

Answers to questions

1. $0.171 + i(-0.214 + 2\pi n)$
2. (a) 0 (b) $1/6$
3. Eigenvalues 2 and $1/2$. Eigenvectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (or any multiples of them)
4. (a) $\left[\frac{1}{3}, \frac{1}{3\sqrt{3}}, \pm \sqrt{\frac{3}{27}} \right]^T$ (b) $\frac{\sqrt{3}}{2}$
5. (a) $\frac{1}{9} - \frac{\omega^2 - 9}{\omega^4 - 2\omega^2 + 81} \sin \omega t - \frac{4\omega}{\omega^4 - 2\omega^2 + 81} \cos \omega t + e^{-t} (A \cos \sqrt{5} t + B \sin \sqrt{5} t)$
 (b) $\frac{1}{9} - \frac{1}{12} \cos 3t$
6. $e^{-t} \sin t$
7. (a) $-1, 1/2$ (b) $e^{-t} - e^{-2t}$ (c) $\begin{cases} 0 & t < 0 \\ (t-1)e^{-t} + e^{-2t} & 0 \leq t \end{cases}$
 (d) $\begin{cases} 0 & t < 0 \\ (t-1)e^{-t} + e^{-2t} & 0 \leq t \leq T \\ Te^{-t} - (e^T - 1)e^{-2t} & t > T \end{cases}$
8. 13,334
9. $\sum_{n=1}^{\infty} \left(-\frac{1}{n} \cos \frac{n\pi}{2} + \frac{2}{n^2 \pi} \sin \frac{n\pi}{2} \right) \sin n\theta =$

$$\frac{2}{\pi} \sum_{m=1}^{\infty} (-1)^{m+1} \frac{\sin(2m-1)\theta}{(2m-1)^2} + \frac{1}{2} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin 2n\theta}{n}$$
10. (b) Saddles at $(0, 0)$, $(\pm 2, 0)$, $(0, -4)$.
 Minimum at $\left(\frac{2}{\sqrt{5}}, -\frac{8}{5} \right)$, Maximum at $\left(-\frac{2}{\sqrt{5}}, -\frac{8}{5} \right)$
12. $O(ndata)$