

ENGINEERING TRIPOS PART 1B

1996

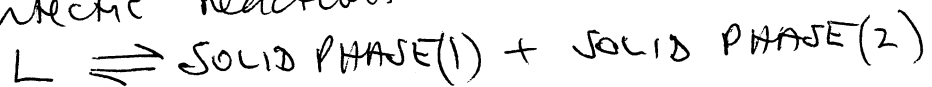
PAPER 3 - MATERIALS

SOLUTIONS

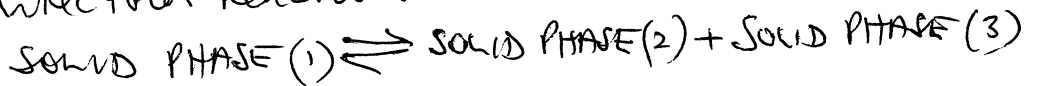
Q1

(a)

(i) Eutectic reaction



(ii) Eutectoid reaction



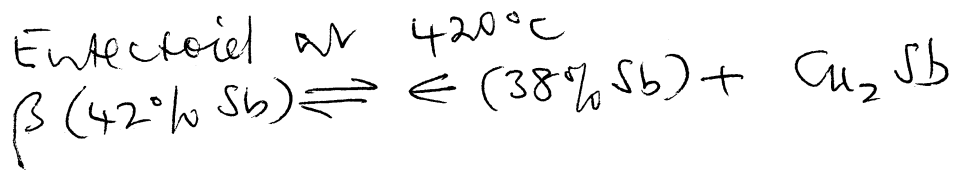
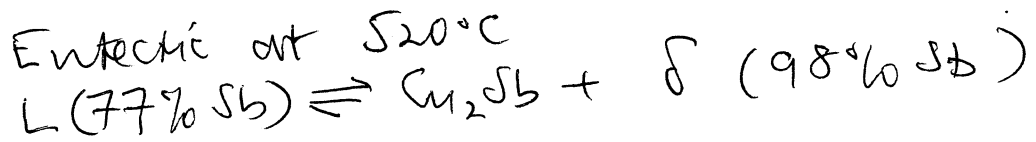
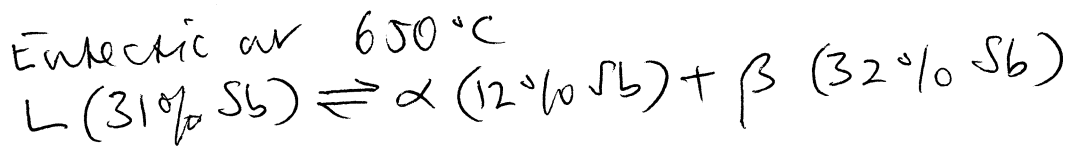
Note that, if the temperature is decreasing, the reactions all go from LEFT to RIGHT. If the temperature is increasing, the reactions all go from RIGHT to LEFT. When any of the reactions is taking place, the three phases are in thermodynamic equilibrium at a fixed temperature. A tie-line joins the composition points of the three phases on the phase diagram.

(b) "X" is an composition of 48 weight % Sb, 52 weight % Cu. From the Materials Java Book, the atomic weights are Sb = 121.75, Cu = 63.54. Thus, moles of Sb = 48 / 121.75 and moles of Cu = 52 / 63.54. The ratio

$$\frac{\text{Moles Cu}}{\text{Moles Sb}} = \frac{52}{63.54} \times \frac{121.75}{48} = \underline{2.08}$$

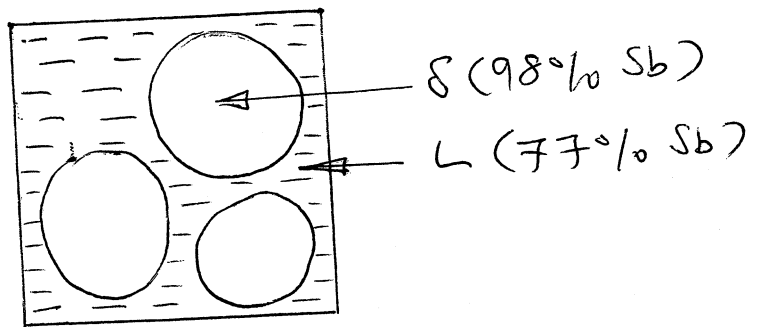
Thus the chemical formula of X is Cu₂Sb

(c)

