ENGINEERING TRIPOS PART 1A

Thursday 7 June 2012 9 to 12

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 **(short)** An L-shaped simply-supported beam is shown in Fig. 1. A frictionless pulley with radius $L/4$ is attached to the beam at point D. A horizontal force of $2P$ is applied at one end of a cable, which travels around the pulley and is attached to the beam at point B. The beam is also loaded with a uniformly distributed load of w (per unit length), where the total distributed load is P (i.e. $P = wL$). The self-weight can be neglected.

(a) Find the reactions at the two supports (A and C). [3]

(b) Draw the shear force and bending moment diagrams on sketches of the structure. Label all salient values and clearly indicate the sign convention used. [7]

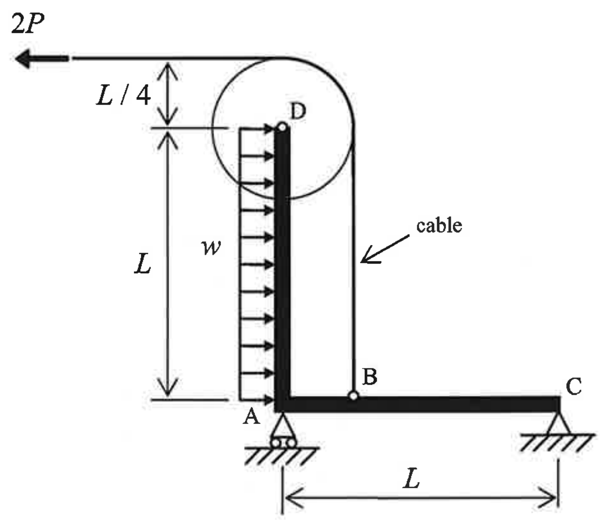

Fig. 1

2 (short) A pin-jointed truss is shown in Fig. 2. All members have the same cross-sectional area A and are made of a linear elastic material with Young's modulus E. The self-weight can be neglected. A vertical load of W is applied to the structure at joint F as shown in the figure.

(a) Find the horizontal displacement at joint A. [7]

(b) Write down the vertical displacement at joint A. [3]

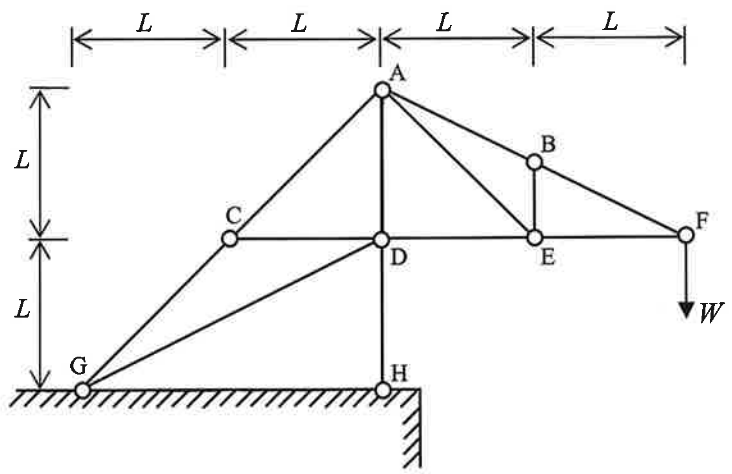

Fig. 2

3 **(short)** A parabolic three-pin arch is shown in Fig. 3. There is a vertical point load P at point B, and a uniformly distributed load w (per unit horizontal length) between C and D, where the total distributed load is P (i.e. $P = wL$). The shape of the arch is described by $y = x^2/(4L)$, where the origin is at point C.

