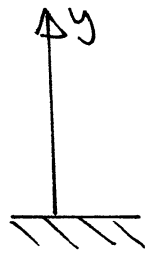
Paper 4 Fluid Mechanics and Heat Transfer, 1996

Solutions



① (i) Hydrostatics $\dfrac{dp}{dy} = -\rho g$

$$\Rightarrow \quad \frac{dp}{dy} = -\left(\frac{p}{RT_0}\right) g$$

$$\Rightarrow \quad \frac{dp}{p} = -\frac{g}{RT_0}\,dy$$

$$\Rightarrow \quad \ln\left(\frac{p}{p_0}\right) = -\frac{g}{RT_0}\,y$$

$$\Rightarrow \quad \underline{\underline{p = p_0\, e^{-\frac{g}{RT_0}y}}}$$

Expand $e^{-\frac{gy}{RT}}$ in Taylor series

$$p = p_0\left[1 - \frac{g}{RT_0}y + \cdots\right]$$

$$\Rightarrow \quad p = p_0 - \frac{p_0}{RT_0}gy + \cdots$$

$$\Rightarrow \quad \underline{\underline{p = p_0 - \rho_0 g y + \cdots}}$$

Need only retain first term in expansion if

$$\frac{gy}{RT_0} \ll 1$$

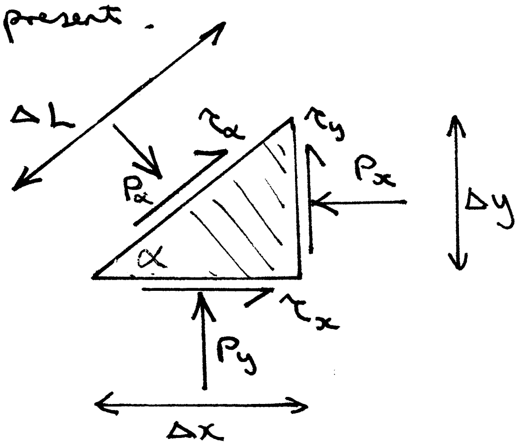
$$\Rightarrow \quad \frac{\rho_0 g y}{p_0} \ll 1$$

$$\Rightarrow \quad \underline{\underline{y \ll \frac{p_0}{\rho_0 g}}}$$

(ii) Pascals law: Pressure same in all directions if the fluid is stationary.

Technical defn. of fluid: Fluid cannot sustain a shear stress and stay at rest.

If fluid is in motion then there will, in general, be shear stresses present.



Forces on element are of order $\Delta y \, p$, $\Delta x \, \tau$ etc.

$$\Sigma F = \Delta m \, a$$

But Δm is of order $\Delta x \Delta y$. Thus, for small element

$$\Sigma F = 0 .$$

Resolve in x direction:

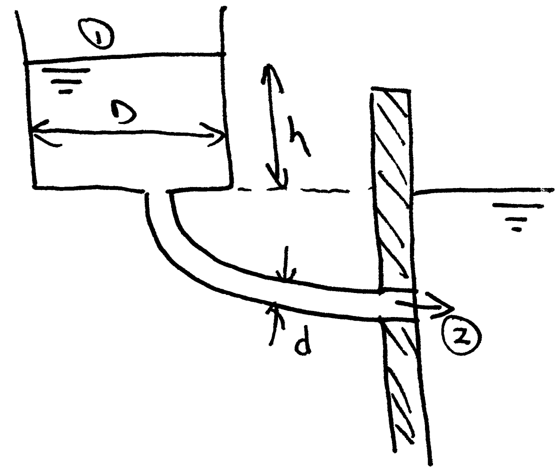
$$-P_x \Delta y + \tau_x \Delta x + (P_\alpha \Delta L) \sin\alpha + (\tau_\alpha \Delta L) \cos\alpha = 0$$

$$\Rightarrow \quad -P_x \Delta y + \tau_x \Delta x + P_\alpha \Delta y + \tau_\alpha \Delta x$$

$$\Rightarrow \quad P_\alpha \neq P_x \quad \text{unless } (\tau_\alpha + \tau_x) = 0$$

