SECTION A

1 **(long)** Figure 1(a) shows the circuit for a source follower amplifier. The FET has small-signal parameters g_m = 5 mS and r_d = 15 kΩ. The source resistor R_S = 6 kΩ, and the gate resistor R_G = 2 MΩ.

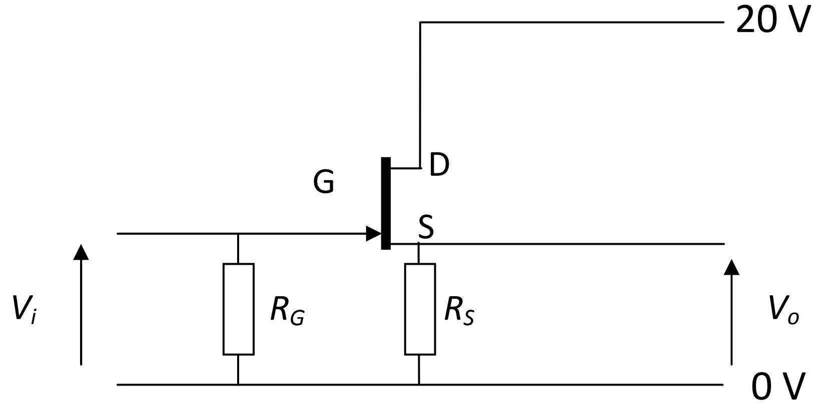
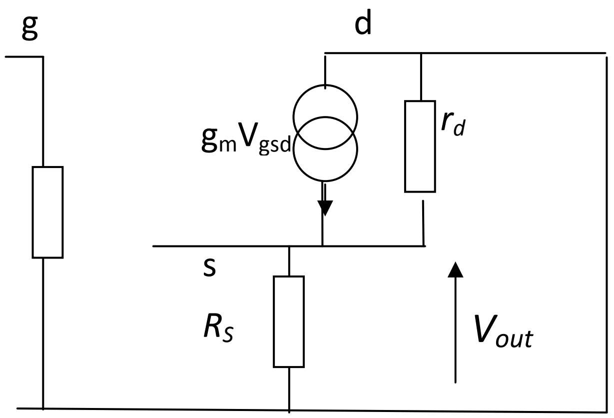


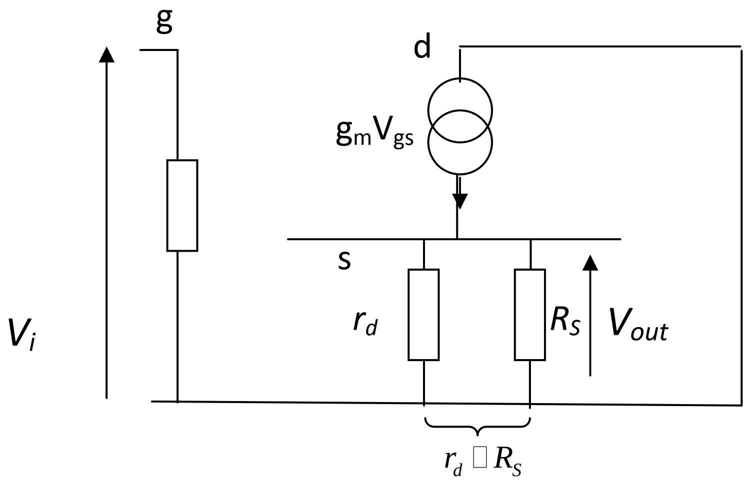
Figure 1(a)

(a) Calculate the gain and output impedance of the circuit. [15]

The small signal circuit can be drawn as:



equivalent to:



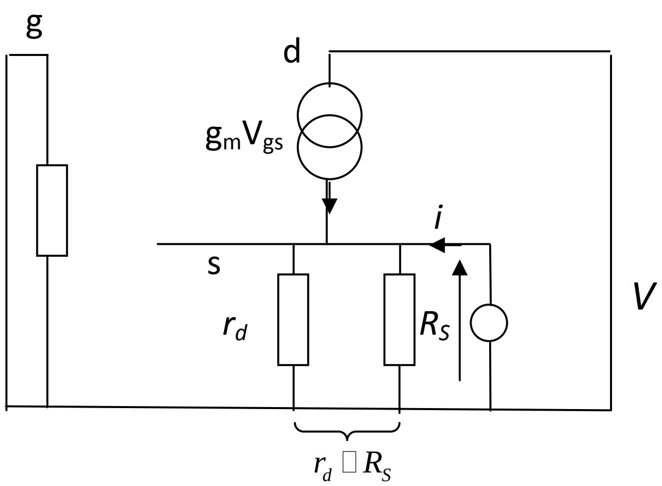
$$\frac{V_{OUT}}{r_d \parallel R_s} = g_m V_{gs}$$

$$V_i = V_{gs} + V_{OUT}$$

$$\frac{V_{OUT}}{r_d \parallel R_s} = g_m \left(V_1 - V_{OUT}\right)$$

$$\frac{V_{OUT}}{V_i} = \frac{g_m r_d \parallel R_s}{1 + g_m r_d \parallel R_s} = 0.955$$

To find R_{OUT} we short-circuit input, and apply signal at output. The equivalent circuit is:



$$V = -V_{gs}$$

$$i = \frac{V}{r_d \parallel R_s} - g_m V_{gs}$$

$$R_{OUT} = \frac{V}{i} = \frac{r_d \parallel R_s}{1 + g_m r_d / R_s} = 191\Omega$$

(b) As a result of electrical interference, noise in the form of a small voltage of frequency 200 Hz is induced in the drain circuit of the FET. The presence of the 200 Hz noise can be modelled by the inclusion of a small signal source V_N in the drain circuit as shown in Figure 1(b).

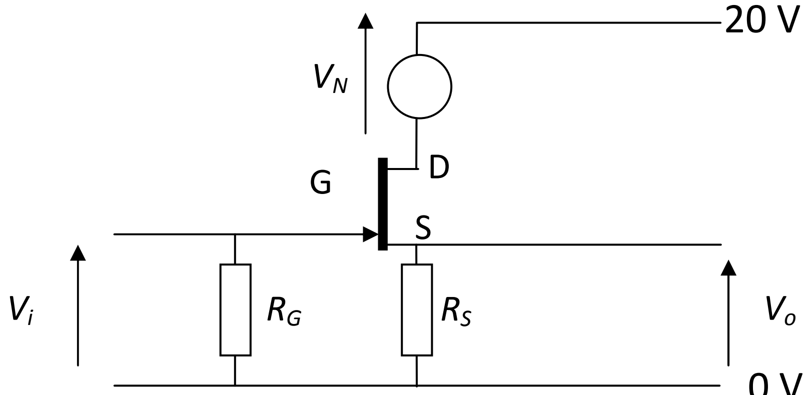
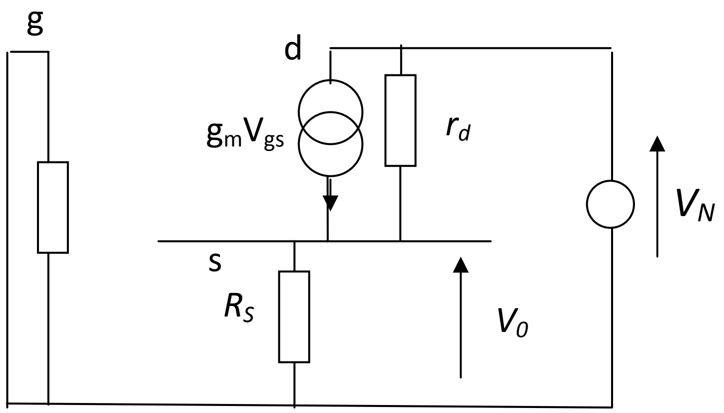


