ENGINEERING TRIPOS PART IA

Thursday 6 June 2013 9 to 12

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 **(short)** A rigid triangular block resting on a rough horizontal surface is shown in Fig. 1. The block is infinitely long (into the page). There is water on the right side of the block, but not the left. The height h of the block and the water is the same. The density of the block is twice the density of water.

(a) Assuming the block does not slide, find the minimum width x for the block to remain stable. [7]

(b) If $x = h$, find the minimum angle of friction needed to prevent sliding. [3]

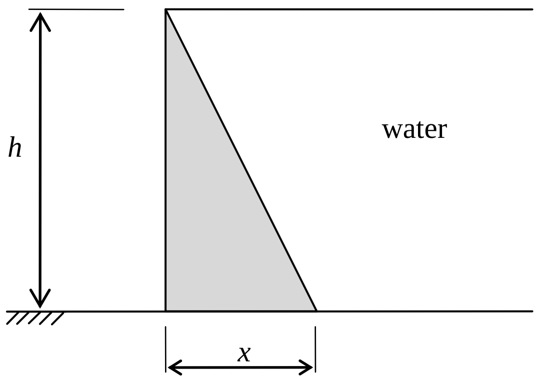

Fig. 1

2 **(short)** A simply-supported steel beam with a vertical point force P is shown in Fig. 2(a). A cross-section of the beam, which is composed of two identical C-shaped members bolted back-to-back, is shown in Fig. 2(b). Both C-shaped members have a uniform thickness of 1 cm. Pairs of bolts are located at a spacing of 0.5 m along the length of the beam. The beam bends about the x-x axis shown in Fig. 2(b). Each bolt has a shear capacity of 1 kN, and the yield stress of the C-shaped members is 355 MPa.

Calculate the maximum vertical point force P that the beam can sustain. [10]

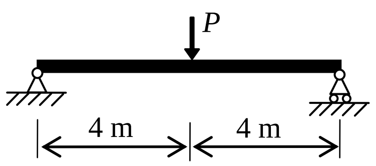

Fig. 2(a)

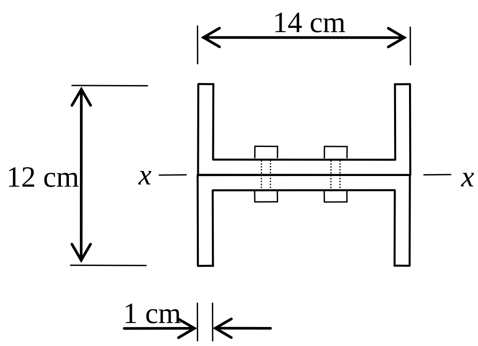

Fig. 2(b)

3 **(short)** A simply-supported bent beam is shown in Fig. 3. There is a uniformly distributed force of $w = 8$ kN m^{-1} acting perpendicular to the beam between points B and C.

(a) Find the reaction forces at supports A and C. [3]

(b) Draw the shear force diagram for the beam, giving salient values. [5]

(c) Sketch the bending moment diagram for the beam. It is not necessary to calculate values. [2]

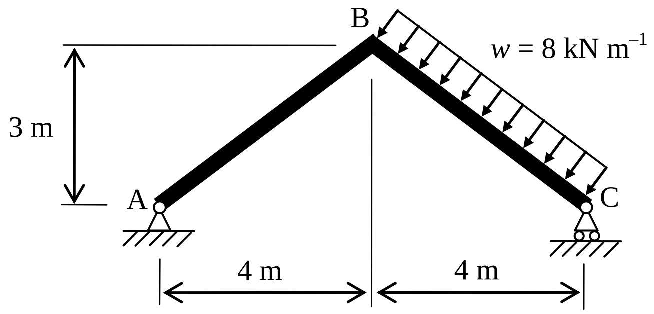

Fig. 3

4 **(short)** The beam shown in Fig. 4 is rigid and is supported by a pin at point A and a pin-jointed truss at point B. The truss is square and has four equal sides of length *L*. All members of the truss have the same cross-sectional area *A* and are made of a linear elastic material with Young's modulus *E*. A vertical force of *P* is applied to the structure at point C. The self-weight can be neglected.

