ENGINEERING TRIPOS PART IA

Wednesday 5 June 2013   9 to 12

Paper 1

MECHANICAL ENGINEERING

Answer all questions.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachment: Copy of Fig. 3(b) for question 3.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 (**short**)

(a) Find an expression for the hydrostatic force acting on a rectangular gate of width w (normal to the page), retaining water as shown in Fig. 1. [5]

(b) The gate is hinged at the bottom with a frictionless hinge. Find an expression for the horizontal restraining force F. [5]

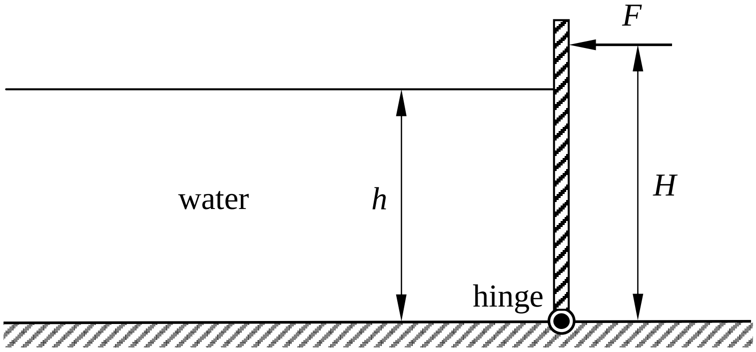

Fig. 1

2 (**short**) Air of density ρ flows through a sudden expansion in a pipe as shown in Fig. 2. The flow is uniform upstream and downstream of the expansion at sections 1 and 2, and the air may be assumed to be incompressible.

Assuming that the pressure p_b on the back-face of the expansion can be approximated as being equal to the upstream pressure p_1, construct an appropriate control volume and find an expression for the dimensionless pressure rise $(p_2 - p_1)/(\rho V_1^2)$ in terms of the area ratio A_1 / A_2. [10]

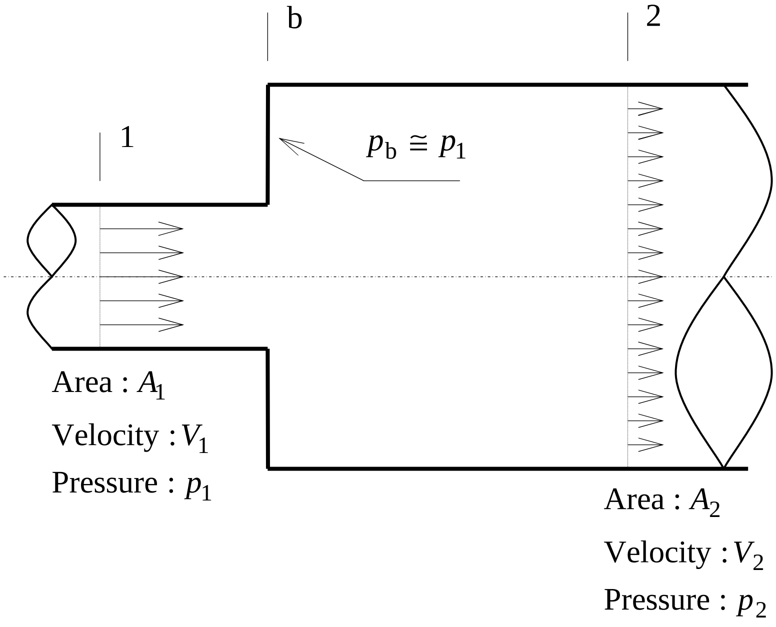

Fig. 2

3 (**long**) Water of density $\rho = 1000\,\mathrm{kg\,m^{-3}}$ discharges from a large tank through a nozzle of circular cross-section, to a horizontal plate a distance $h_1 = 10\,\mathrm{m}$ beneath it, as shown in Fig. 3(a). The cross-sectional area of the nozzle exit A_1 is $0.002\ \mathrm{m}^2$. The vertical distance between the top surface of the water and the nozzle exit is $h_2 = 5\,\mathrm{m}$. The cross-sectional area A_2 is much greater than A_1. The flow may be treated as frictionless throughout.