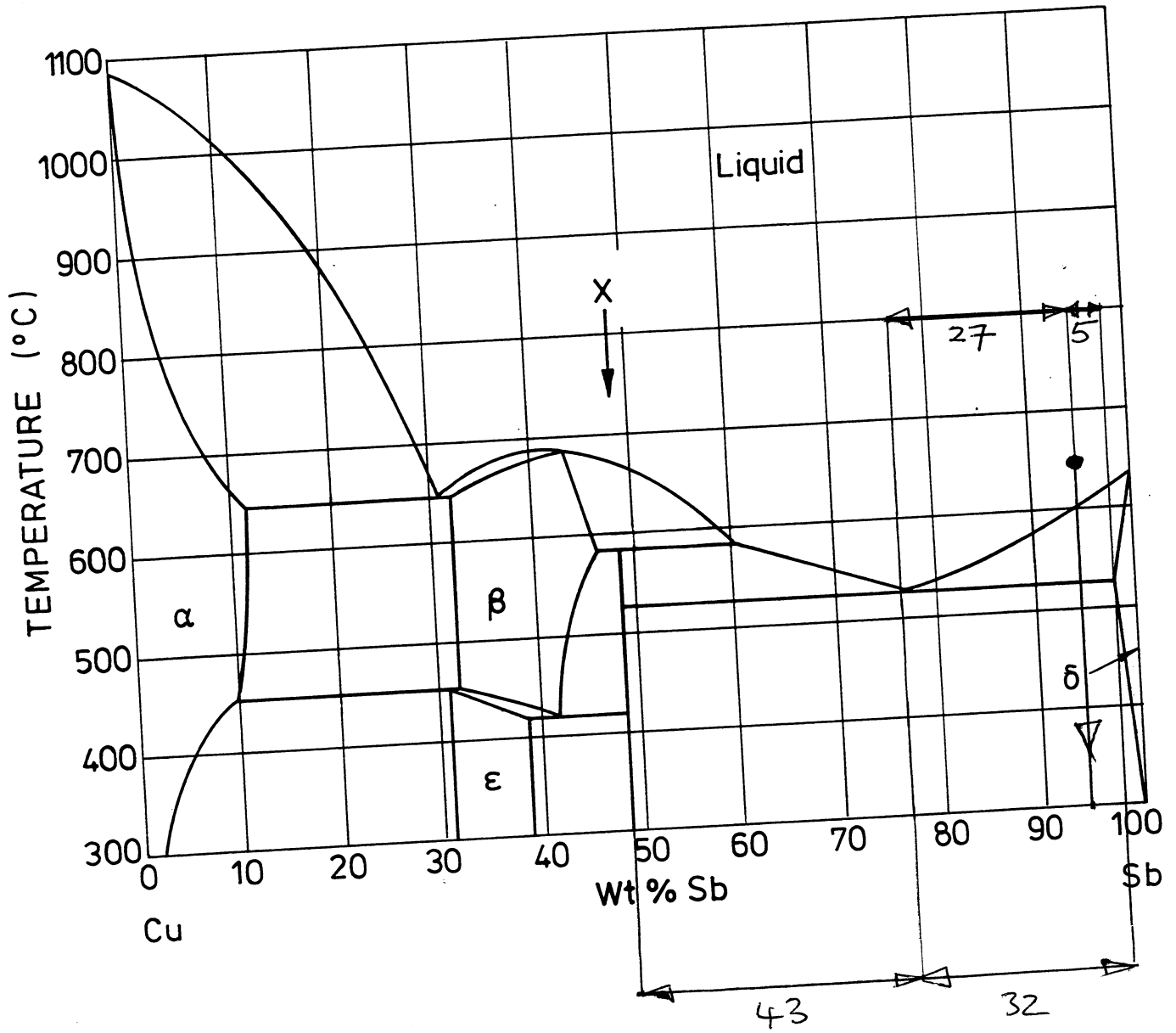


(d)