2 (i)

Bernoulli $1 \to 2$.



$$p_1 + \tfrac{1}{2}\rho V_1^2 + \rho g(h+w)$$
$$= p_2 + \tfrac{1}{2}\rho V_2^2 + \rho g(0)$$

$p_1 = p_a$, $\quad p_2 = \rho g w + p_a$

$$\Rightarrow \quad \tfrac{1}{2}\rho V_1^2 + \rho g(h+\cancel{w}) = \cancel{\rho g w} + \tfrac{1}{2}\rho V_2^2$$

$$\Rightarrow \quad \tfrac{1}{2}\rho(V_2^2 - V_1^2) = \rho g h$$

$$\Rightarrow \quad V_2^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] = 2gh$$

$$\Rightarrow \quad V_2^2\left[1 - \left(\frac{d}{D}\right)^4\right] = 2gh$$

$$\Rightarrow \quad V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{d}{D}\right)^4}}$$

But $\quad Q = A_2 V_2 = A_1\left(-\frac{dh}{dt}\right)$

$$\Rightarrow \quad \frac{dh}{dt} = -\frac{A_2}{A_1} V_2$$

$$\Rightarrow \quad \frac{dh}{dt} = -\left(\frac{d}{D}\right)^2 \sqrt{\frac{2gh}{1-(d/D)^4}}$$

Integrate

$$h^{-1/2}\, dh = -\left(\frac{d}{D}\right)^2 \sqrt{\frac{2g}{1-(d/D)^4}}\, dt$$

$$\Rightarrow \quad 2\left(\sqrt{h} - \sqrt{h_0}\right) = -\left(\frac{d}{D}\right)^2 \sqrt{\frac{2g}{1-(d/D)^4}}\; t$$

Let t_{final} be time to drain.

$$\Rightarrow \qquad \tau_{final} = 2\sqrt{\frac{1-(d/D)^4}{2g}}\left(\frac{D}{d}\right)^2\sqrt{h_0}$$

(ii) Extended Bernoulli $1 \to 2$

$$\underbrace{\left[0 + \tfrac{1}{2}\rho V_1^2 + \rho g(h+w)\right]}_{\text{Initial Energy}} - \underbrace{\left[\rho g w + \tfrac{1}{2}\rho V_2^2 + 0\right]}_{\text{Final Energy}} = \frac{fl}{d}\tfrac{1}{2}\rho V_2^2$$

$$\Rightarrow \qquad \tfrac{1}{2}\rho V_1^2 - \tfrac{1}{2}\rho V_2^2 - \frac{fl}{d}\tfrac{1}{2}\rho V_2^2 + \rho g h = 0$$

$$\Rightarrow \qquad V_2^2\left[1 + \frac{fL}{d}\right] - V_1^2 = 2gh$$

$$\Rightarrow \qquad V_2^2\left[1 + \frac{fL}{d} - \left(\frac{A_2}{A_1}\right)^2\right] = 2gh$$

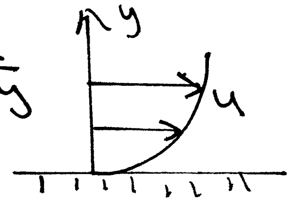
$$\Rightarrow \qquad V_2 = \sqrt{\frac{2gh}{1 + \frac{fL}{d} - \left(\frac{d}{D}\right)^4}}$$

Rest of analysis carries over as before but with $1 - \left(\frac{d}{D}\right)^4$ replaced by, $1 + \frac{fL}{d} - \left(\frac{d}{D}\right)^4$.