(a) Find the reaction forces at supports A and E. [4]

(b) Find the location within segment DE where the magnitude of the bending moment is maximum. [6]

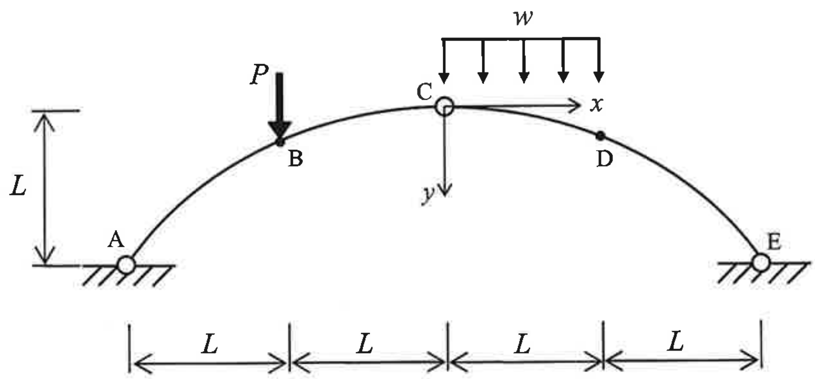

Fig. 3

4 **(short)** A simply supported beam with a uniform bending stiffness *EI* and two uniformly distributed loads is shown in Fig. 4. The distributed loads have magnitudes of *w* (per unit length) but are applied in opposite directions. The self-weight can be neglected.

Find an equation which describes the vertical deflection between points B and C as a function of x. [10]

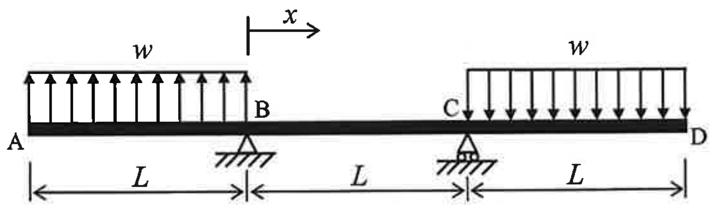

Fig. 4

5 **(long)** An elastic strut with uniform bending stiffness EI is shown in Fig. 5(a). At end A the strut is fixed. At end B the strut is restricted from rotating, but it may translate horizontally and vertically. A vertical force P is applied at the top of the strut. A moment M_o is generated at the bottom end of the strut in the buckled configuration. Assume that the strut is initially straight and that buckling occurs in the x-y plane.

(a) Draw the buckled shape and a free-body diagram of the strut alone with reaction forces. [3]

(b) The horizontal displacement of the buckled shape is defined to be $v(x)$, where x is the distance from end A. Derive an equation for the buckled shape. [9]

(c) By applying boundary conditions, derive an equation for the Euler buckling load P_E. [7]

(d) How does the Euler buckling load for the strut in Fig. 5(a) compare with the Euler buckling load for the strut in Fig. 5(b) which cannot displace horizontally at point D? [2]

(e) A stiff beam is supported by two fixed-ended struts as shown in Fig. 5(c). The struts are steel and have the following properties:

Young's modulus,	$E = 210$ GPa
Second moment of area,	$I = 100$ cm^4
Cross-sectional area,	$A = 10$ cm^2
Yield stress,	$\sigma_y = 275$ MPa

(i) Assuming the beam is rigid and that it remains perfectly horizontal, and that the struts are perfectly straight, calculate the failure load W of the structure. [3]

(ii) Now consider a more realistic case where the struts are not perfectly straight. Assume that Fig. 5(d) shows experimental results for the strut configuration in Fig. 5(a), and that the curve labelled lower bound is representative of the imperfection of the struts in the frame in Fig. 5(c). *Estimate* a new failure load for the frame in Fig. 5(c). (Note that r is the radius of gyration.) [6]

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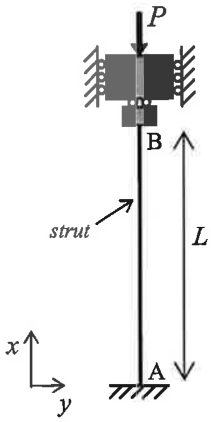

Fig. 5(a)

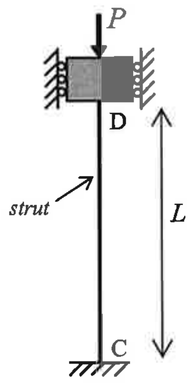

Fig. 5(b)