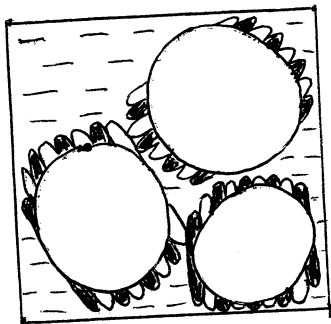
When the alloy composition line meets the liquidus line (at 600°C) crystals of the δ -phase begin to form from the liquid. As the temperature falls, the proportion of δ increases and the proportion of liquid decreases. The compositions of the liquid and δ phases become progressively less rich in Sb, following the ends of the two-phase $L + \delta$ tie line. The following sketch shows the microstructure at the eutectic temperature of 520°C before the eutectic reaction starts.



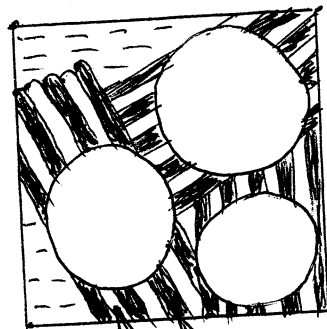
Using the lever law, the weight fraction of liquid is $5/(27+5) = 0.156$. The weight fraction of δ is $27/(27+5) = 0.844$.