Figure 1(b)

Draw the small signal equivalent circuit for determining the component of the output voltage that arises as a result of the noise source. [5]

We use superposition with V_i short circuited. The equivalent circuit becomes:



$$V_0 = -V_{gs}$$

$$\frac{V_0}{R_s} = g_m V_{gs} + \frac{(V_n - V_0)}{r_d}$$

$$V_0 = \frac{V_n}{r_d \left(\frac{1}{r_d} + \frac{1}{R_s} + g_m \right)}$$

(c) Determine the maximum amplitude of V_N in Figure 1(b), if the noise component of the amplifier's output is not to exceed 30 μV. [10]

$$\frac{V_N}{r_d\left(\frac{1}{r_d}+\frac{1}{R_s}+g_m\right)} < 30\times10^{-6}V$$

$$V_N < 2.34mV$$

2 **(long)** Consider the amplifier circuit in Figure 2.

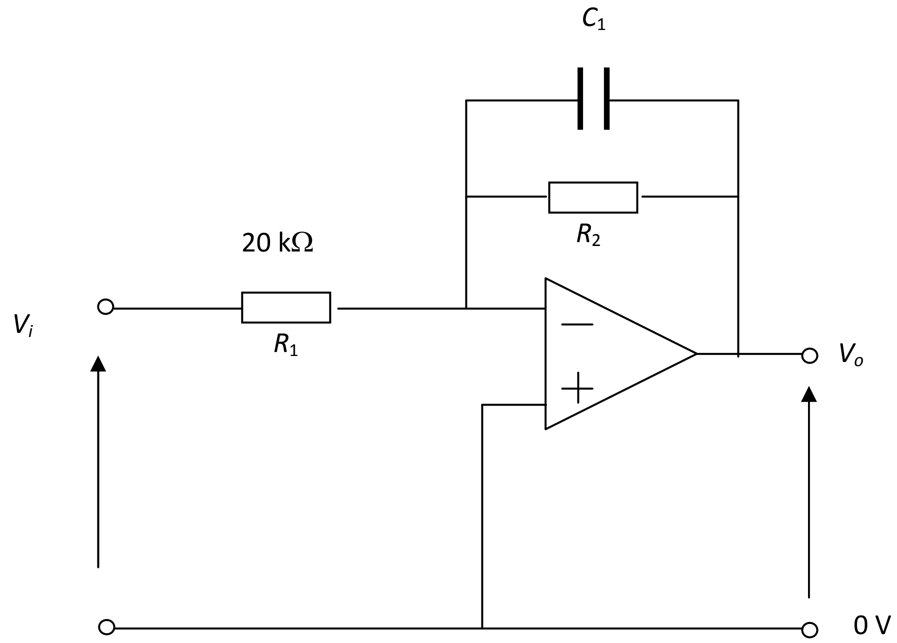


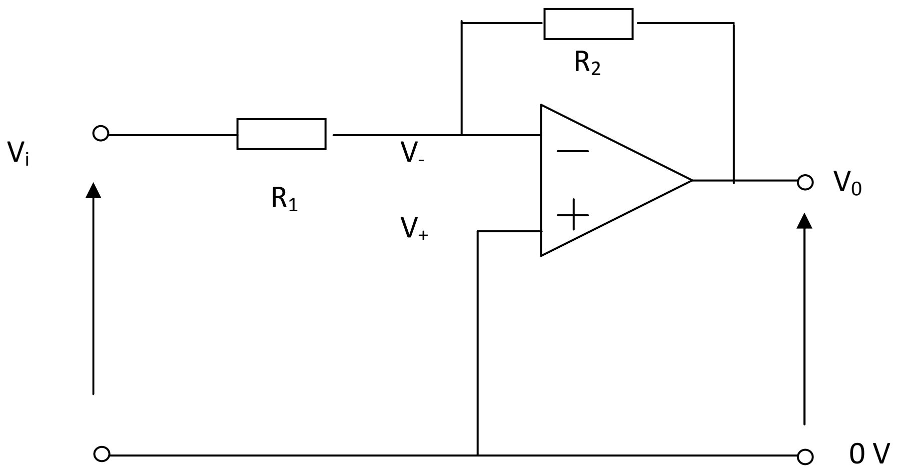
Figure 2

(a) At a mid-band frequency where C_1 may be considered an open circuit, calculate the value of R_2 required to give a voltage gain of 50 dB between input and output. The operational amplifier may be considered ideal. [8]

$$\text{Voltage Gain} = 20\log_{10}\left(\frac{V_o}{V_i}\right)$$

$$\frac{V_0}{V_i} = 10^{\frac{50}{20}} = 316.3$$

We ignore the capacitor for mid-band frequency. We get a standard inverting op-amp circuit. $V_- \approx 0$ comprising a "virtual earth":



Summing currents at - node:

$$\frac{V_i - 0}{R_1} = \frac{0 - V_0}{R_2}$$

$$\frac{V_0}{V_i} = GAIN = -\frac{R_2}{R_1} = -316.3$$

Thus: $R_2 = 6.33 M\Omega$

(b) What is the mid-band input impedance of the circuit? [2]

Virtual earth at V- node means that the input impedance is just $R_1 = 20 K\Omega$

(c) Calculate the value of C_1 required to have a 3 dB high frequency cut-off of 6kHz, i.e., where the circuit gain drops to $\frac{1}{\sqrt{2}}$ of its mid-band value. [5]

Considering the effect of C_1 on the gain:

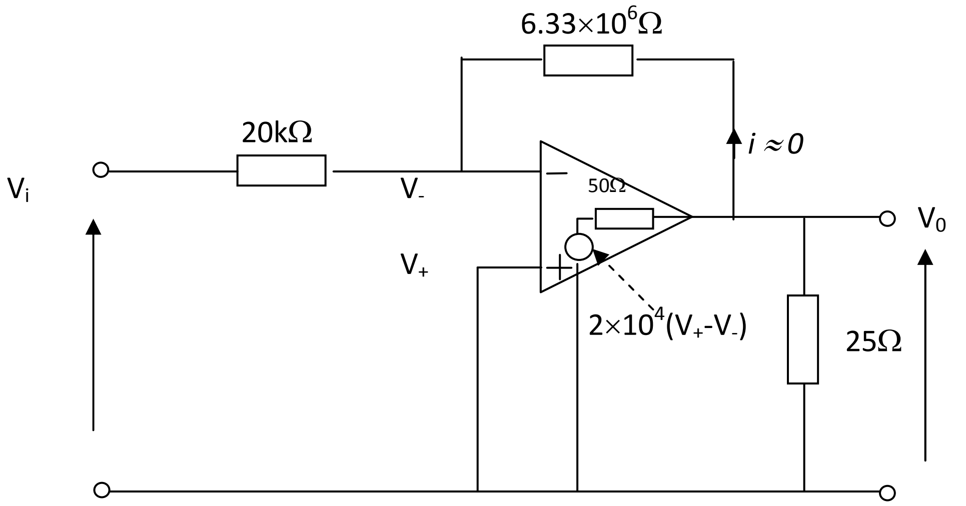
$$\text{GAIN} = \frac{-R_2 \parallel C_1}{R_1} = -\frac{\frac{R_2}{R_1}}{(1 + j\omega C_1 R_2)}$$

This drops to $\frac{1}{\sqrt{2}}$ of its mid-band value when $\omega C_1 R_2 = 1$

$$C_1 = \frac{1}{2\pi \cdot 6 \times 10^3 \cdot 6.33 \times 10^6} = 4.19 pF$$

(d) If a practical operational amplifier has an open loop voltage gain of 20,000 and an output impedance of 50 Ω, but is otherwise ideal, what is the actual mid-band voltage gain in dB when the circuit drives a load impedance of 25 Ω? [15]

Let us consider the effects of finite open loop gain and output impedance:



Since the load impedance is much smaller than the feedback impedance, the current through R_2 may be neglected if summing currents at the output node

Summing currents at the V_- node:

$$\frac{V_i - V_-}{2\times10^4} = \frac{V_- - V_0}{6.33\times10^6} \qquad (1)$$

Summing currents at output:

$$\frac{V_0}{25} \approx i \approx \frac{2\times10^4\left(V_+ - V_-\right) - V_0}{50} \qquad (2)$$

But $V_+ = 0$. Then:

$$2V_0 \approx -2\times10^4 V_- - V_0$$

$$V_0 \approx -\frac{2\times10^4}{3}V_-$$

$$V_- \approx -\frac{3}{2\times10^4}V_0$$

Substituting into (1)

$$V_0 = -\frac{\frac{633}{2}}{\left(\frac{3}{2\times10^4}+1+\frac{3\times633}{4\times10^4}\right)} = -302V_i$$

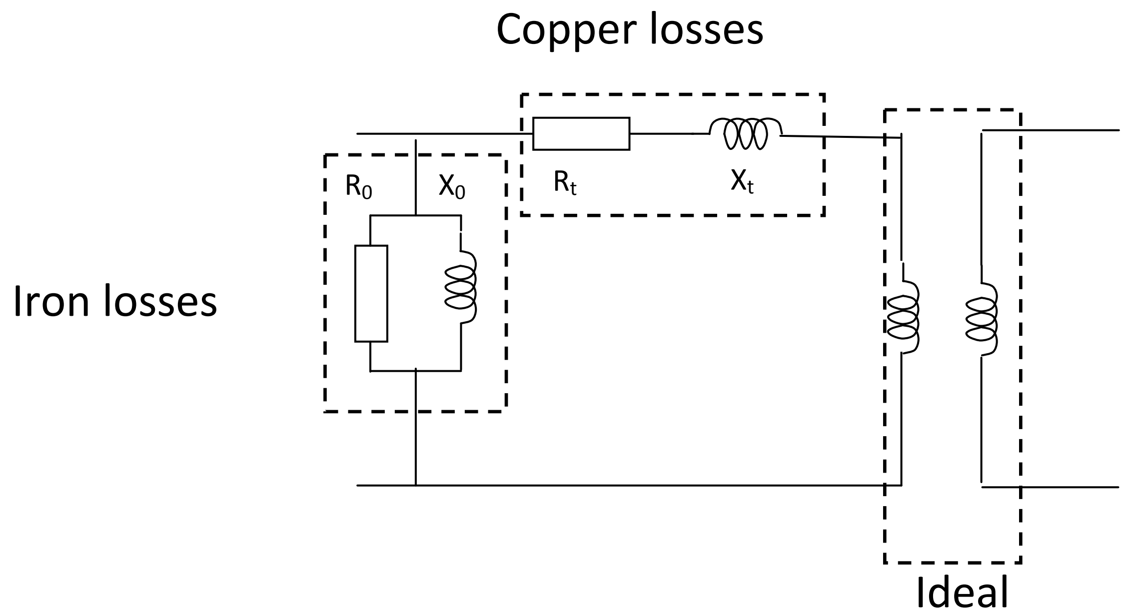
GAIN = $20\log_{10}(302) = 49.6$

3 **(short)**

(a) What are copper loss and iron loss in the context of power transformers? Indicate how each of these corresponds to elements of a simple transformer equivalent circuit, and explain how each may be measured. [3]

Copper loss: resistance of windings+ leakage inductance of windings.

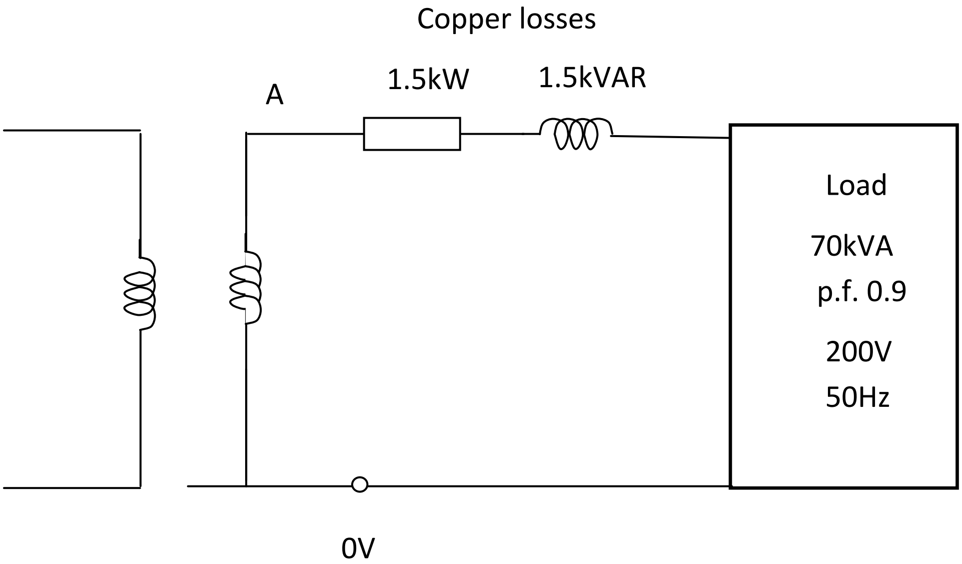
Iron loss: Hysteresis in iron magnetisation curves causes power loss, and the shape also causes some apparent inductive loss. Eddy currents also cause similar effects. (distinguishable by changing frequency).