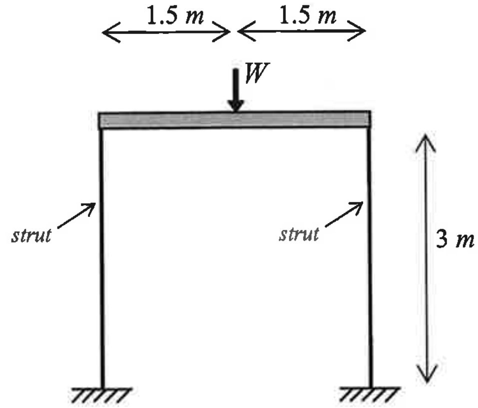

Fig. 5(c)

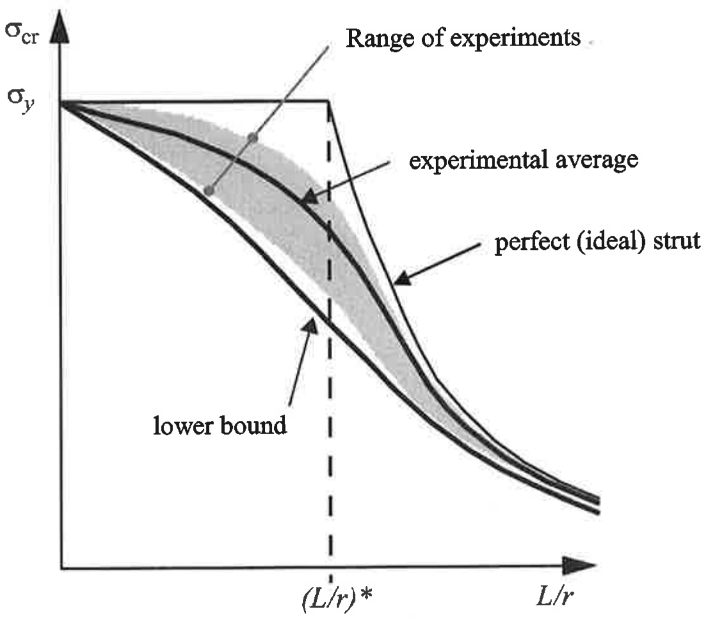

Fig. 5(d)

6 **(long)** A *light* cable which supports two masses (m_1 and m_2) is shown in Fig. 6. The cable is attached to the wall at point A and mass m_2 at point D. Mass m_1 is attached to a frictionless pulley which is supported by the cable at point B. At location C, the cable is draped over a non-smooth circular pipe which cantilevers from the wall beyond (the axis of the pipe is perpendicular to the axis of the cable). Assume the cable is long enough that mass m_2 will never hit the pipe at point C.

(a) If the system is in static equilibrium, find the tension in each segment of the cable. [3]

(b) For $m_1 = m_2$, determine the minimum coefficient of friction μ required to maintain static equilibrium for a given angle β. (This may require more than one expression.) [9]

(c) Assume the non-smooth pipe is a thin-walled circular section with radius R and thickness t ($R >> t$), and that the pipe has a total length of L. The cable is draped over the mid-length of the pipe. For $m_1 = m_2$ and $\beta = 30^\circ$:

(i) Find the maximum bending moment and shear force in the pipe. [5]

(ii) Find the maximum tensile stress in the pipe and locate where this stress occurs (sketch if necessary). [6]

(iii) Find the maximum shear stress in the pipe and locate where this stress occurs (sketch if necessary). [7]

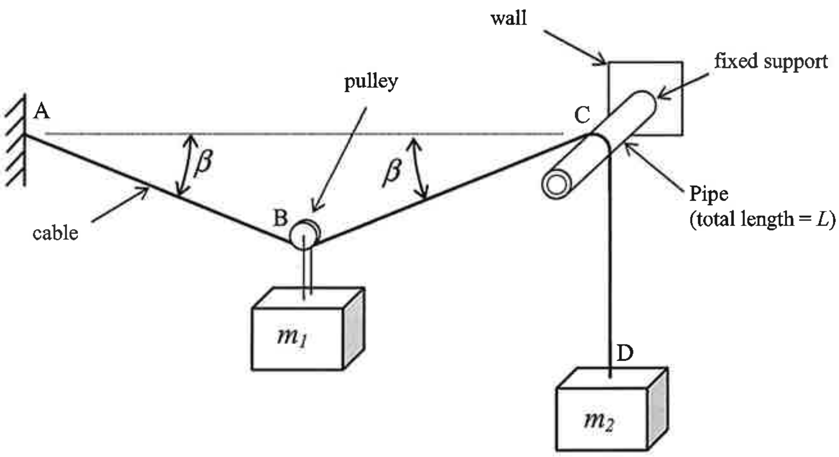

Fig. 6

SECTION B

7 **(short)** Two identical flat metallic blocks are placed in contact, with nominal contact area A_n, and loaded by a normal force P. The surfaces have hardness H and the shear strength of asperities is τ_s.