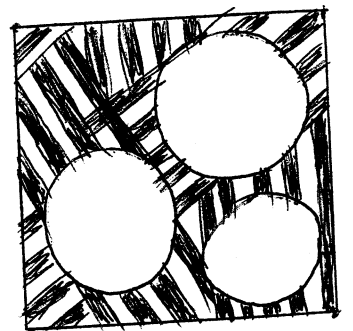
The sequence at 520°C is as follows



Eutectic Start



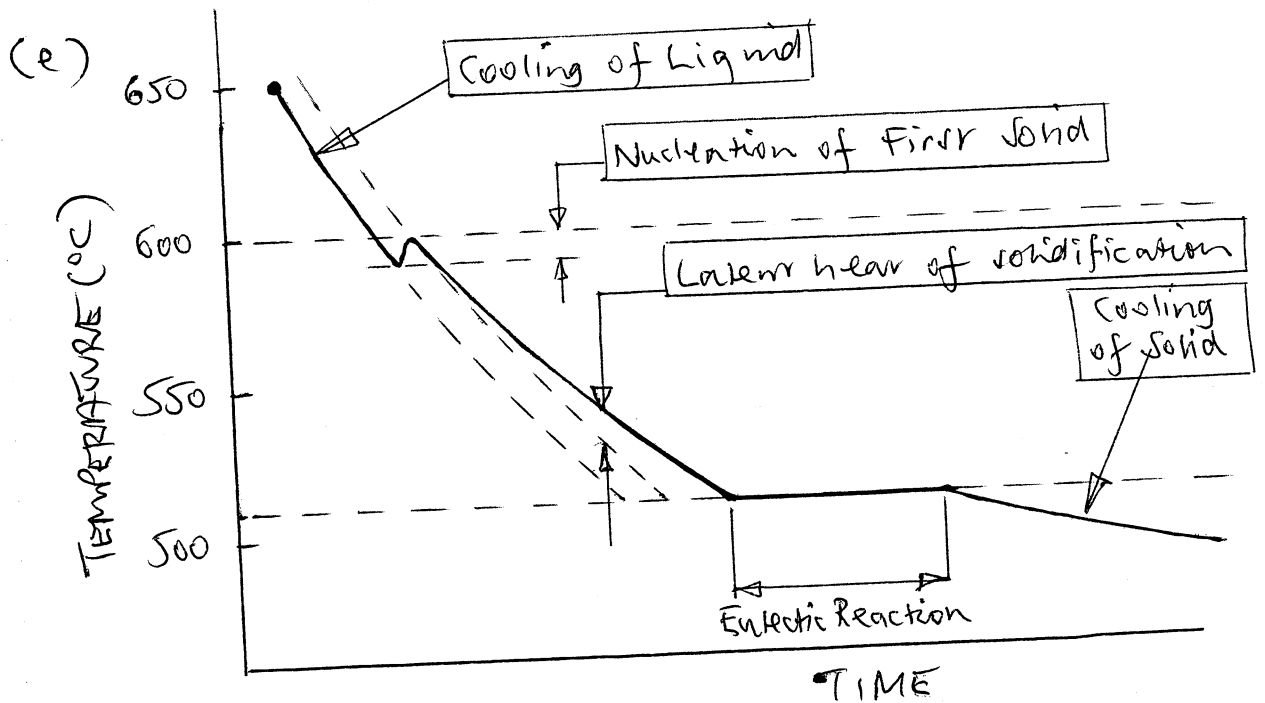
δ
Cu₂Sb



Eutectic finish

~~III~~
In the eutectic microstructure, the weight fraction of the δ phase is $43/(43+32) = 0.573$, and the weight fraction of the Cu_2Sb phase is $32/(43+32) = 0.427$.

Below $520^\circ C$ the microstructure does not change significantly. However, owing to the decreasing solubility of Cu in δ with decreasing temperature the weight fraction of δ decreases slightly, and the weight fraction of Cu_2Sb increases slightly as room temperature is approached. At room temperature the δ phase is pure Sb. These small adjustments require solid-state diffusion, and hence a very slow cooling rate.



Q2

(a) Ceramics are intrinsically hard materials: the ionic or covalent bonds present a very large lattice resistance to the motion of dislocations. The penalty for this high hardness is a low fracture toughness: the plastic zone size at the tip of a crack in a ceramic material is very small, and fracture is easily initiated by crack-like defects. These originate in several ways. Sintered or hot-pressed products contain pores on the scale of the powder (grain) size. Thermal stresses caused by cooling or thermal cycling can generate small cracks. Cracks also appear when a ceramic is loaded, nucleated by the elastic anisotropy of the grains or by easy slip on a single slip system. If the longest microcrack in a given component has length $2a_m$ then the tensile strength is given by

$$\sigma_{TS} = \frac{K_{Ic}}{\sqrt{\pi a_m}}$$

σ_{TS} can be improved by increasing K_{Ic} or decreasing a_m .

a_m can be decreased by using powders of a controlled small size and pressing them under tightly controlled conditions to give a high density hot-pressed ceramic with small, rounded residual pores.

K_{Ic} can be increased using the following methods:

(i) Use a fine dispersion of second-phase particles. These help to pin the advancing crack front. Particles may be inert or can be

⑤

made to undergo energy-absorbing transformations as in partially stabilised zirconia (PSZ).

(ii) Use fibre strengthening, e.g. glass-fibre reinforced cement.

(iii) Bond ceramic particles together with a tough matrix, e.g. tungsten carbide (cobalt) cements.

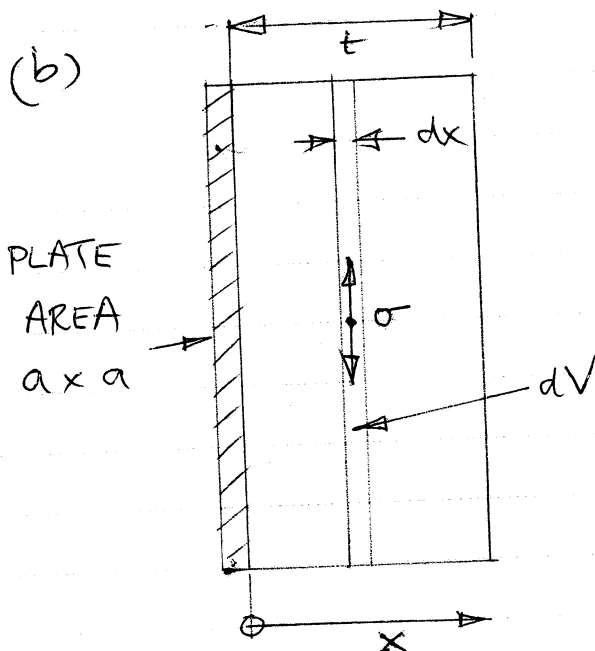
Ceramics are approximately 15 times stronger in compression than in tension.

In tension, cracks propagate in an unstable manner by fast, brittle fracture. In compression cracks propagate in a stable manner and twist out of their original orientation until they are parallel with the loading direction. Fracture is caused by the slow extension of many cracks to form a crushed zone. It is the average crack size \bar{a} which determines the crushing strength.