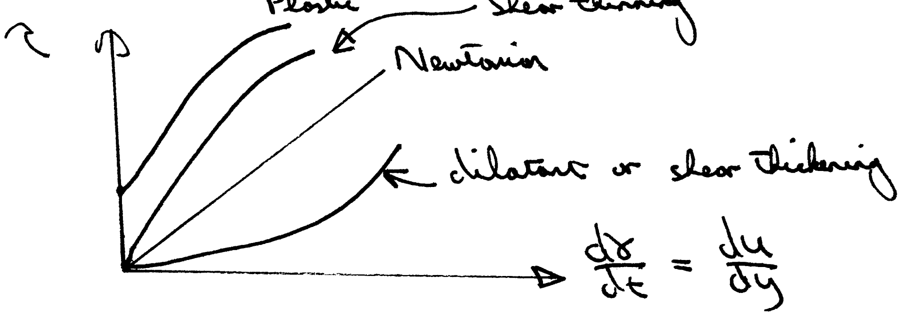
$$\tau_{final} = 2\sqrt{\frac{1 + \frac{fL}{d} - \left(\frac{d}{D}\right)^4}{2g}}\left(\frac{D}{d}\right)^2\sqrt{h_0}$$

Neglect friction if $\frac{fL}{d} \ll 1$.

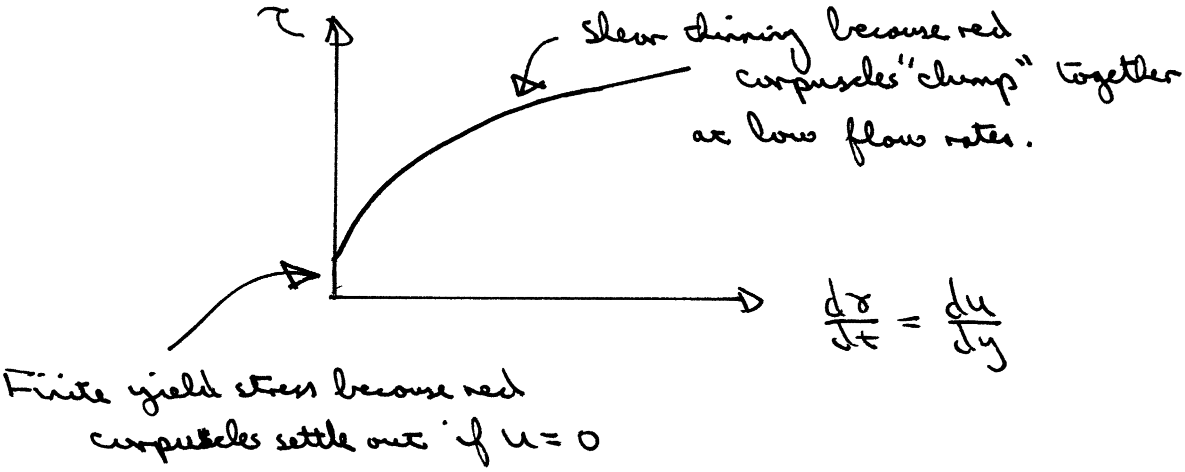
③ (i) Newtons law of viscosity: $\tau = \mu \frac{du}{dy}$



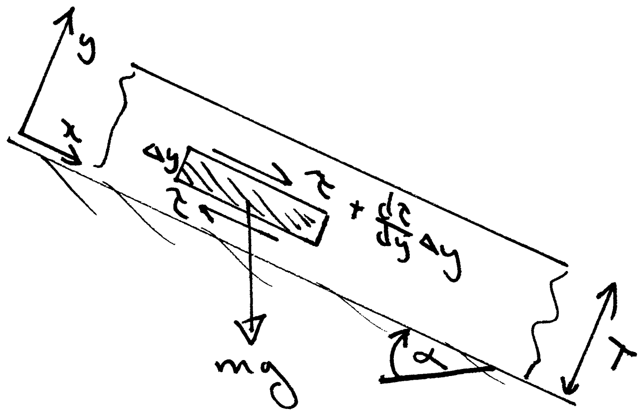
Other types of behavior are:



One non-Newtonian fluid is blood



(ii)



Force balance:

$$x: \quad mg\sin\alpha + \left(\tau + \frac{d\tau}{dy}\Delta y\right)\Delta x - \tau\Delta x = 0$$

$\rho\Delta x\Delta y$

$$\Rightarrow \quad \rho \cancel{\Delta x} \cancel{\Delta y}\, g \sin\alpha + \frac{d\tau}{dy} \cancel{\Delta x} \cancel{\Delta y} = 0$$

$$\Rightarrow \quad \underline{\frac{d\tau}{dy} = -\rho g \sin\alpha}$$

$$\tau = \mu \frac{du}{dy} \Rightarrow \mu \frac{d^2u}{dy^2} = -\rho g \sin\alpha$$

$$\Rightarrow \quad \tau = \mu \frac{du}{dy} = -\rho g \sin\alpha (y + C)$$

But : $\tau = 0$ at $y = T$ (surface)

$$\Rightarrow \quad \tau = \rho g \sin\alpha (T - y)$$

$$\Rightarrow \quad \frac{du}{dy} = \frac{\rho g \sin\alpha}{\mu}(T - y)$$

$$\Rightarrow \quad u = \frac{\rho g \sin\alpha}{\mu}\left(Ty - \frac{y^2}{2}\right) + C$$

But : $u = 0$ at $y = 0 \Rightarrow C = 0$

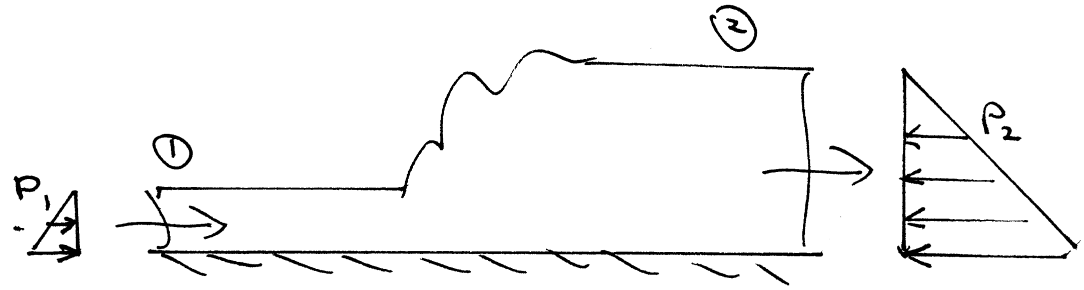
$$\Rightarrow \quad \underline{u = \frac{\rho g \sin\alpha}{\mu}\, y\left(T - \frac{y}{2}\right)}$$

$$Q = \int_0^T u\, dy\, w = w \frac{\rho g \sin\alpha}{\mu}\left(\frac{T^3}{2} - \frac{T^3}{6}\right) = \frac{w \rho g \sin\alpha\, T^3}{3\mu}$$

$$\Rightarrow \quad \underline{T = \left(\frac{3\mu Q}{w \rho g \sin\alpha}\right)^{1/3}}$$

(iii) $$T = \left(\frac{3 \times 10^{-6} \times 45 \times 10^{-6}}{0.3 \times 9.81 \times \sin 60}\right)^{1/3} = \underline{0.760 \text{ mm}}$$

4 (i) Pressure distribution is hydrostatic because there is no vertical acceleration in fluid so the vertical force balance on an element of fluid is pressure versus weight i.e. hydrostatic.