(a) Briefly explain what is meant by the true area of contact (A_t). How is it influenced by the hardness H? [4]

(b) Derive an expression for the coefficient of friction μ between the two surfaces in terms of τ_s and H. Show that μ is independent of the choice of metal if $\tau_s = k$, the material shear yield strength. Under what circumstances would the assumption $\tau_s = k$ break down? [6]

8 **(short)**

(a) An amorphous thermoplastic is cold drawn into fibres for use in a fibre reinforced composite. Briefly describe the microstructural changes occurring during cold drawing. What is the effect of cold drawing on (i) the stiffness, (ii) the strength and (iii) the ductility of the polymer? [5]

(b) The fibres (modulus E_f) are arranged in parallel in a matrix (modulus E_m) as shown in Fig. 8. Show that the elastic modulus of the composite in the x-direction is given by:

$$E = E_f V_f + E_m \left(1 - V_f\right)$$

where V_f is the volume fraction of fibres. [5]

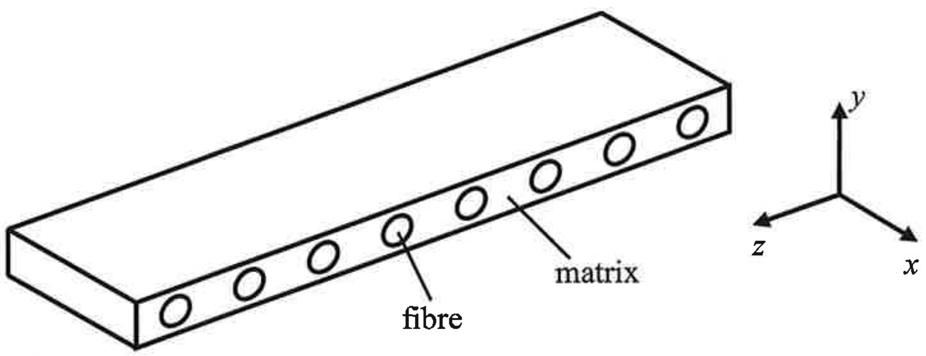

Fig. 8

9 **(long)**

(a) Briefly describe the physical meaning of (i) the stress intensity factor (K) and (ii) the strain energy release rate (G) in fracture mechanics problems. For each quantity, explain the condition for crack propagation. [6]

(b) A large rectangular steel plate ($K_{IC} = 60$ MPa $\sqrt{\text{m}}$, $\sigma_y = 320$ MPa) contains a long weld (Fig. 9(a)). Within the weld, the fracture toughness is βK_{IC} ($\beta \leq 1$). The plate is subjected to stresses σ and $\sigma/2$ as shown. NDT reveals through-thickness cracks of lengths $2a_1$ and $2a_2$ oriented as shown in Fig. 9(a).

(i) Derive an expression for the range of β that would result in the crack oriented parallel to the weld propagating first. [5]

(ii) If $\beta = 0.75$ and $a_1 = a_2 = 15$ mm, calculate the stress σ required for the plate to fracture. Comment on the validity of your analysis. [5]

(c) The plate is replaced by a fibre reinforced composite version of thickness t subjected to the same external stresses. Close inspection of a crack oriented as shown in Fig. 9(b) reveals fibres bridging the crack tips. This results in a constant resistance stress σ_f (N m^{-2}) to crack opening over a length b as shown in Fig. 9(c).