We can measure iron losses by working at the rated voltage, but with open circuit output (when copper losses $\rightarrow$ 0)

We can measure copper losses at the rated current, but with short circuit output (when iron losses $\rightarrow$ 0)

(b) A transformer consumes real and reactive power of 3kW and 3 kVAR respectively when providing its full load of 70 kVA with a lagging power factor of 0.9 at 200 V and 50 Hz. Under these conditions it may be assumed that copper and iron losses are equal, that the reactive power of the magnetising reactance and leakage reactances are equal, and that supply has very low impedance. Calculate the output voltage under no-load conditions. [7]



$S = \left(P^2 + Q^2\right)^{\frac{1}{2}}$ $\qquad S_{LOAD} = 70kVA$

$P_{LOAD} = S.\cos\varphi = 63kW$

$Q_{LOAD} = S\sin\varphi = S\sqrt{1-\cos^2\varphi} = 70\sqrt{1-(0.9)^2} = 30.51kVAR$

$S = VI$

$I = \dfrac{70\times10^3}{200} = 350A$

Losses are split equally between copper and iron. For voltage calculations the iron losses may be neglected, since in the simple model they occur in parallel with the transformer input. The copper losses are then 1.5kW and 1.5kVAR.

At point A, total power = (63+1.5) kW

Total reactive power $Q_{Tot} = (30.51+1.5)kVAR$

$S = \sqrt{P^2 + Q^2} = 72kVA$

But I = 350A

So $VA = \dfrac{72100}{350} = 206V$

This will be the no load voltage, since with no load the copper losses have no effect.

4 **(short)** A battery, modelled as a 12 V e.m.f. in series with a resistance of 2 Ω, is being charged by a constant current source through the network shown in Figure 4. The multirange ammeter A_m used to measure the battery current drops a voltage of 1 V at full scale deflection on all ranges. Here it is set to its 10 A range and reads 1 A.

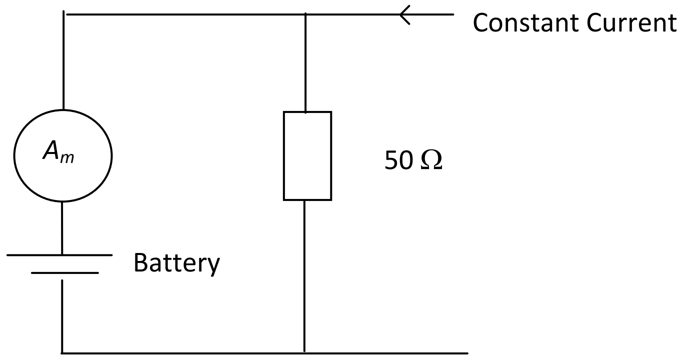
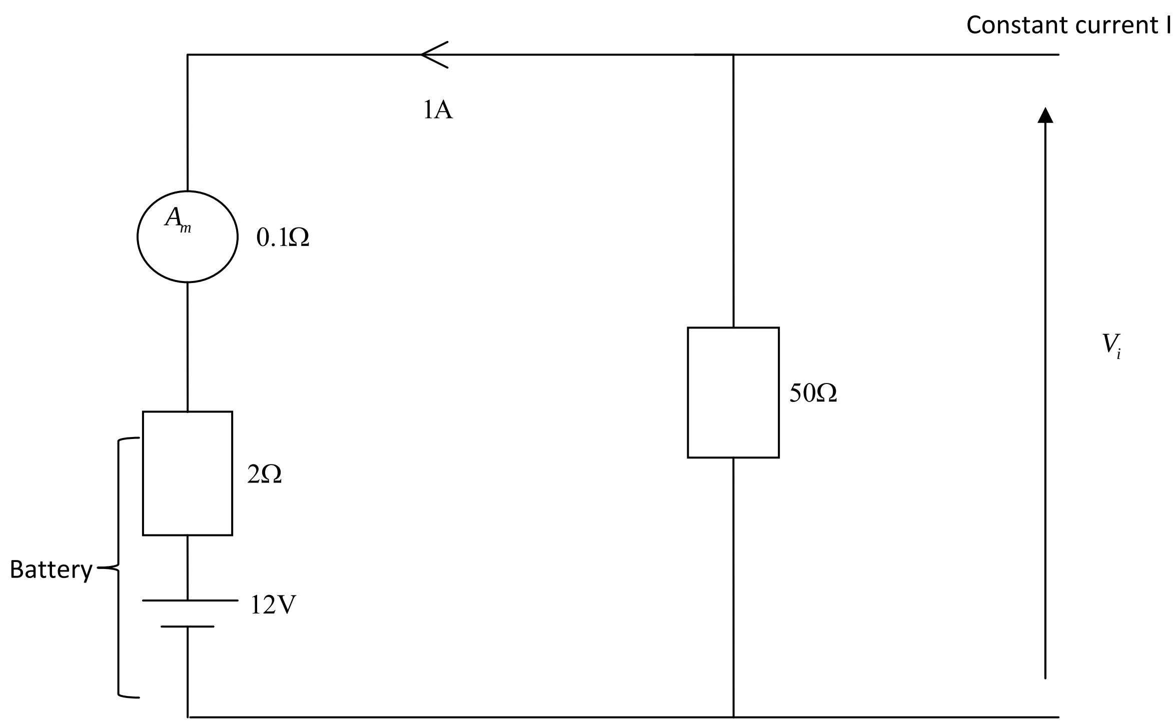


Figure 4

(a) What is the value of the constant current supply? [5]

The ammeter A_m, dropping 1V and on its full range 10A, is modelled as a 0.1Ω resistance:



When A_m reads 1A, for left hand circuit, the KVL gives:

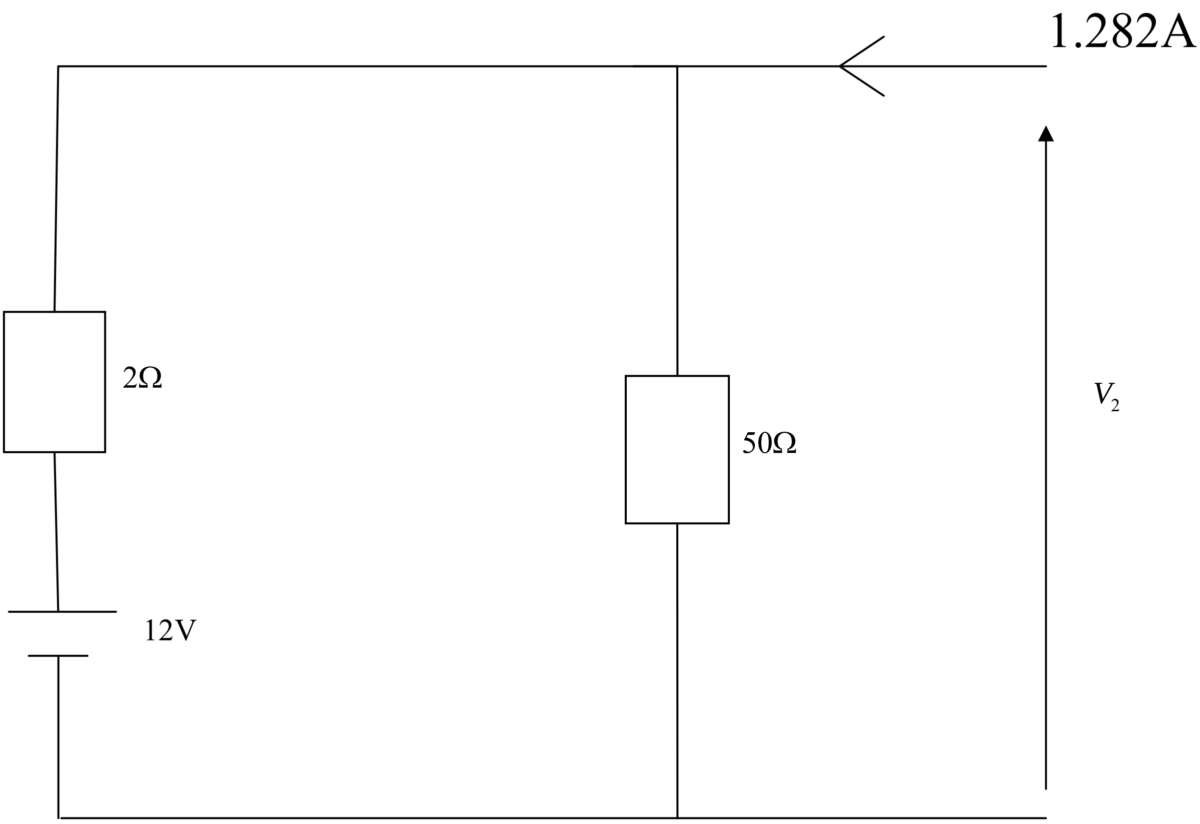
$V_i = 12 + 1(2 + 0.1) = 14.1V$

Thus, current in $50\,\Omega$ resistor = $\frac{14.1}{50} = 0.282A$

So I=1.282A

(b) What would be the current into the battery if the ammeter was replaced with a wire of zero resistance? [5]

With the ammeter replaced the circuit becomes



The new voltage V_2 across the circuit can be derived from KCL:

$$\frac{V_2}{50} + \frac{V_2 - 12}{2} = 1.282$$

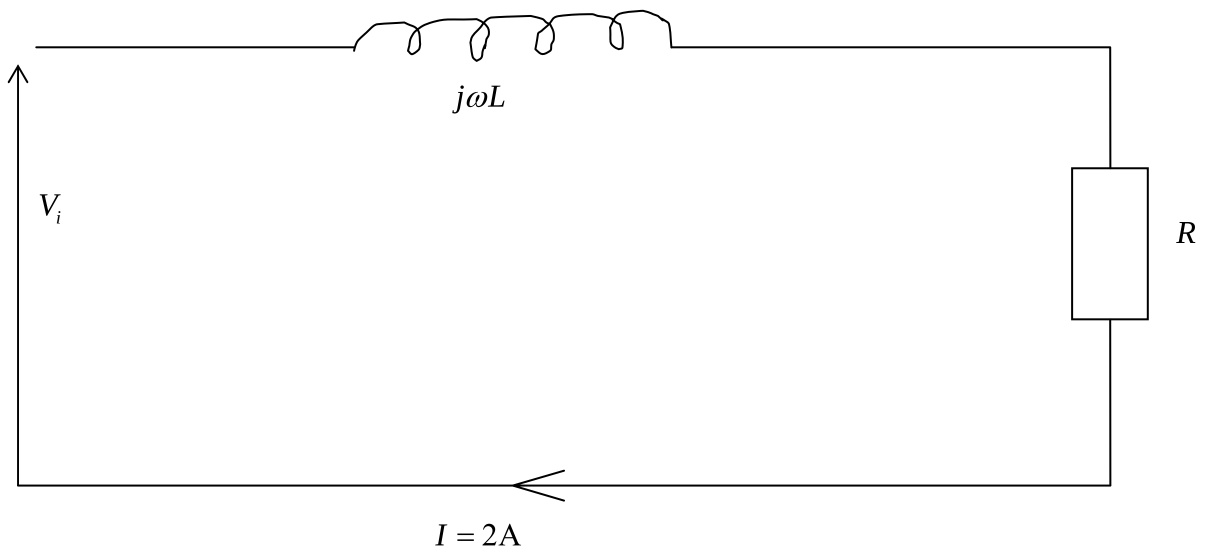
$V_2 = 14V$

Current through battery

$$\frac{14 - 12}{2} = 1\text{A}$$

5 **(short)** A small lab-scale crane lifts a mass of 2 kg at a speed of 0.5 ms^{-1}. There are no motor power losses, and the crane's 50 Hz AC motor can be modelled as an inductance *L*=50mH in series with a resistor *R*, where the latter models the conversion of the electrical input power to the mechanical output power of the motor.

(a) If the input current is 2 A when lifting this load, what is the power factor of this circuit? [5]



mgv= $2kg \times 0.5\frac{m}{s} \times 9.81 = 9.81W$

$VARS = I^2 \omega L$

$= I^2 .2\pi .50 \times 10^{-3} .50Hz$

=62.8

$VA = \sqrt{W^2 + VAR^2}$

=63.6

Power Factor = $\frac{W}{VA} = \frac{9.81}{63.6} = 0.154$

(b) If the crane is driven by a higher voltage AC supply through an ideal step-down transformer with a turns ratio of 30:1, what capacitance should be placed across the transformer's high voltage terminals to give the circuit a power factor of unity? [5]

Transformer ratio $\frac{1}{30}$ means $\frac{2}{30}A =$ current in for 2A out

Input voltage = $30 \times V_i$

As $\omega L = 2\pi.50.50 \times 10^{-3} = 15.7\Omega$

$$\omega LI = 31.4V$$

IR=4.9V

$V_i = \sqrt{31.4^2 + 4.9^2} = 31.8V$

$\cos\varphi = \dfrac{9.8}{2.318} = 0.154$

=31.8

$\cos\varphi = \dfrac{9.8}{2 \times 31.8} = 0.154$

Input voltage = 954V

A capacitor C in parallel across the input has

$VARS = \dfrac{V^2}{\dfrac{1}{\omega C}} = V^2 \cdot 2\pi \cdot 50 \cdot C = 62.8$