(a) Find the vertical displacement at point B. [8]

(b) Find the vertical displacement at point C. [2]

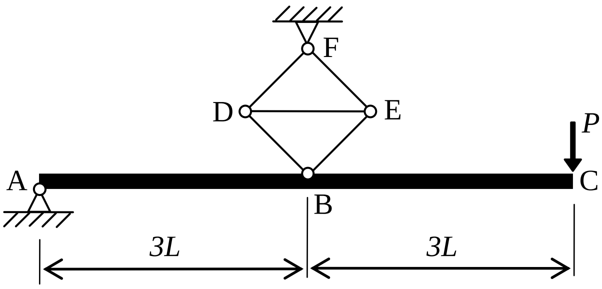

Fig. 4

5 **(long)** The cross-section of a bi-metallic beam is shown in Fig. 5(a). The top layer is made of aluminium alloy (Young's modulus, E = 70 GPa) and the bottom layer is made of steel (E = 210 GPa). Each layer is 4 mm thick. The beam is straight when the temperature is 0 ^{o}C. When a portion of the composite beam is at θ ^{o}C, that portion has a change in curvature due to temperature of:

$$\Delta\kappa = 0.001\ \theta\ [\text{rad m}^{-1}]$$

The self-weight of the beam can be neglected.

(a) Find the bending stiffness EI of the beam. [5]

(b) Consider the cantilevered beam shown in Fig. 5(b). If the temperature is uniformly raised from 0 ^{o}C to 20 ^{o}C, find the vertical deflection at the end of the cantilever. [5]

(c) Consider the same beam but with a roller support added at its end, as shown in Fig. 5(c). The temperature is again raised from 0 ^{o}C to a uniform 20 ^{o}C.

(i) Find the vertical reaction at point A. [4]

(ii) Find the location of the maximum vertical deflection of the beam, and sketch the deflected shape. [10]

(d) For the beam shown in Fig. 5(c), find a temperature distribution other than $\theta(x)$ = 0 ^{o}C that causes no vertical displacements anywhere. [6]

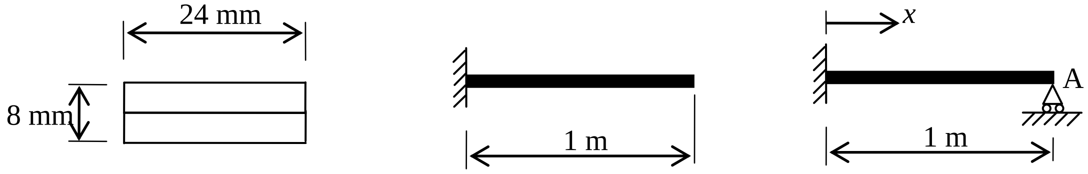

Fig. 5(a) Fig. 5(b) Fig. 5(c)

6 **(long)** Three different structures, each subjected to a vertical force P, are shown in Fig. 6. Assume that all members are initially straight, have negligible self-weight, and that buckling occurs in the x-y plane (the plane of the page).

(a) The rigid strut shown in Fig. 6(a) is restrained by a linear spring of stiffness k_1 [N m^{-1}] at mid-height, and a rotational spring with stiffness K [N m rad^{-1}] at the base. Calculate the buckling load. [10]

(b) A flexible structure made of two struts with bending stiffness EI is shown in Fig. 6(b). The structure has a pin at points A, B, and C, and is restrained by a linear spring of stiffness k_2 [N m^{-1}] at point B. There are no rotational springs.

(i) Sketch the governing buckling modes. [4]

(ii) Find the critical buckling load for each mode, and an equation describing the values of k_2 for which each mode governs. [12]

(c) The structure in Fig. 6(c) is identical to that in Fig. 6(b), apart from the removal of the pin joint at point B. Determine the range of possible governing buckling loads for this case. [4]

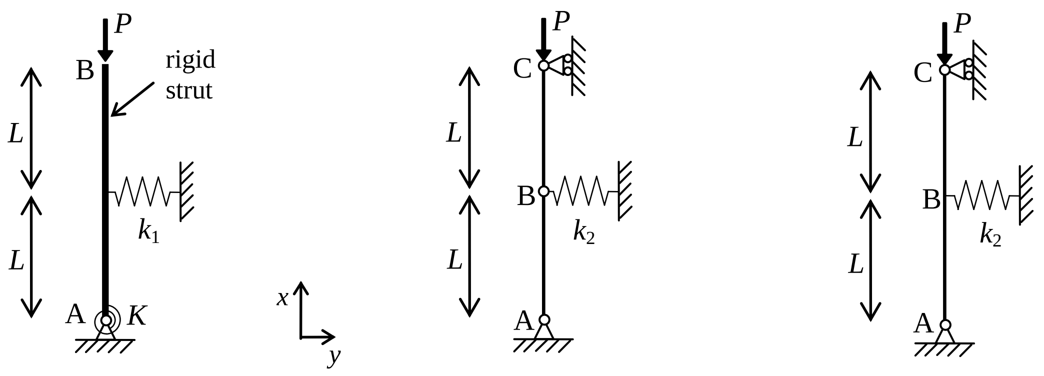

Fig. 6(a) Fig. 6(b) Fig. 6(c)

