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Paper 1 - Mechanics

ANSWERS

1. $\frac{a}{\sqrt{2}}$; $\underline{v}_R = (q\omega_2 - r\omega_1)\underline{j}$; $\underline{a}_R = (-q\omega_2^2 + 2r\omega_1\omega_2)\underline{i} - r\omega_1^2\underline{k}$

2. $\frac{10}{\sqrt{3}}$ m s^{-1}

3. ~34 rad s^{-2}

4. $\frac{a+r}{r}\theta$; 0.342

5. $\frac{\sqrt{1099}ma}{32}$ $x/L = 0.4046$

6. (i) B, 90° and C, 233° from the out of balance at A

   (ii) 36.3 N

   (iii) 0.072 kg at A and C at angular positions 214° and 34° repectively from out of balance at A.

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Paper 2 - Structures

ANSWERS

1. b) (i) 3.66 kN; (ii) 10.6°; (iii) 10.6°

2. a) 4960 kNm, 4520 kN, 2500 kN/m^2

   b) $\sigma_1 = 140$ N/mm^2, $\sigma_2 = 21$ N/mm^2, $\sigma_3 = 0$
   (σ_1 at 20.8° from hoop direction.)
   $\varepsilon_1 = 636 \times 10^{6}$, $\varepsilon_2 = -100 \times 10^{-6}$, $\varepsilon_3 = -230 \times 10^{-6}$

   c) -27.6 MN $\leq$ axial force $\leq$ 43.3 MN

3. c) (i) 31.6 kN/m (ii) No change

4. b) $\lambda = 43$; d) 1.84 mm

5. 7640 kN

6. (ii) $\dfrac{5}{12}\dfrac{WL^3}{EI} - \dfrac{\alpha\ \Delta T\ L}{3}$

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Paper 3 - Materials

ANSWERS

1.

(b) Cu_2Sb.

(c) Eutectic at 650°C: L(31% Sb) = α(12% Sb) + β(32% Sb).
Eutectic at 520°C: L(77% Sb) = Cu_2Sb + δ (98% Sb).
Eutectoid at 420°C: β (42% Sb) = ε(38% Sb) + Cu_2Sb.

2.

(b) Volume = 10.6×10^6 mm^3.
Distance between cracks ≈ 230 mm.

3.

(b) Time = 2.25 h. Hardness = HV960.

4.

(a) Merit index for external-pressure buckling = $(E^{1/2} / \rho)$.
Merit index for yield or compressive brittle failure = (σ_f / ρ).
Optimum material is alumina, with a mass of 2.02 tonne and a wall thickness of 41 mm.
Limiting failure mechanisms: buckling for alumina, glass and alloy steel; compressive yielding for titanium alloy and aluminium alloy.

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Paper 4 - Fluid Mechanics and Heat Transfer

ANSWERS

1. (a) $p = p_o exp[-gy/RT_o]$

2. (a) $t = 2h_o^{\frac{1}{2}} (D/d)^2 [(1 - d^4/D^4)/2g]^{\frac{1}{2}}$
   (b) $fL/d << 1$

3. (a) Blood
   (c) 0.760 mm

4. (b) $h_2 = 4.61m$
   $V_2 = 3.12\ m/s$
   (c) $5.54\ m/s$

5. -

6. (a) $T = 825K$
   (b) $T = 1369°C$

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Paper 5 - Electrical Engineering

ANSWERS

1. i) Class A amplifier: Output resistance = R_C; Gain = $-\frac{h_{fe}R_C}{h_{ie}}$

ii) Class B amplifier: Input resistance = $(h_{fe} + 1)\,R_L + h_{ie}$; Gain = $\frac{(h_{fe}+1)R_L}{(h_{fe}+1)R_L + h_{ie}}$

Voltage at input to Class B amplifier = $-\frac{(h_{fe}+1)R_L + h_{ie}}{(h_{fe}+1)R_L + h_{ie} + R_C} \cdot \frac{h_{fe}R_C}{h_{ie}} v_i$

2. i) Diff. gain = $-\frac{h_{fe}R_C}{R_B + h_{ie}}$ ii) Common-mode gain = $-\frac{h_{fe}R_C}{R_B + h_{ie} + 2(h_{fe}+1)R_T}$

$R_B = 350\ \Omega$; $R_T = 22.4\ k\Omega$; $V_{CC} = +19$ V; $V_{EE} = -897$ V.
Constant current source ≅ 40 mA; CMRR is then infinite.

3. P = 6575 W; Q = +6889 VAR; Capacitor value = 42.4 μF; P = 2847 W;
Q = -1423 VAR; Line voltage at load 2 = 421.8 V

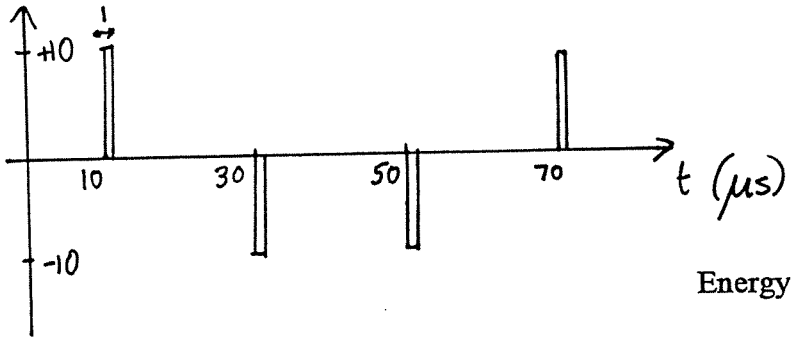
4. (c) Speed = 1048 rpm (d) Field voltage = 80 V; Speed = 2455 rpm;
Efficiency = 76.2 %