$C = 2.2 \times 10^{-7} F$

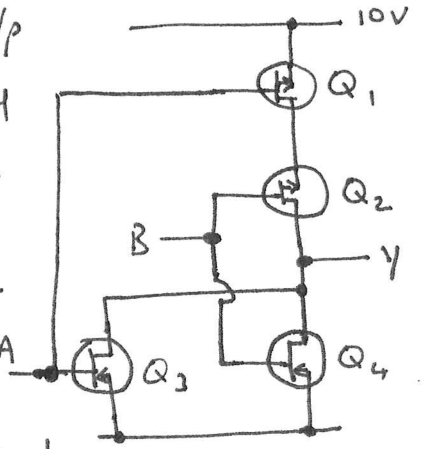
Section B

6(a)(i) **NMOS**. Upper transistor acts to mimic a resistive load (see lecture notes)

By inspection acts as inverter: $V_{out} = \overline{V_{in}}$

(ii) **CMOS**

Input A	Input B	Q_1	Q_2	Q_3	Q_4	o/p
L	L	ON	ON	OFF	OFF	H
L	H	ON	OFF	OFF	ON	L
H	L	OFF	ON	ON	OFF	L
H	H	OFF	OFF	ON	ON	L



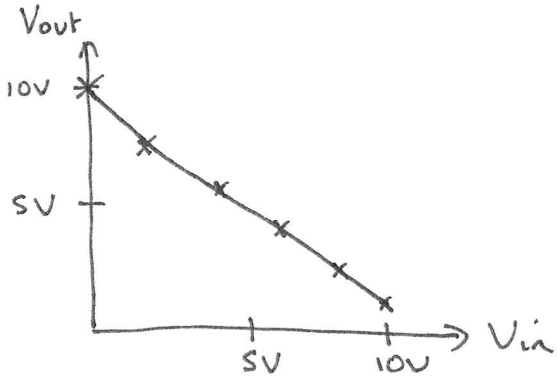
$\Rightarrow$ NOR $\quad Y = \overline{A+B}$

N.B. $Y = \overline{A}.\overline{B}$ also accepted.

(b) For top transistor, $V_{GS} = V_{DS}$

See attached solution on characteristic

$V_{in} = V_{GS}$	$V_{out} = V_{DS}$
0V	10V
2V	7.5V
4V	5.9V
6V	4.2V
8V	2.4V
10V	1V

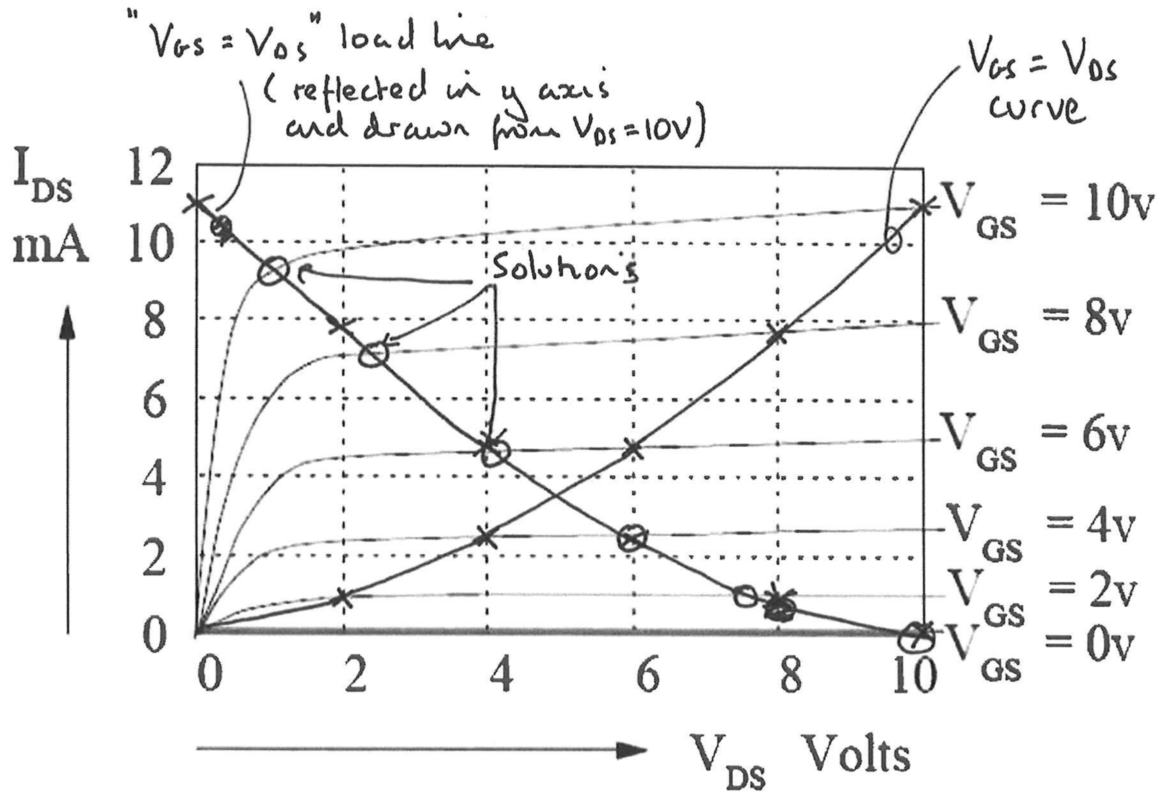


N.B. The slope of the $V_{GS} = V_{DS}$ is approximately the same as a 900 Ω resistor (see lecture notes). A load line based on this, with some explanation, would have been accepted as a reasonable approximation.

(c) Several reasons acceptable (2 for full marks)

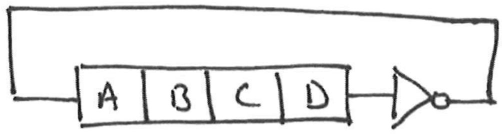
e.g.
- only consumes power when switching
- transistor is smaller than resistor
- better transfer characteristic (more abrupt).

PART IA Paper 3 Q6 Candidate Number ____________________



NOTE: The graph in Figure 6(c) is reproduced here. You should use this to help you complete Question 6. You must attach this sheet to your answer.

7 (a)



Start 1st sequence with A = B = C = D = 0, 2nd sequence with missing value eg. 0010

Sequence 1

A	B	C	D
0	0	0	0
1	0	0	0
1	1	0	0
1	1	1	0
1	1	1	1
0	1	1	1
0	0	1	1
0	0	0	1

then repeats

Sequence 2

A	B	C	D
0	0	1	0
1	0	0	1
0	1	0	0
1	0	1	0
1	1	0	1
0	1	1	0
1	0	1	1
0	1	0	1

then repeats.

