iron. Calculate the new magnetic flux density in the air gap. [5]

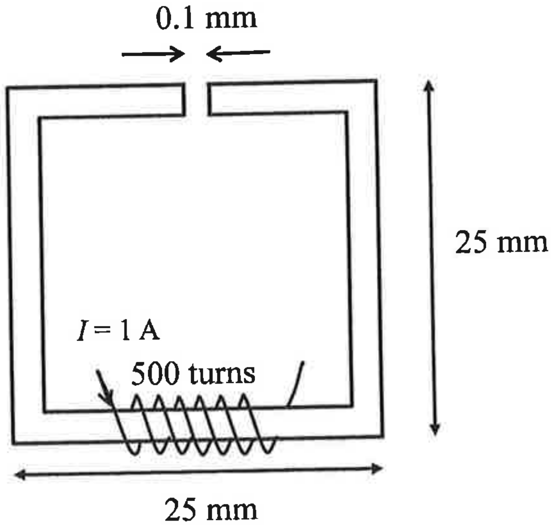

Fig. 4

(TURN OVER

12 **(long)**

(a) A sphere, radius R, has been charged with a non-uniform, but spherically symmetric, charge distribution such that the volume charge density at a radius r, $\rho(r)$, is given by

$$\rho(r) = \rho_0 \left(1 - {}^{2r}/_{3R}\right), \qquad \text{for } r \le R,$$
$$\rho(r) = 0, \qquad \text{for } r > R.$$

(i) Derive an expression for the total charge contained in the sphere. [8]

(ii) Using Gauss's law, obtain expressions for the electric field for $r > R$ and for $r \le R$. [10]

Within the sphere the relative permittivity is ε_r but 1 elsewhere.

(b) A capacitor is constructed from two concentric conducting spherical shells which are separated by air. The inner and outer shells have radii of 1 cm and 5 cm respectively.

(i) Calculate the capacitance. [8]

(ii) A potential difference of 220 V is applied between the two shells. Calculate the magnitude of the charge on each shell. [4]

END OF PAPER

PART IA

2011 Paper 3 SECTION A

2 (a) $R_2 = 2k\Omega$

$V_{DD} = 37V$

(b) Input impedance = 1MΩ
Output impedance = 11.1kΩ
Gain=-111.1

(c) $R_{LOAD} = 11.1k\Omega$

(d) $C = 36nF$

3 (a) $P_{LINE} = 2355W$

$V = 251.2V$

(b) P_{Line}= 1.701 kW

4 (b) $R_o = 1352\Omega$

$X_0 = 457.5\Omega$

Turns Ratio=2

$R_t = 1.39\Omega$

$X_t = 8.21\Omega$

5 (a) $V_{Th} = V_{OC} = \frac{R_2}{R_1 + R_2}V$

$R_{Th} = \frac{R_1 R_2}{R_1 + R_2}$

(b) $\overline{I} = 20.26\angle 80.25A$

$\hat{I}_C = 28.65A$

PART IA

Short Answers Paper 3 – 2011

Section B

6. a) $F = B.\bar{C}.\bar{D} + A.D + \bar{B}.D$

c) $F = B.\bar{C}.\bar{D} + A.D + \bar{B}.D + A.B.\bar{C}$

7. a) 2 bistables

c) $J_A = \overline{Q_B}$, $J_B = 1$, $K_A = \overline{Q_B}$, $K_B = 1$

8 a) i) 00, 7F, 1, 1, 1

ii) FF, 7F, 0, 0, 0

b) 900 ns

9 a)

$Z_0 = Y_0$, $Z_1 = X_0 . \overline{Y_1} + \overline{X_0} . Y_1$, Z_2

b) $Z_2 = (X_1 + X_0).(X_1 + Y_1).(\overline{X_1} + \overline{X_0} + \overline{Y_1})$

c)

$Z_0 = Y_0$, $Z_1 = \overline{\left(\overline{Y_0 . \overline{Y_1}}\right) . \left(\overline{\overline{X_0} . Y_1}\right)}$,

Section C

10 b) 0.031 m

11 a) 3.14 T

b) 1.3 T

12 a) i) $\frac{2}{3}\pi\rho_0 R^3$

ii) $\rho_0 R^3 / 6\varepsilon_0 r^2$, $\rho_0 / \varepsilon_0 \varepsilon_r \left(\frac{1}{3} + \frac{r}{6R}\right) r$

b) i) 1.4 pF

ii) 0.31 nC