SECTION B

7 **(short)**

(a) Describe the chemical reactions taking place during the formation of a metal oxide layer in dry air. Explain the role of the free energy of oxidation. [6]

(b) Sketch the change in mass with time of a metallic component undergoing oxidation with (i) parabolic growth, and (ii) linear growth. In each case, explain the shape of the curve. [4]

8 **(short)**

(a) Using sketches to support your answer, explain (i) the difference between an edge and a screw dislocation, and (ii) how dislocations enable the plastic deformation of materials. [6]

(b) Explain briefly two microstructural features that affect the mobility of dislocations in a material. In each case state one key microstructural parameter that can be controlled during processing to influence the yield strength of the material. [4]

9 **(short)**

(a) Sketch the body-centred cubic (BCC) unit cell, and hence show that the side length of the unit cell (a) is related to the atomic radius (R) as follows:

$$a = \frac{4R}{\sqrt{3}}$$ [5]

(b) Using the atomic properties given in the Materials Data Book, calculate the theoretical density of face-centred cubic (FCC) aluminium at 20 °C. [5]

10 **(short)** A ceramic column with cross-sectional area A and density ρ is suspended vertically from the top, as shown in Fig. 7.

(a) Given that the column is loaded only by self-weight, derive an expression for the tensile stress variation in the column $\sigma(y)$. [3]

(b) Hence, by using the statistics of fracture equations in the Materials Data Book, derive an expression for the survival probability P_S of a column of length L. [7]

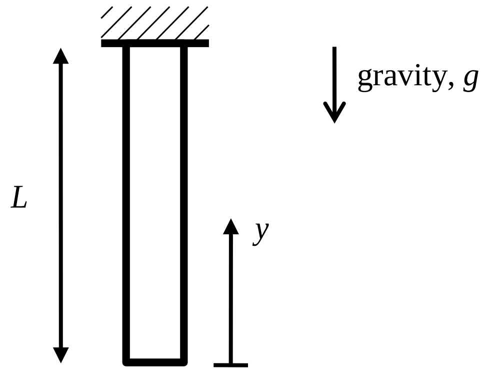

Fig. 7

11 **(long)**

(a) With the aid of sketches briefly explain how atomic bonding determines the Young's modulus of (i) metals and (ii) thermoplastics. [8]

(b) An experiment is performed in order to measure Young's modulus (E) and the coefficient of thermal expansion (α) of a material. The test specimen (a cube of side length d) is compressed elastically between rigid, frictionless, flat platens by a small amount Δd, as shown in Fig. 8(a). With the platens' separation held fixed, the temperature is then increased by ΔT.

Derive an expression for the force F required to hold the platens in position. Sketch the variation in F with ΔT, indicating how E and α can be determined. What limits the maximum temperature rise ΔT that can be applied? [10]

(c) A thin layer of adhesive of thickness h joins two large, flat blocks as shown in Fig. 8(b). The adhesive is cured at 50 °C. At the end of curing, the adhesive is stress free and is perfectly bonded to the blocks. The assembly is then cooled down to 25 °C, during which the thickness h remains constant. The adhesive can be assumed to be elastic with E = 2.5 GPa, Poisson's ratio ν = 0.40 and $\alpha = 10^{-5}$ °C^{-1}. The blocks can be assumed rigid, with $\alpha = 0$.

(i) Explain why the x-, y- and z-components of stress in the adhesive at 25 °C are equal. [3]

(ii) Hence, calculate the magnitude of these stresses at 25 °C. [9]

(cont.

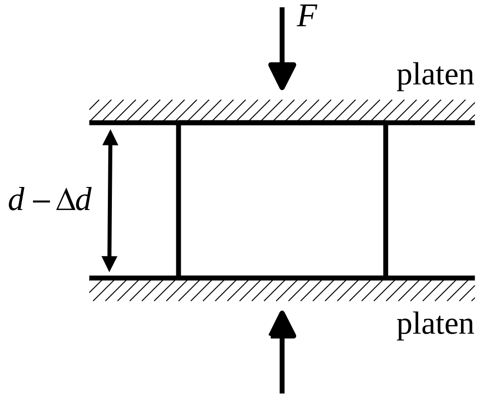

Fig. 8(a)

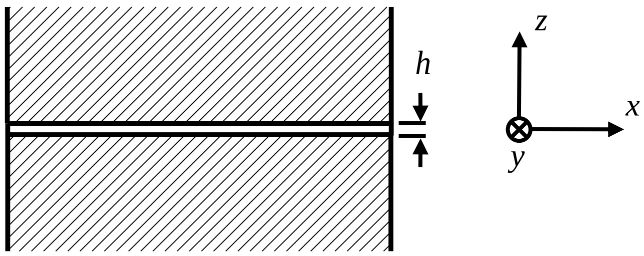

Fig. 8(b)