(a) Find the flow velocity V_1 and the mass flow rate $\dot{m}$ at the nozzle exit, station 1. [6]

(b) A non-dimensional pressure coefficient is defined as

$$C_p = (p - p_{atm}) / 0.5 \rho V_1^2$$

Calculate the values of C_p on the stagnation streamline in the centre of the jet at the nozzle exit 1 and at the stagnation point 0. Show that the pressure coefficient $C_{p,0}$ at point 0 is a function of the height ratio h_2 / h_1 only. [10]

(c) On Fig. 3(b), sketch roughly the C_p variation along the stagnation streamline from 1 to 0 and along a streamline on the surface of the plate from 0 to a large r value. [8]

(d) Calculate the value of the pressure gradient normal to the streamline on the edge of the spreading stream at point E where the radius of curvature is $R_C = 0.1\,\mathrm{m}$. Neglect the small height difference between E and the plate. [6]

(Cont.

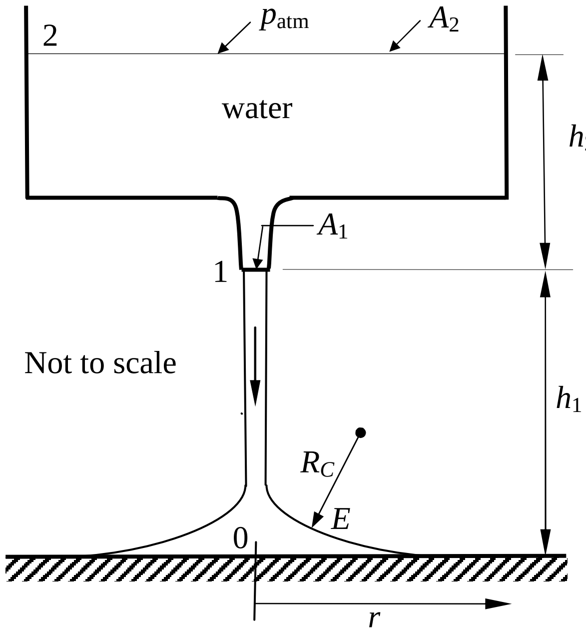
p_{atm}
A_2
2
water
h_2
A_1
1
Not to scale
h_1
R_C
E
0
r

Fig. 3(a)

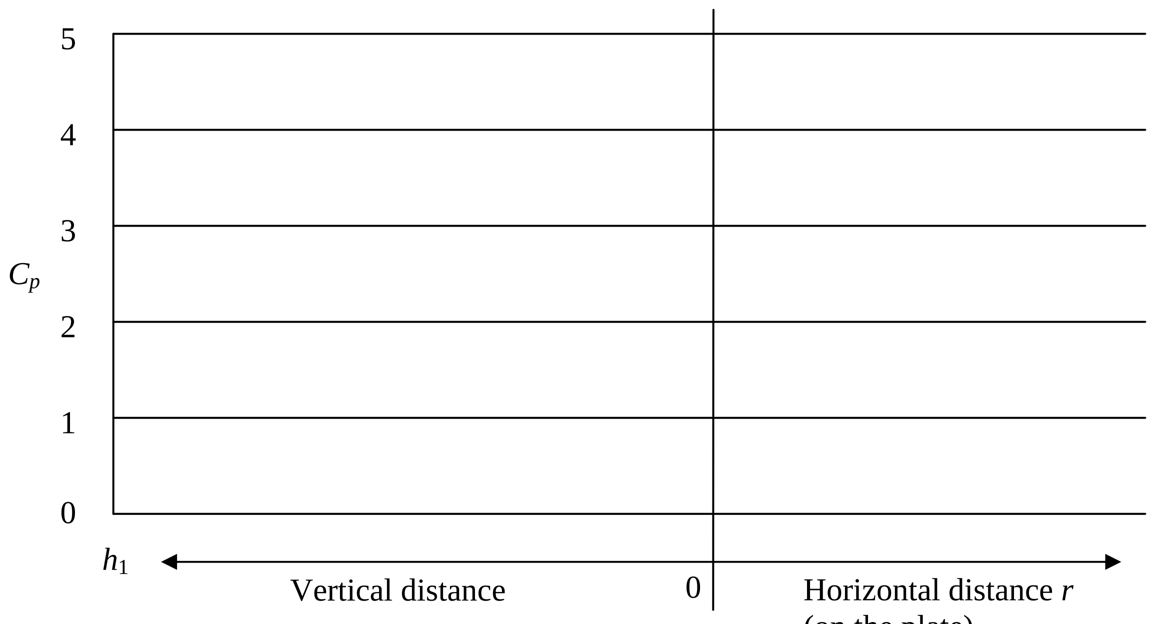
5
4
3
2
1
0
C_p
h_1
Vertical distance
0
Horizontal distance r
(on the plate)