(i) Given that four point forces F (N) located a distance $\pm x$ from the crack centre line (Fig. 9(d)), induce a stress intensity factor:

$$K = \frac{F}{t\sqrt{\pi a}} \frac{2a}{\sqrt{a^2 - x^2}} \quad ,$$

show that the stress intensity factor due to the bridging fibres is given by

$$K = -\frac{2a\sigma_f}{\sqrt{\pi a}} \left[\frac{\pi}{2} - \sin^{-1}\left(\frac{a-b}{a} \right) \right] \quad .$$ [8]

(ii) What is the *total* stress intensity factor for the cases $b = 0$ and $b = a$? [4]

(iii) With the aid of a sketch, discuss briefly the physical basis of the toughening effect of the fibres. [2]

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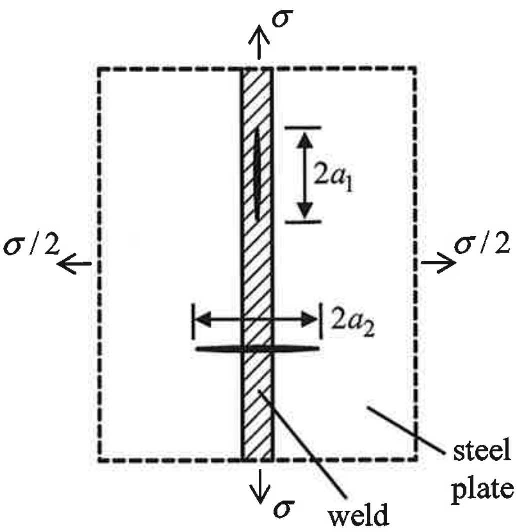

Fig. 9(a)

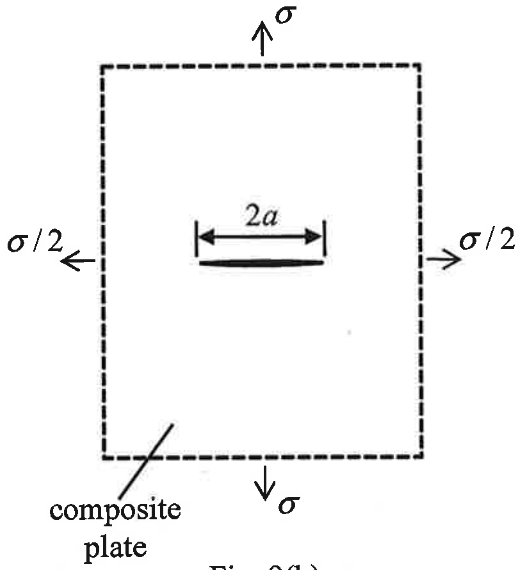

Fig. 9(b)

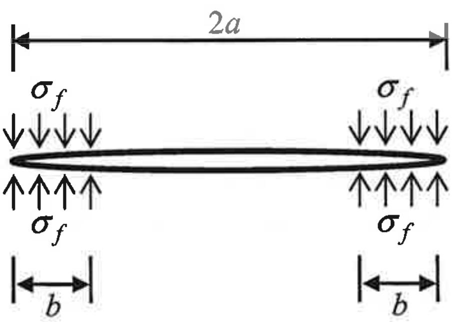

Fig. 9(c)

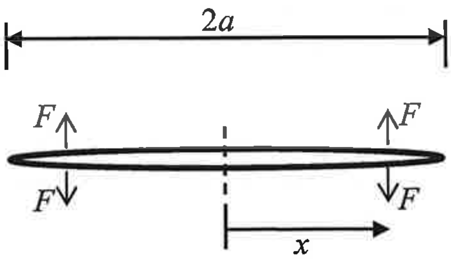

Fig. 9(d)

10 **(long)** A mast has length $L = 5$ m and a square cross-section with side length d, as shown in Fig. 10(a). Wind loading induces a uniform pressure P $(\mathrm{N\,m^{-2}})$ on one face.

(a) The mast is to be as light as possible, while ensuring the tip deflects less than a specified amount when subjected to the wind loading. The side length d may be varied. Assume that the mast is rigidly supported at its base.