5. P_{gen} = 41.15MW; Q_{gen} = 43.3 MVAr; P_{feeder} = 1.15MW; V_{gen} = 23.9 kV;
Factor = 9.2

6. $\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t}$; $\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t}$; $\frac{\partial^2 V}{\partial z^2} - LC\frac{\partial^2 V}{\partial t^2} = 0$; $v = 1/\sqrt{LC}$

i) $\rho_V = -1$ (short-cicuited end), $\rho_V = +1$ (open-circuited end)

ii)



Energy = 2.4 μJ

7. $\overline{\mathbf{H}} = \mathbf{u}_y \frac{E_0}{\eta_0} \exp j(\omega t - \beta z)$

$\overline{\mathbf{P}} = \frac{1}{2}\mathbf{E} \times \mathbf{H}^* = \mathbf{u}_z \frac{E_0^2}{2\eta_0}$ for plane electromagnetic waves traveling in the z direction.

i) Reflected E-field = $\frac{\sqrt{\mu_r} - 1}{\sqrt{\mu_r} + 1} E_i$; Transmitted E-field = $\frac{2\sqrt{\mu_r}}{\sqrt{\mu_r} + 1} E_i$ ii) 10.9 %

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Paper 6 - Information Engineering

ANSWERS

1. a) $\frac{1}{s^2+2s+K_p}$, $\frac{K_p}{s^2+2s+K_p}$,
   b) $\zeta = 1/\sqrt{K_p}$, $K_p = 1$

   (i) 0
   (ii) 0
   (iii) 1
   (iv) -2

2. b) $T_f = 2$, $K_d = 3.4$, $K_p = 1.44$
   c) 8.5°, 0.0235°

3. a) $K_p = 0.25$, $GM = 14.8$ (23.5dB)
   b) $\alpha \leq 0.09$

4. b)

   (i) $-0.5 < K < 4$
   (ii) $K = 2$

5. -

6. a) 600 k bit/sec
   b) 74dB, 98dB, 20kHz, 15kHz

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Paper 7 - Mathematical Methods

ANSWERS

1. Volume, $V = \frac{7a^4}{12}$
2. $\mathbf{p} = \frac{k\mathbf{r}}{\mathbf{r}^3}$, $\text{curl}\mathbf{p} = \text{div}\mathbf{p} = 0$
   Work done, $W = -\frac{k}{30}$
3. Area of ellipse, $A = \pi ab$
4. Accuracy of approximation $= O(\Delta t)^2$
5. $I_o = 7.58 \times 10^{-10}\text{A}, \quad V_s = 0.036V$
6. i). $Y(\omega) = \frac{1}{2}[X(\omega - \omega_o) + X(\omega + \omega_o)]$
   ii) $Y(\omega) = \frac{1}{T}\sum_n sinc\left(\frac{n\omega_o\tau}{2}\right) X(\omega - n\omega_o)$
7. $f_b = 10kHz$, $\Delta f = 19.5Hz$ $H(\omega) = e^{-j\omega T} sinc^2\left(\frac{\omega T}{2}\right)$
8. $W \geq 1242g$

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Paper 8 - Selected Topics

ANSWERS

1. (b) Overall depth = 675 mm, d = 600 mm
   (c) 11300 mm^2 in tension face
   (d) 7540 mm^2 in compression face, 18840 mm^2 in tension face,

2. (a) 20.7 %
   (b) 17.1 %, 0.46
   (c) 2.24 m

3. (a) 24.7 kPa
   (b) 50.1 kPa
   (c) 9.68 kPa

4. (a) 220 rpm
   (d) 1 Mb

6. (c) k=64x10^6 N/m, 465 Hz, 590 Hz
   (d) 530 Hz, 680 Hz

7. (a) 259.8 K, 46.5 kPa
   (b) 805.5 K, 1840 kPa
   (c) 945.5 K, 332.6 kPa
   (d) 50.92 kPa, 422 m/s
   (e) 5.75

8. (a) Gross Thrust = $(m_a + m_f)V_j$ =Thrust on test bed
   Net Thrust = $(m_a + m_f)V_j - m_a V_1$ =Thrust in flight
   (b) $\dfrac{\dot{m}_a\sqrt{c_p T_{02}}}{D^2 P_{02}}$ or $\dfrac{\dot{m}_f LCV}{\sqrt{c_p T_{02}} D^2 P_{02}}$ or $\dfrac{P_{04}}{P_{02}}$
   (d) 236.7 kg/s, 1425 K
   (e) 31.28 kN

11. (b) $f_{\min} = \exp\left[-\dfrac{k_g \pi dh}{G}\right]$
    (c) 0.571

12. (b) $L_{\min} = HG\left[1 - \dfrac{c_{gT}}{c_{gB}}\right]$
    (c) $L_{\min} = 1.2\ m^3/s$, $d = 1.37$ m

13. (a) 5×10^{21} m^{-3}
(b) 4 μm
(c) 3.2 mA, 8×10^{7} Wm^{-2}
(d) -3.77 V

14. (c) 41 n^2 eV

15. (b) 5 μm, 20 μm
(d) 2.8 s
(e) 13.8 ms^{-1}