

Answers

2(a) $r^4 e^{4i\theta}$; $\sqrt{r} e^{i\theta/2 + in\pi}$ for $n = 0, 1$; $\ln r + i\theta + 2n\pi i$ for any integer n

(b) $i, -2, -3$

3(a) $\left| \frac{Al + Bm + Cn - D}{\sqrt{A^2 + B^2 + C^2}} \right|$ (b) 2

4 $\lambda = 1, 1, -1$ e-vectors $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ (eigenvectors are not unique)

5 (b) $2 \frac{\sinh kx_1}{k}$ (c) (i) $\frac{d^2 y}{dx^2} = k$ (ii) $y = k \frac{x^2}{2}$

6 $\sin t$ for $0 \leq t \leq \pi$, 0 otherwise

7 (a) $f(t) = \frac{1}{\pi} + \frac{1}{2} \cos t + \frac{2}{\pi} \sum_{m=1}^{\infty} (-1)^{m+1} \frac{\cos(2mt)}{4m^2 - 1}$ (b) $\frac{2}{35\pi}$

8 £ 0.4

9 (a) $F(s)G(s)$ (b) $\frac{t}{2} - \frac{1}{4} + \frac{1}{4} e^{-2t}$ (d) $y = 0, \dot{y} = 0, \ddot{y} = 1$ at $t = 0$

10 (a) $\frac{3}{\sqrt{5}}$ (b) $y = \frac{1}{2} x$