(i) Show that the performance index for the mast material is given by:

$$M = \frac{E^{2/3}}{\rho} \quad .$$

[5]

(ii) Using the selection charts in the Materials Data Book, identify the three most suitable material classes. Suggest one practical limitation of each. [6]

(b) A material is now to be selected so that the mast is as cheap as possible. Subjected to a wind loading $P = 1\ \mathrm{kN\,m^{-2}}$, the stress everywhere in the mast must not reach yield *and* the tip deflection δ must not exceed 50 mm. Again, assume that the mast is rigidly supported at its base. Using the data in Table 10, choose between steel and aluminium alloy. Identify the active constraint. [9]

(c) In fact, the base of the mast is not rigid, but can be modelled as a rigid plate sitting on an elastic foundation. The foundation has a square cross section of side length b and height h, as shown in Fig. 10(b). This foundation is constrained so that no lateral expansion is possible in the x or y directions, but is otherwise free to deform. Neglecting wind loading, and assuming a vertical force W due to the weight of the mast, derive an expression for the vertical deflection of the rigid plate supporting the mast. [10]

Material	Young's modulus (GPa)	Yield stress (MPa)	Density ($\mathrm{kg\,m^{-3}}$)	Cost (£ $\mathrm{kg^{-1}}$)
Steel	210	300	7800	0.5
Aluminium alloy	70	250	2700	0.9

Table 10

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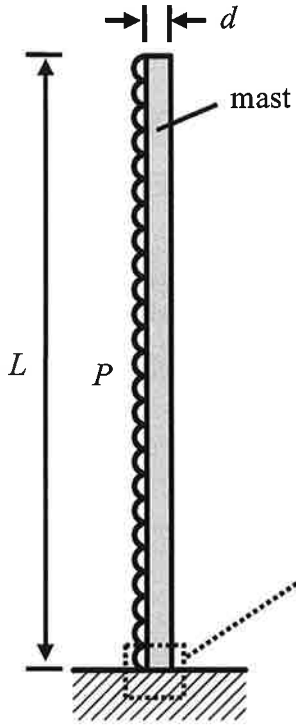

Fig. 10(a)

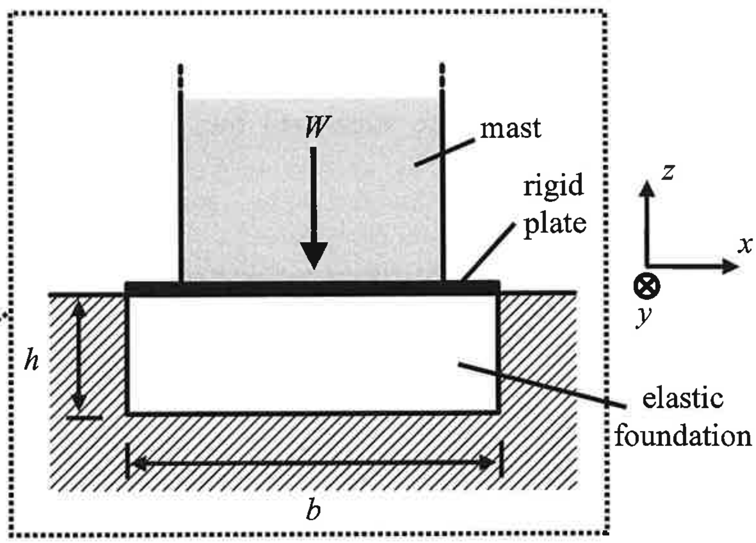

Fig. 10(b)

11 **(short)**

(a) Briefly describe the types of atomic bonding present in (i) metals, (ii) ceramics and (iii) polymers. Hence account for the differences in Young's modulus of the materials listed in Table 11. [6]

(b) Aluminium has an FCC atomic structure. Sketch the FCC unit cell, and show the location of (i) a tetrahedral hole and (ii) an octahedral hole. [4]

Material	**Young's modulus (GPa)**
Aluminium	70
Alumina	300
Polyethylene (PE)	0.8

Table 11

12 **(short)** A specimen of medium carbon steel is tested to failure in a tensile testing machine. The resulting nominal stress - nominal strain curve is shown in Fig. 12. The gauge section of the test specimen is initially cylindrical in shape.