The compressive strength is given by

$$\sigma_c = C \frac{K_{Ic}}{\sqrt{\pi \bar{a}}}$$

where the constant $C \approx 15$,



$$\sigma(x=t) = \frac{\alpha \Delta T E}{(1-\nu)}$$

$$= \frac{10^{-4} \times 170 \times 40}{(1-0.3)}$$

$$= 0.97 \text{ MPa}$$

$$\sigma(x) = \frac{(0.97 \text{ MPa}) \times x}{t}$$

$$P_s + P_f = 1$$

Since $P_f = 0.5$, $P_s = 0.5$ as well.

For the foam layer,

$$0.5 = \exp \left\{ -\frac{1}{V_0} \int_V \left(\frac{\sigma}{\sigma_0} \right)^m dV \right\}$$

$$dV = a^2 dx$$

$$0.5 = \exp \left\{ -\frac{1}{V_0 \sigma_0^8} \int_V \left[\frac{(0.97 \text{ MPa})x}{t} \right]^8 a^2 dx \right\}$$

$$= \exp \left\{ -\frac{a^2}{V_0 \sigma_0^8 t^8} (0.97 \text{ MPa})^8 \int_0^t x^8 dx \right\}$$

$$= \exp \left\{ -\frac{a^2 t}{V_0 \sigma_0^8} \frac{(0.97 \text{ MPa})^8}{9} \right\}$$

For the tensile stress, $\int_V dV = V_0$ and σ is constant. Thus

$$0.5 = \exp \left\{ -\left(\frac{\sigma}{\sigma_0} \right)^8 \right\}$$

Equating the terms in brackets gives

$$\frac{a^2 t}{V_0 \sigma_0^8} \frac{(0.97 \text{ MPa})^8}{9} = \left(\frac{\sigma}{\sigma_0} \right)^8$$

$$a^2 t = 9 V_0 \left(\frac{\sigma}{0.97 \text{ MPa}} \right)^8$$

$$a^2 t = 9 \times 25 \times 25 \times 100 \text{ mm}^3 \left(\frac{1.4 \text{ MPa}}{0.97 \text{ MPa}} \right)^2$$
$$= \underline{10.6 \times 10^6 \text{ mm}^3}$$

Since $t = 200 \text{ mm}$, $a = \underline{230 \text{ mm}}$

Q3

(a) Refer to Figures 4.7, 5.1(b) and 5.2 in the Materials Data Book.

(i) Heat to about 50°C above the eutectoid temperature of 723°C to austenitize the steel. Quench in a salt bath to about 650°C , hold for at least 100s and cool to room temperature. Alternatively, air cool from the austenitizing temperature (normalising).

(ii) Austenitize, quench to about 400°C , hold for at least 200s and cool to room temperature.

(iii) Austenitize, quench to room temperature in oil or water, note that, because M_f is below 0°C for a eutectoid steel, the sample should be removed from the quenching medium and cooled using liquid nitrogen to complete the transformation to martensite. If this is not done, the structure will contain retained austenite.

(iv) Reheat the martensite to a temperature in the range 250 to 500°C . Tempering times are typically 1 hour.

Structures

(i) Pearlite. Alternating plates of ferrite (α iron, with a body-centred cubic structure) and iron carbide (Fe_3C). Forms as nodules.

(ii) Bainite. Isolated rods or stringers of Fe_3C in a ferrite matrix.

(iii) Martensite. Body-centred tetragonal (distorted body-centred cubic) iron containing 0.8 weight % carbon dissolved in supersaturated interstitial solid solution.

(iv) Tempered martensite, Carbon precipitates out of solution to give small discrete precipitates of Fe_3C . The distortion of the matrix is reduced, the matrix structure tending towards undistorted (body-centred cubic) iron. The structure produced by tempering is not pearlite!

(b) Referring to Figure 5.2 in the Materials Data Book we can see that an HV of 800 requires a carbon content of 0.5 mass %.