Fig. 3 (b)

4 **(short)** A compressor is used to increase the density of air entering a car engine. Ambient air at 100 kPa and 300 K enters the compressor at a volumetric flow rate of 250 litres per second. The power input to the compressor is 20 kW.

(a) Calculate the air mass flow rate and the specific work input to the compressor. [3]

(b) Assuming the compression is adiabatic and reversible, and neglecting changes in kinetic and potential energy, calculate the temperature and pressure at the exit from the compressor. [5]

(c) Find the ratio of outlet to inlet air density. [2]

5 **(short)** Two streams of air are mixed in the device shown in Fig. 4. At inlet 1 the mass flow rate is $\dot{m}_1$ = 2 kg s^{-1} and the temperature and velocity are T_1 = 600 K and V_1 = 100 m s^{-1}, respectively. At inlet 2 the mass flow rate is $\dot{m}_2$ = 3 kg s^{-1} and the temperature and velocity are T_2 = 300 K and V_2 = 50 m s^{-1}, respectively. The exit velocity is V_3 = 125 m s^{-1}, and the entire system is at constant pressure.

(a) Assuming the flow is adiabatic, calculate the temperature of the air at the exit. [5]

(b) Calculate the rate of entropy generation within the mixing device. [5]

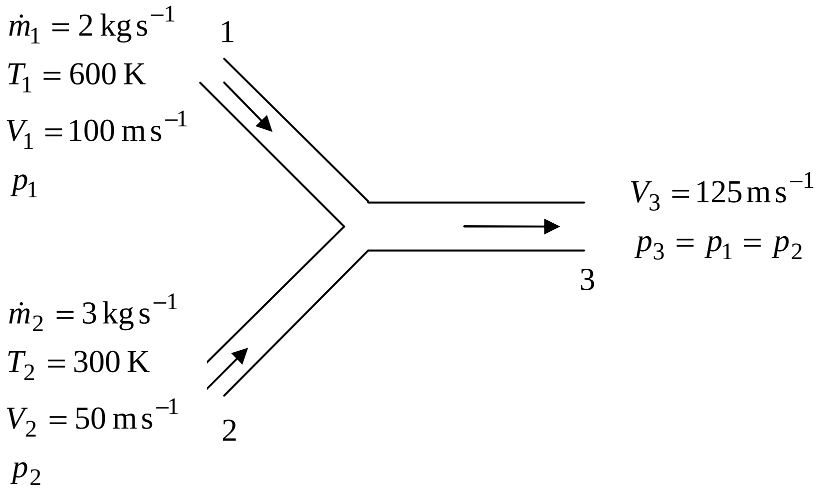

Fig. 4

6 **(long)** The performance of a spark ignition engine can be approximated by the air-standard Otto cycle. A modification to the Otto cycle takes into account the fact that part of the heat addition occurs after the volume has started to increase, *i.e.*, after the piston has started to move downwards. Therefore, consider a cycle identical to an Otto cycle except that the first 2/3 of the heat addition is at constant volume and the remaining 1/3 is at constant pressure.