(a) Account for the shape of the nominal stress - nominal strain curve, illustrating your answer with sketches of the test specimen where appropriate. [5]

(b) Derive an expression for the true stress σ_t in terms of σ_n and ε_n stating any assumptions. Calculate the true stress and true strain when the test specimen has reached its tensile strength. [5]

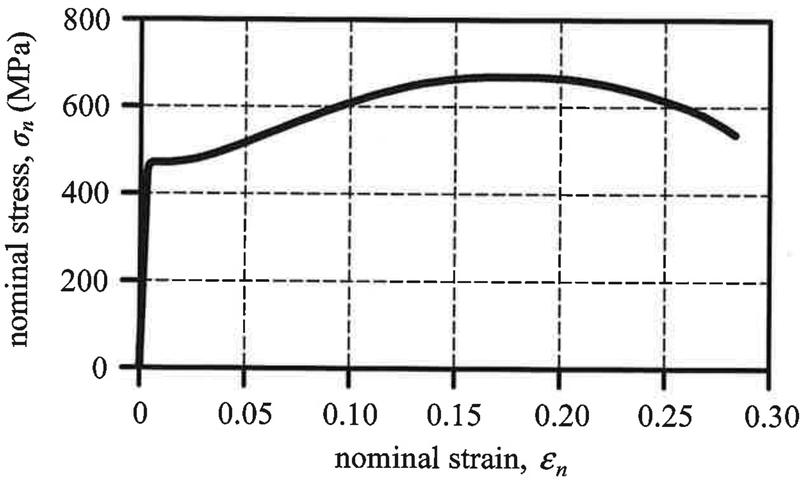

Fig. 12

END OF PAPER

2012 PART 1A P2 - Structures and Materials Dr M DeJong

1. (a) $V_A = 2P$, $V_C = -2P$, $H_A = P$; (b) -
2. (a) $\left(5+8\sqrt{2}\right)*WL/AE$; (b) $-5*WL/AE$
3. (a) $V_A = 7P/8$, $V_E = 9P/8$, $H_A = 5P/4$, $H_E = -5P/4$; (b) $x = 7L/5$
4. $v(x) = \dfrac{1}{EI}\left(-\dfrac{wLx^3}{8} + \dfrac{wL^2x^2}{4} - \dfrac{wL^3}{12}\right)$
5. (a) - ; (b) $v(x) = \dfrac{M_o}{P}\left(1-\cos\sqrt{\dfrac{P}{EI}}x\right)$; (c) $P_E = \dfrac{\pi^2 EI}{L^2}$; (d) $P_{E,Fig.5b} = 4P_{E,Fig.5a}$;

   (e) i) $W_{\text{failure}} = 461$ N ; ii) $W_{\text{failure}} \approx 250$ N
6. (a) $T_{AB} = T_{BC} = \dfrac{m_1 g}{2\sin\beta}$, $T_{CD} = m_2 g$; (b) $\mu = \left|\dfrac{\ln(2\sin\beta)}{\beta+\pi/2}\right|$;

   (c) i) $V_{max} = \sqrt{3}mg$, $M_{max} = \sqrt{3}mgL/2$;

   ii) $\sigma_{max} = \dfrac{\sqrt{3}}{2}\dfrac{mgL}{\pi R^2 t}$ at extreme fiber at support ;

   iii) $\tau_{max} = \dfrac{\sqrt{3}mg}{\pi R t}$ at neutral axis between support and cable

Part IA 2011-12 Paper 2 Section B: short answers

Q7. (b) $\mu = \frac{\tau_s}{H}$

Q9. (b) (i) $\beta < \frac{1}{2}\sqrt{\frac{a_1}{a_2}}$ (ii) σ = 276 MPa

(c) (ii) $K = \sigma\sqrt{\pi a}$, $K = \left(\sigma - \sigma_f\right)\sqrt{\pi a}$

Q10. (a) (ii) ceramics, composites, natural materials (b) steel (£38.95)

(c) deflection $\delta = \frac{Wh}{Eb^2}\left(\frac{1-\nu-2\nu^2}{1-\nu^2}\right)$

Q12. (b) $\sigma_t = \sigma_n\left(1+\varepsilon_n\right)$, σ_t = 791 MPa