(TURN OVER

12 **(long)** A load-carrying element of an aeroplane wing can be modelled as a cantilever beam of length L which, during flight, experiences a uniformly distributed load w (N m^{-1}) as shown in Fig. 9(a). The cross-section of the beam is shown in Fig. 9(b), and has a second moment of area

$$I = \frac{bd^3}{12}(1 - \lambda^4)$$

(a) Laboratory fatigue experiments performed at zero mean stress show that the beam fails in fatigue following Basquin's law with $C_1 = 650$ MPa and $\alpha = 0.07$ (as defined in the Materials Data Book). During flight, the beam experiences cyclic loading with mean load w = 40 kN m^{-1} and load range Δw = 5 kN m^{-1}. Calculate the total number of cycles to failure for a beam with dimensions: $b = 0.4$ m, $d = 0.2$ m, $\lambda = 0.8$, $L = 5$ m. The tensile strength $\sigma_{ts} = 400$ MPa. [10]

(b) A material (with Young's modulus E and density ρ) is to be selected for the beam in order to minimise mass, while ensuring the tip deflects less than a critical amount during flight. The shape of the cross-section is to be taken into consideration.

(i) Show that the shape factor ϕ for the beam is given by:

$$\phi = \frac{I}{I_{\mathrm{REF}}} = \frac{d}{b}\left(\frac{1 + \lambda^2}{1 - \lambda^2}\right)$$

where the reference cross-section is a solid square. [8]

(ii) Taking the cross-sectional area A to be a free variable, show that, in order to minimise mass, the following shape-dependent performance index should be maximised:

$$M = \frac{\sqrt{E\phi}}{\rho} \quad [8]$$

(iii) Aluminium alloy and wood are considered for the beam. Discuss (without detailed calculation) which is likely to offer the lighter solution. [4]

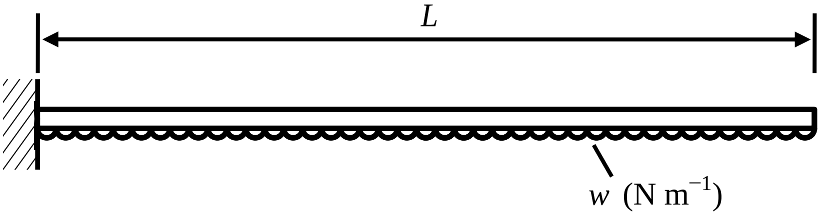

Fig. 9(a)

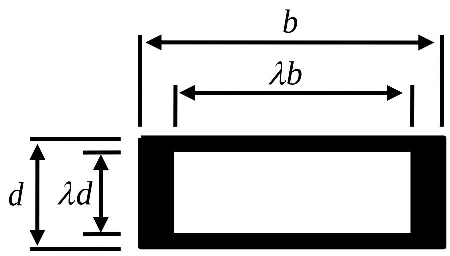

Fig. 9(b)

END OF PAPER

Answers to questions

1. (a) $x = h/2$ (b) λ = atan(1/3)

2. P = 5.6 kN

3. (a) V_A = 12.5 kN H_A = 24 kN V_C = 19.5 kN

4. (a) $\delta_B = \left(4+2\sqrt{2}\right)\dfrac{PL}{AE}$ (b) $\delta_C = 2\delta_B$

5. (a) EI = 116.5 Nm^2 (b) $\delta = -0.01$ m (c) V_A = 3.5 N $x = 2/3$

   (d) $\theta(x) = C(1-x)$

6. (a) $P_{cr} = \dfrac{k_1 L}{2} + \dfrac{K}{2L}$ (b) $P_1 = \dfrac{k_2 L}{2}$ $P_2 = \dfrac{\pi^2 EI}{L^2}$ $k_{2,cr} = \dfrac{2\pi^2 EI}{L^3}$

   (c) $\dfrac{\pi^2 EI}{4L^2} < P_{cr} < \dfrac{\pi^2 EI}{L^2}$

9. (b) ρ = 2699 kg/m^2

10. (a) $\sigma = \rho g y$ (b) $P_s(V) = \exp\left[-\left(\dfrac{\rho g}{\sigma_0}\right)^m \dfrac{AL^{m+1}}{V_0(m+1)}\right]$

11. (b) $F = Ed^2\left(\dfrac{\Delta d}{d} + \alpha\Delta T\right)$ (c) $\sigma = -\dfrac{E\alpha\Delta T}{(1-2\nu)}$ = 3.13 MPa

12. (a) $N_f = 3.50 \times 10^7$ cycles

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