N.B. 16 variations – all present

(b) Sequence 1

A	B	C	D	Output
0	0	0	0	0
1	0	0	0	1
1	1	0	0	2
1	1	1	0	3
1	1	1	1	4
0	1	1	1	5
0	0	1	1	6
0	0	0	1	7

CD \ AB	00	01	11	10
00	0	X	2	1
01	7	X	X	X
11	6	5	4	X
10	X	X	3	X

So $0 = \bar{A}.\bar{D}$

$1 = A.\bar{B}$

$2 = B.\bar{C}$

$3 = C.\bar{D}$

8.

```
main   movlw 0x31 ;   moves 0x31 into W                       ~1
       movwf FSR ;    moves (address) 0x31 into               ~1
                      FSR (to set up indirect addressing)     ~2
       call sr ;      calls subroutine labelled sr
       decf FSR ;     decrements FSR (now                     ~1
                      pointing to 0x30)
       call sr ;      calls subroutine                        ~2
end    sleep ;        ends programme                          ~1
.......
sr     rrf INDF ;     rotates right FSR contents              ~1
                      (effectively ÷2)
       movlw 0x10 ;   move 0x10 into W                        ~1
       addwf INDF ;   add 10 to contents of FSR               ~1
                      (i.e. original no./2 + 16 (decimal))
       return ;       return to main programme                ~2
```

(a) contents of 0x30 = $\frac{20}{2} + 16 = 26$ (0x1A)

" " 0x31 = $\frac{50}{2} + 16 = 41$ (0x29)

" " W = 16 (0x10) as last time it is changed is in the 2nd call of the subroutine

(b) main programme 8 cycles

subroutine 5 cycles.

subroutine called twice

$\Rightarrow$ total run time = $8 + (2 \times 5)$ cycles

$= 18 \times 50\text{ns}$

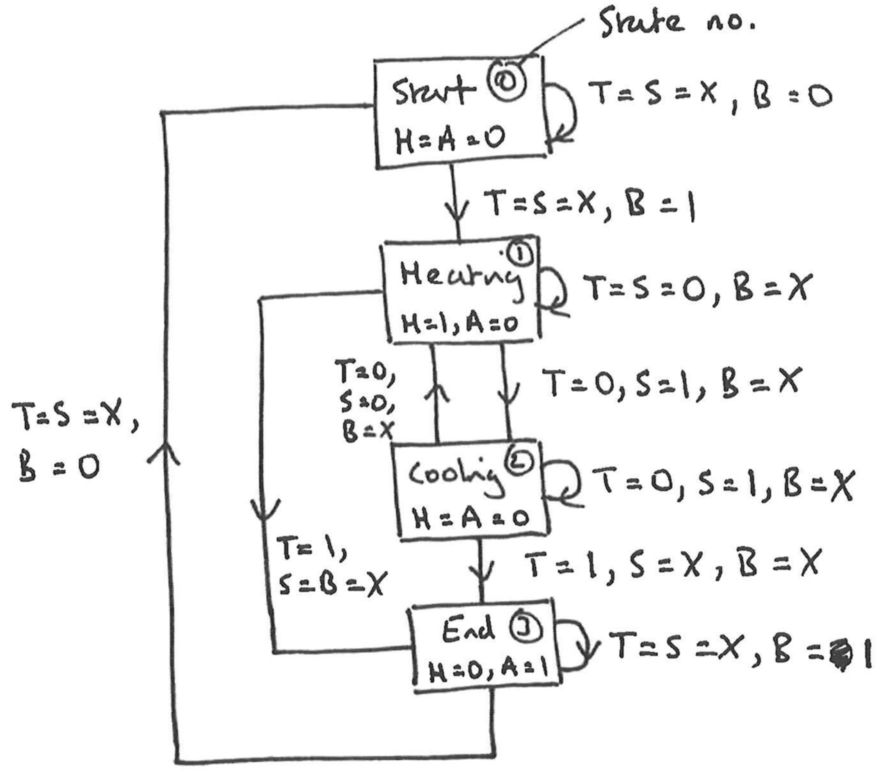
$= 0.9\ \mu\text{s}$ or 900ns.

Clock = 20MHz

$\Rightarrow$ 1 cycle = 1 period

$= \frac{1}{20 \times 10^6} = 50\text{ns}$

9 (a) Inputs T, S, B Outputs H, A.



(b) 4 states ⇒ 2 bistables

State Allocation

State	X	Y
0	0	0
1	0	1
2	1	0
3	1	1

(other allocations are obviously valid)

(c) State Transition Table:

Inputs			Current State		Next State					
T	S	B	Q_x	Q_y	Q_{x+1}	Q_{y+1}	J_x	K_x	J_y	K_y
X	X	0	0	0	0	0	0	X	0	X
X	X	1	0	0	0	1	0	X	1	X
0	0	X	0	1	0	1	0	X	X	0
0	1	X	0	1	1	0	1	X	X	1
1	X	X	0	1	1	1	1	X	X	0
0	0	X	1	0	0	1	X	1	1	X
0	1	X	1	0	1	0	X	0	0	X
1	X	X	1	0	1	1	X	0	1	X
X	X	0	1	1	0	0	X	1	X	1
X	X	1	1	1	1	1	X	0	X	0

(d) Outputs

Q_x	Q_y	H	A
0	0	0	0
0	1	1	0
1	0	0	0
1	1	0	1

$\Rightarrow$ $H = \bar{Q}_x . Q_y$

$A = Q_x . Q_y$

J_x

B = 0

Q_xQ_y \ TS	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	X	X	X	X
10	X	X	X	X

B = 1

Q_xQ_y \ TS	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	X	X	X	X
10	X	X	X	X

$$J_x = S.Q_y + T.Q_y$$

K_x

B = 0

Q_xQ_y \ TS	00	01	11	10
00	X	X	X	X
01	X	X	X	X
11	1	1	1	1
10	1	0	0	0

B = 1

Q_xQ_y \ TS	00	01	11	10
00	X	X	X	X
01	X	X	X	X
11	0	0	0	0
10	1	0	0	0

$$K_x = \bar{B}.Q_y + \bar{T}.\bar{S}.\bar{Q}_y$$

J_y

B = 0

Q_xQ_y \ TS	00	01	11	10
00	0	0	0	0
01	X	X	X	X
11	X	X	X	X
10	1	0	1	1

B = 1

Q_xQ_y \ TS	00	01	11	10
00	1	1	1	1
01	X	X	X	X
11	X	X	X	X
10	1	0	1	1

$$J_y = B.\bar{Q}_x + T.Q_x + \bar{S}.Q_x$$

K_y

B = 0

Q_xQ_y \ TS	00	01	11	10
00	X	X	X	X
01	0	1	0	0
11	1	1	1	1
10	X	X	X	X

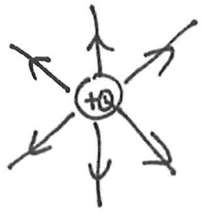
B = 1

Q_xQ_y \ TS	00	01	11	10
00	X	X	X	X
01	0	1	0	0
11	0	0	0	0
10	X	X	X	X

$$K_y = \bar{B}.Q_x + \bar{T}.S.\bar{Q}_x$$

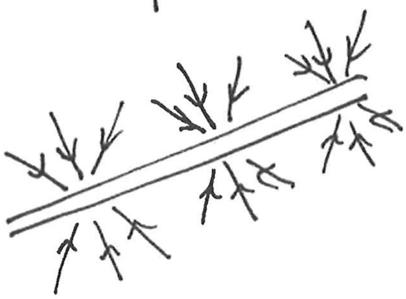
Section C

10 a) i)