(a) Sketch the modified cycle on a pressure-volume diagram and label the following points: 1 - beginning of compression; 2 - end of compression; 3 - end of constant-volume heat addition; 3′ - end of constant-pressure heat addition; 4 - end of expansion. [3]

(b) Consider the modified cycle where the equivalent heat input from the fuel is 1200 kJ per kg air (q_{2-3} = 800 kJ kg^{-1} and $q_{3-3'}$ = 400 kJ kg^{-1}), the compression ratio is $r_v = 9$, and the temperature and pressure before compression are T_1 = 300 K and p_1 = 100 kPa, respectively.

(i) Find the temperature and specific volume after compression. [5]

(ii) Find the maximum pressure of the cycle. [6]

(iii) Find the maximum temperature of the cycle. [6]

(iv) Find the heat rejected from the cycle. [5]

(c) Calculate the specific work output from the cycle. Also, calculate the maximum possible specific work output from a thermodynamic cycle operating with the same heat input and the same maximum and minimum temperatures. Comment on the difference between the two values. [5]

SECTION B

7 **(short)** Figure 5 shows a system of two identical frictionless pulleys, cables and three weights, all at rest. The two weights A and B, of mass *m* and 2*m* respectively, are on the ground and a third weight C , of mass M , is suspended. The combined mass of the pulleys and cables is negligible compared to m and M . When C is released from rest, A and C accelerate, while B just lifts off the ground with negligible acceleration.

(a) Show that the ratio of the tensions in the cables, T_1 / T_2, is equal to the ratio of the accelerations of C and A, a_2 / a_1. [5]

(b) Determine the ratio M / m . [5]

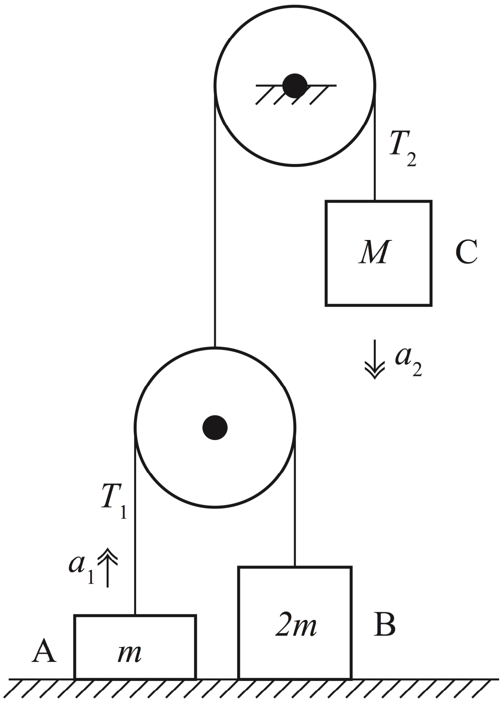

Fig. 5

8 **(short)**

(a) Find, from first principles, the polar moment of inertia of a uniform disc of mass m and radius a about a fixed axis through its centre of mass, G . [4]

(b) Find its moment of inertia about a fixed axis that is perpendicular to the disc and goes through a point that is halfway between its centre and its edge, indicated on Fig. 6 as point A . [2]

(c) The disc is in the vertical plane, and pivots about an axis through A that is perpendicular to the plane of the disc. Initially, the centre of mass G of the disc is at the same height as the pivot axis, and then the disc is released from rest. Find the initial angular acceleration of the disc. [4]

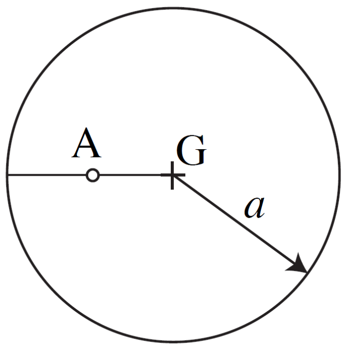

Fig. 6

9 **(long)** With reference to Fig. 7, a space ship is in a geostationary orbit of radius R_{P} around the Earth, such that it always stays above the same point on the equator as the Earth rotates about its axis with angular velocity ω .