We note that $x = 1.0$ mm. Thus

$$0.5 = 1.05 \{ 1 - \text{erf}(y) \} + 0.15$$

$$\frac{0.35}{1.05} = 1 - \text{erf}(y)$$

$$\text{erf}(y) = 1 - \frac{0.35}{1.05} = 0.67$$

From Fig. 3, $y = 0.68$

$$\frac{x}{2\sqrt{Dt}} = 0.68$$

$$\frac{x^2}{4Dt} = (0.68)^2$$

$$t = \frac{x^2}{4D(0.68)^2}$$

$$= \frac{10^{-6} \text{ m}^2}{4 \times 6.68 \times 10^{-11} \text{ m}^2 \text{ s}^{-1} (0.68)^2}$$

$$= \underline{8094 \text{ s}} = \underline{2.25 \text{ h}}$$

From Figure 5.12 in the materials data Book we can see that martensite containing 1.2 weight % carbon has an HV of 960. Since $C_s = 1.20$ weight %, the surface hardness will be HV 960.

Q4

(a)

$$(i) \quad p_b = 0.3E \left(\frac{t}{r} \right)^2$$

$$m_b = 4\pi r^2 t \rho$$

$$\left(\frac{t}{r} \right)^2 = \frac{p_b}{0.3E}$$

$$\frac{t}{r} = \left(\frac{p_b}{0.3E} \right)^{1/2}$$

$$t = \left(\frac{p_b}{0.3E} \right)^{1/2} r$$

$$m_b = 4\pi r^3 \rho \left(\frac{p_b}{0.3E} \right)^{1/2} = 22.9 r^3 p_b^{1/2} \left(\frac{\rho}{E^{1/2}} \right)$$

Merit index is $\left(\frac{E^{1/2}}{\rho} \right)$

$$p_f = 2\sigma_f \left(\frac{t}{r} \right)$$

$$m_f = 4\pi r^2 t \rho$$

$$\left(\frac{t}{r} \right) = \frac{p_f}{2\sigma_f}$$

$$t = \frac{p_f r}{2\sigma_f}$$

$$m_f = \frac{4\pi r^3 p_f \rho}{2\sigma_f} = 2\pi r^3 p_f \left(\frac{\rho}{\sigma_f} \right)$$

Merit index is $\left(\frac{\sigma_f}{\rho} \right)$

$$(ii) m_b = 22.9 r^3 p_b^{1/2} \left(\frac{\rho}{E^{1/2}} \right)$$

$$t_b = \frac{m_b}{4\pi r^2 \rho}$$

$$m_f = 2\pi r^3 p_f \left(\frac{\rho}{\sigma_f} \right)$$

$$t_f = \frac{m_f}{4\pi r^2 \rho}$$

Set $r = 1 \text{ m}$ and $p_b = p_f = 200 \text{ MPa}$,
 Values for ρ , E and σ_f are to be taken from
 the table of data.

<u>Material</u>	<u>m_b</u> (tonne)	<u>t_b</u> (mm)	<u>m_f</u> (tonne)	<u>t_f</u> (mm)	<u>Mechanism</u>
Al_2O_3	2.02	41	0.98	20	Buckling
Glass	3.18	97	1.63	50	Buckling
Alloy steel	5.51	56	4.90	50	Buckling
Ti alloy	4.39	74	4.92	83	Compressive
Al alloy	3.30	97	6.79	200	Compressive

Optimum material is Al_2O_3 with a mass
 of 2.02 tonne.

(iii)

For each material, the limiting failure mechanism
 is shown which requires the largest value of t .

(b)

Processing routes

Al_2O_3 - hot isostatic pressing (HIP). Al_2O_3
 powder is first compressed to make a "green"

compact with near-net shape. This is then "canned" with a foil of refractory metal. The canned compact is heated in a furnace at high temperature while a high external pressure is applied using an inert gas. In practice, the size of pressure vessel and furnace required to make a sphere 2 m in diameter by the HIP process make it economically and technically prohibitive. An alternative might be to build-up the sphere from small, separately-formed "bricks" since the structure is designed to operate under a compressive stress.

Glass. This can be melted and hot formed using a suitable mould. Once the glass has cooled, it should in principle be possible to remove the mould core from the interior of the sphere by way of the access aperture. Technical problems include preventing cracking of the glass during cooling, and selecting a mould material which is able to withstand the high temperature whilst being unreactive with respect to the glass.