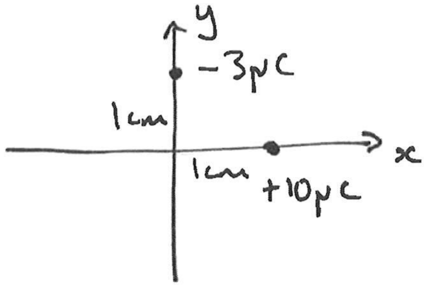
Spherically symmetric

ii)



Radially symmetric

b)



i) $|\underline{E}| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

$$\underline{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{10\times10^{-6}}{(10^{-2})^2}(-\underline{i}) + \frac{3\times10^{-6}}{(10^{-2})^2}\,\underline{j}\right)$$

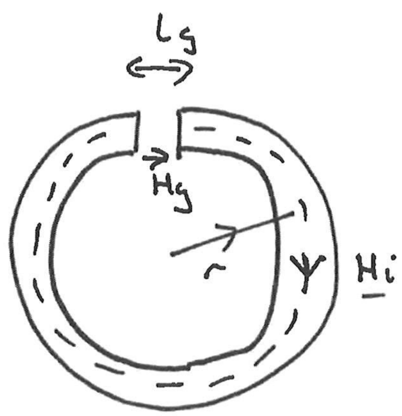
$$= -898\,\underline{i} + 269\,\underline{j} \text{ MV/m}$$

ii) $\underline{F} = q\underline{E}$

$$= -2\times10^{-6}\,(-898\,\underline{i} + 269\,\underline{j})\times10^{6}$$

$$= 1800\,\underline{i} - 539\,\underline{j} \text{ N}$$

11 a)



Ampère's Law $\quad H_g l_g + H_i l_i = NI$

$$H_g = \frac{NI - H_i l_i}{l_g}$$

Flux continuity $\quad B_g \cancel{A} = B_i \cancel{A}$ ← area

$$B_g = \mu_0 H_g$$

$$\Rightarrow \quad B_i = \frac{\mu_0 NI}{l_g} - \mu_0 \frac{l_i}{l_g} H_i$$

$$= \frac{4\pi \times 10^{-7} \times 400 \times 4}{2 \times 10^{-3}} - \frac{4\pi \times 10^{-7} \times 2\pi \times 0.2}{2 \times 10^{-3}} H_i$$

$$= 1.005 - 7.9 \times 10^{-4} H_i$$

Drawing load line on curve in data book

$$\Rightarrow B_i \sim 0.9T$$

b) Faraday $\quad emf = (-) \frac{d\phi}{dt}$ (ignore the −)

$$= N \frac{\Delta\phi}{\Delta t} = \frac{10 \times B \times A}{\Delta t}$$

Since 10 turn coil is small wrt area of air gap, neglect fringing effects.

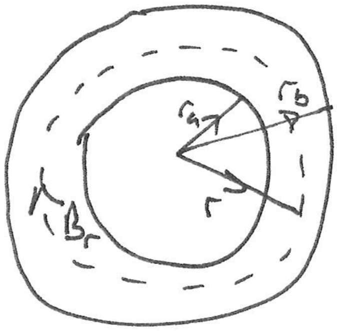
$$= \frac{10 \times 0.9 \times (10^{-3})^2}{2 \times 10^{-3}}$$

$$= 4.5 \text{ mV}$$

12 a) i) Solid area of pipe $= \pi(r_b^2 - r_a^2)$

$$\Rightarrow J = \frac{I}{\pi(r_b^2 - r_a^2)}$$

ii)



Ampère's Law

$$\oint \underline{B} . \underline{dl} = \mu_0 I' \text{ (where } I' \text{ is current enclosed by line of integration)}$$

For path shown $|\underline{B}_r| 2\pi r = \mu_0 I'$

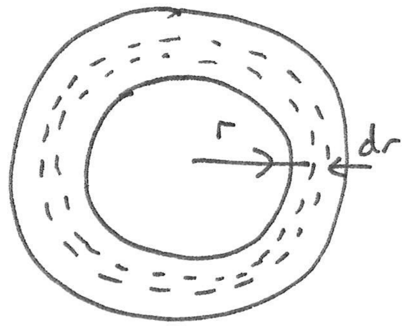
$$|\underline{B}_r| = \frac{\mu_0 I'}{2\pi r}$$

For two simple cases

$r < r_a$, $I' = 0 \Rightarrow |\underline{B}_r| = 0$

$r > r_b$, $I' = I \Rightarrow |\underline{B}_r| = \frac{\mu_0 I}{2\pi r}$

For $r_a < r < r_b$, need to calculate enclosed current



$$dI' = J\, 2\pi r\, dr$$

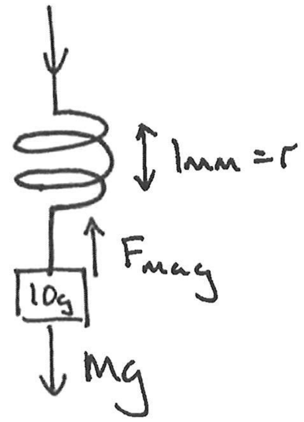
$$= \frac{2\pi I r\, dr}{\pi(r_b^2 - r_a^2)}$$

$$I' = \frac{2I}{(r_b^2 - r_a^2)} \int_{r_a}^{r} r\, dr$$

$$= \frac{2I}{(r_b^2 - r_a^2)} \left[\frac{r^2}{2}\right]_{r_a}^{r} = \left(\frac{r^2 - r_a^2}{r_b^2 - r_a^2}\right) I$$

$$\Rightarrow \text{for } r_a < r < r_b \quad |\underline{B}_r| = \frac{\mu_0}{2\pi r}\left(\frac{r^2 - r_a^2}{r_b^2 - r_a^2}\right) I$$

b)



NB wires attract due to self induced magnetic force so F_{mag} direction ↑

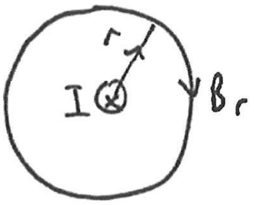
For spring constant k

$$mg - F_{mag} = kr$$

$$F_{mag} = mg - kr$$

$$= 10\times10^{-3}\times 9.81 - 30\times10^{-3}$$

$$= 0.0681\ N.$$



From Ampère's law

$$B_r\, 2\pi r = \mu_0 I$$

$$B_r = \frac{\mu_0 I}{2\pi r}$$

The two coils are equivalent to two parallel wires, each of length $l = \pi d$, seperated by 1mm

$$F_{mag} = B_r . I . l$$

$$= \frac{\mu_0 I}{2\pi r} . I . \pi d = \frac{\mu_0 I^2 d}{2r}$$

$$\Rightarrow \quad I^2 = \frac{2r}{\mu_0 d} F_{mag}$$

$$= \frac{2\times10^{-3}}{4\pi\times10^{-7}\times 0.05} \times 0.0681$$

$$= 2167\ A^2$$

$$\Rightarrow \quad I = 46.6\ A$$