(a) Find the radius of the geostationary orbit, R_{P} , in terms of the radius of the Earth *R*, the gravitational acceleration at the surface of the Earth *g* and the angular velocity ω . [10]

(b) The pilot of the space ship is instructed to climb to a higher circular orbit of radius R_{A} . The speed of the space ship is v_1 in the inner circular orbit and v_2 in the outer circular orbit. Show that $v_1 / v_2 = \sqrt{x}$, where $x = R_{\mathrm{A}} / R_{\mathrm{P}}$. [4]

(c) The orbital transfer is accomplished by a short burn at point P changing the speed to v_{P} , and another short burn at point A changing the speed from v_{A} to the final v_2 . Both impulses are tangential to the orbits. Write down the equations relating the speeds v_{A} and v_{P} that express the principles of conservation of energy and angular momentum for the transfer orbit, shown by the dashed line in Fig. 7 . [6]

(d) Hence, show that, in terms of the ratio of the orbital radii x , the fractional change in speed at point P is given by $(v_{\mathrm{P}} - v_1)/v_1 = \sqrt{2x/(x+1)} - 1$. Find also the fractional change in speed at A, $(v_2 - v_{\mathrm{A}})/v_2$, as a function of x . [10]

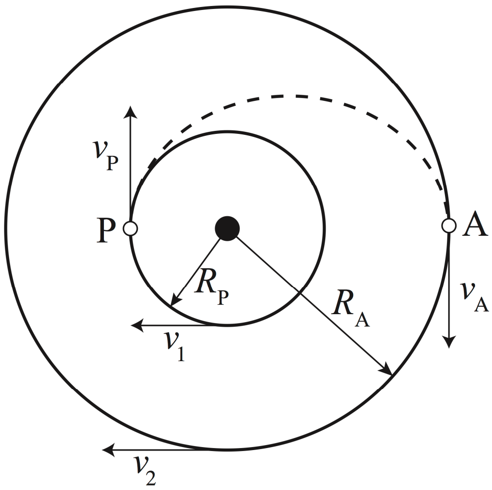

Fig. 7

(TURN OVER

10 **(short)** A particle P attached to a light elastic string is set in motion in a horizontal plane as shown in Fig. 8. At the instant shown its radial and angular motion with respect to origin O is defined by $r = 1$ m, $\dot{r} = 1\ \mathrm{m\,s^{-1}}$, $\ddot{r} = 0\ \mathrm{m\,s^{-2}}$, $\dot{\theta} = \sqrt{3}\ \mathrm{rad\,s^{-1}}$, $\ddot{\theta} = \sqrt{3}\ \mathrm{rad\,s^{-2}}$.

(a) What are the velocity and acceleration of P at the instant shown? [4]

(b) Show that, at the instant shown, the component of acceleration of P along the path is $3\ \mathrm{m\,s^{-2}}$. Determine the instantaneous radius of curvature of the path. [6]

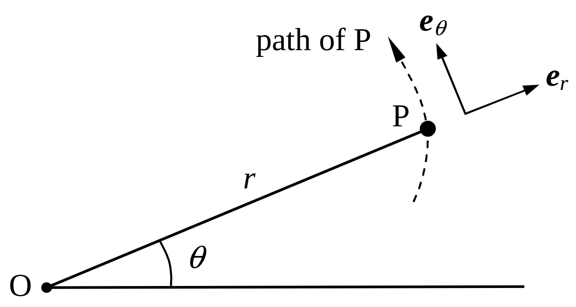

Fig. 8

11 **(short)** Consider the spring-dashpot system shown in Fig. 9. A light rigid frame is attached on one side to a wall by a spring of stiffness k and a viscous dashpot of rate λ, and on the other side to a viscous dashpot of rate 2λ. Point O is forced harmonically by an external force f.

(a) Derive a differential equation relating the displacements x and y. [4]

(b) If $k = 10\ \mathrm{N\,m^{-1}}$ and $\lambda = 0.1\ \mathrm{N\,s\,m^{-1}}$, find the amplitude of x such that the amplitude of y is 10 mm at a forcing frequency of 10 Hz. [4]

(c) Find the amplitude of the force f that must be applied at point O to produce the above displacements. [2]