Alloy steel. Hot-form two hemispheres. Weld the halves together with a multi-run full penetration circumferential weld, machine-out the aperture. A problem is that the heat of the welding process will degrade the mechanical properties in the vicinity of the weld (heat-affected zones). It may be necessary to heat-treat the whole sphere after welding to achieve uniform strength. Alternatively, use casting with a sand core and mould. Both core and mould can readily be destroyed afterwards.

Titanium alloy

Aluminium alloy

As for alloy steel. However, both these alloys are relatively reactive and special precautions are required to avoid oxidation / combustion when welding, casting and machining.

In practice, a pressure hull made from either alumina or glass is likely to be prohibitively expensive and technically problematical. The lighter hull combined with the ability to use orthodox fabrication technology is probably titanium alloy (mass 4.92 tonnes). This has corrosion resistance as an added bonus.

Q5

(a) The high yield strength of cold-drawn copper is caused by a high dislocation density produced as a result of work-hardening. The annealing process results in recrystallization (the nucleation and growth of new grains having a low dislocation density). The annealed copper is therefore softer. See IB materials experiment on "Annealing of Metals".

(b) Refer to the Al-Cu phase diagram, Figure 4.3 in the Materials Data Book. Below 500°C θ -phase (CuAl_2) starts to precipitate out. During slow cooling there is sufficient time for the nucleation and growth of θ . The θ precipitates strengthen the alloy (giving $\sigma_y = 130 \text{ MPa}$). However, because the precipitates are coarse and widely-spaced the resistance to dislocation motion is not great. If the alloy is quenched, θ will not form - instead, a supersaturated solid solution of Cu in Al is produced. On ageing, precipitates containing Al and Cu precipitate ("GP zones and later θ'' "). These are small, closely spaced and interact strongly with dislocations to give the much higher σ_y of 450 MPa .

(c) The value of 3 GPa at -200°C is caused by van der Waals bonding, since the rubber is well below its glass transition temperature, T_g . At T_g , the van der Waals bonds "melt". Above T_g , the modulus of rubber is determined by the ease with which random lengths of the polymer chain can be straightened-out (entropy elasticity). Entropy elasticity is very much a second-order effect compared to secondary bonding, hence the drastic decrease in modulus to 3 MPa.

In contrast to rubber, which only has the occasional covalent cross-link between polymer chains, epoxy resin is a heavily cross-linked polymer, with a high density of covalent cross-links (network polymer, or thermoset). At low temperature the van der Waals bonds make little difference to the already high modulus of 10 GPa.

Q6

The parts of a motor car which have yet to be replaced with polymers or polymer-based composites are generally those which are subjected to one or more of the following: high stresses; high temperatures; high contact forces; impact loadings. Polymers are inferior to metals in most such applications, having comparatively poor mechanical properties (especially at high temperature) and being poor conductors of heat. This is to some extent offset by their low density, ease of forming, and resistance to corrosion. Composites can be made which have better mechanical properties, by incorporating particles, chopped fibres or continuous fibres of stiff materials, e.g. glass or carbon fibre.

Engine.

Could replace sump and rocker cover by chopped-fibre polymers (low stress but moderately high temperature environment). Could replace connecting rods by continuous carbon-fibre reinforced polymers, but fatigue likely to be the limiting factor. Generally, components too hot, highly stressed or dimensionally critical to permit replacement.

Gearbox

Could replace gear teeth and bearings, but the size of the unit would probably go up considerably to keep the stresses acceptable. Gearbox casing could be made from chopped-fibre composite.

Drive shafts

Could be made from helically wound CFRP

to resist the driving torque. Freedom from corrosion an advantage here. Significant weight savings possible, but production cost likely to be adverse initially.

Suspension

CFRP / GFRP coil and leaf springs are potentially attractive for weight saving and corrosion resistance.

Wheels

Injection-moulded CFRP / GFRP wheels should withstand static loads. Good corrosion resistance. Potential problems from impact. Not obvious that a composite wheel would be lighter than a pressed-steel wheel, given the considerable extent of redesign likely in the replacement.

D.R.H. Jones
June 1996

END OF SOLUTIONS