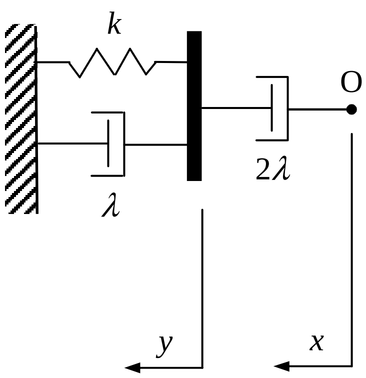

Fig. 9

12 **(long)** Figure 10 shows a schematic diagram of a heavy instrument placed on a floor-mounted table. The mass of the instrument is m while that of the table is $4m$. The two masses are connected by a spring of stiffness k and the table suspension is modelled as a spring of stiffness $2k$. The displacements of the floor, table and instrument are y, x_1 and x_2 respectively.

(a) Derive the equation of motion for this system. [6]

(b) If the floor is fixed, find the resonant frequencies of the system and sketch the corresponding mode shapes. [10]

(c) During an earthquake the floor moves harmonically with frequency ω such that $y = y_0 \cos \omega t$. Derive expressions for x_1 and x_2, each as a function of ω. Sketch the variation of the displacement ratios x_1 / y_0 and x_2 / y_0 with ω. [14]

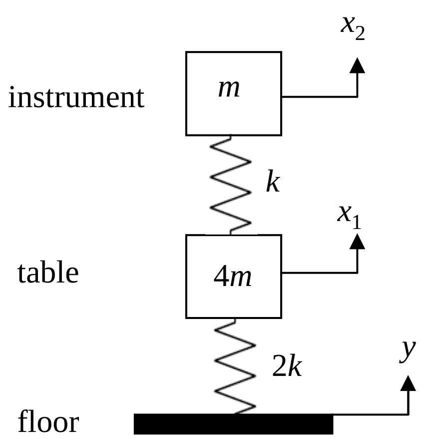

Fig. 10

END OF PAPER

Answers to questions

1. (a) $\dfrac{\rho g w h^2}{2}$ (b) $\dfrac{\rho g w h^3}{6H}$

2. $\dfrac{A_1}{A_2}\left(1-\dfrac{A_1}{A_2}\right)$

3. (a) 9.91 m/s 19.81 kg/s (b) 0.0 3.0 (c) 2.94 MPa/m

4. (a) 0.29 kg/s –68.9 kJ/kg (b) 368.5 K 205.5 kPa (c) 167 %

5. (a) 415.0 K (b) 236.9 J/K/s

6. (b) 0.0957 m^3/kg 722.5 K 5514 kPa 2235 K –505.2 kJ/kg
   (c) 694.8 kJ/kg 1039 kJ/kg

7. (a) $\dfrac{a_1}{a_2} = \dfrac{T_2}{T_1} = 2$ (b) $M = 8m$

8. (a) $\dfrac{ma^2}{2}$ (b) $\dfrac{3ma^2}{4}$ (c) $\ddot{\theta} = \dfrac{2g}{3a}$

9. (a) $R_P = \left(\dfrac{GM}{\omega^2}\right)^{1/3}$ (c) $mv_P R_P = mv_A R_A$ $\dfrac{mv_P^2}{2} - \dfrac{GMm}{R_P} = \dfrac{mv_A^2}{2} - \dfrac{GMm}{R_A}$
   (d) $\dfrac{v_2 - v_A}{v_2} = 1 - \sqrt{\dfrac{2}{x+1}}$

10. (a) $\dot{\mathbf{r}} = \hat{\mathbf{e}}_r + \sqrt{3}\,\hat{\mathbf{e}}_\theta$ $\ddot{\mathbf{r}} = -3\hat{\mathbf{e}}_r + 3\sqrt{3}\,\hat{\mathbf{e}}_\theta$ (b) $\dfrac{4m}{3\sqrt{3}}$

11. (a) $ky + 3\lambda\dot{y} = 2\lambda\dot{x}$ (b) 16.98 mm (c) 0.118 N

12. (b) $\omega^2 = (7 \pm \sqrt{17})\dfrac{k}{8m}$ 0.64 –0.39 (c) $x_1 = \dfrac{(k - m\omega^2)2ky}{\Delta}$ $x_2 = \dfrac{2k^2 y}{\Delta}$

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