$$\sum F_x = \sum_{OUT} \dot m u_x - \sum_{IN} \dot m u_x$$

$$\int_0^{h_1} P_1\,dy - \int_0^{h_2} P_2\,dy = \underbrace{(\rho h_1 V_1)}_{\dot m} V_2 - (\rho h_1 V_1) V_1$$

$$\Rightarrow \quad \tfrac{1}{2}\cancel{\rho} g h_1^2 - \tfrac{1}{2}\cancel{\rho} g h_2^2 = \cancel{\rho} h_1 V_1 (V_2 - V_1)$$

$$\Rightarrow \quad \underline{\underline{\frac{g}{2}\left(h_1^2 - h_2^2\right) = h_1 V_1 (V_2 - V_1)}}$$

(ii) Need to elliminate V_2 using continuity $h_1 V_1 = h_2 V_2$

$$\frac{g}{2}\left(h_1^2 - h_2^2\right) = h_1 V_1^2 \left(\frac{V_2}{V_1} - 1\right) = h_1 V_1^2 \left(\frac{h_1}{h_2} - 1\right)$$

$$\Rightarrow \quad \left(h_1^2 - h_2^2\right) = 2\frac{V_1^2}{g h_1}\frac{h_1^2}{h_2}(h_1 - h_2)$$

$$\Rightarrow \quad (h_1 + h_2) = 2\frac{V_1^2}{g h_1}\frac{h_1^2}{h_2}$$

$$\Rightarrow \quad \underline{\underline{\left(\frac{h_2}{h_1}\right)\left(1 + \frac{h_2}{h_1}\right) = 2\frac{V_1^2}{g h_1}}}$$

For $h_1 = 1.5$, $V_1 = 9.6$, $2Fr^2 = 12.53$

$$\left(\frac{h_2}{h_1}\right)^2 + \frac{h_2}{h_1} - 12.53 = 0$$

$$\Rightarrow \frac{h_2}{h_1} = 3.07$$

$$\Rightarrow \underline{\underline{h_2 = 4.61 \text{ m}}}$$

(iii) For dynamic similarity, need Fr same in lab test.

$$\Rightarrow \frac{V_{test}}{\sqrt{h_{test}}} = \frac{V_{real}}{\sqrt{h_{real}}}$$

$$\Rightarrow V_{test} = V_{real}\sqrt{\frac{h_{test}}{h_{real}}} = 9.6\sqrt{\frac{1}{3}}$$

$$= \underline{\underline{5.54 \text{ m/s}}}$$

5 (i)

$$T - T_w = (T_0 - T_w)\left(1 - \frac{9}{5}\left(\frac{r}{R}\right)^2 + \frac{4}{5}\left(\frac{r}{R}\right)^3\right)$$

T_0 = centre-line temp.

$$Nu = \frac{hD}{k} \quad , \quad h = \frac{\dot{Q}}{(T_0 - T_w)A} = \frac{-k\left(\frac{\partial T}{\partial r}\right)_w}{T_0 - T_w}$$

$$\left(\frac{\partial T}{\partial r}\right)_w = (T_0 - T_w)\left[-\frac{18}{5}\frac{1}{R} + \frac{12}{5}\frac{1}{R}\right]$$

$$= -\frac{(T_0 - T_w)}{R}\frac{6}{5}$$

$$\Rightarrow \quad h = \left(\frac{-k}{(T_0 - T_w)}\right)\left(-\frac{6}{5}\frac{(T_0 - T_w)}{R}\right)$$

$$= \frac{6}{5}\frac{k}{R}$$

$$Nu = \frac{hD}{k} = \frac{6}{5}\frac{\cancel{k}}{\cancel{R}}\frac{2\cancel{R}}{\cancel{k}} = \frac{12}{5} = \underline{\underline{2.4}}$$

(ii)

$$\frac{\partial T}{\partial r} = (T_0 - T_w)\left(-\frac{18}{5}\frac{r}{R^2} + \frac{12}{5}\frac{r^2}{R^3}\right)$$

$$\Rightarrow \left.\frac{\partial T}{\partial r}\right)_0 = 0 \checkmark$$ This must be true to avoid a cusp in the temperature profile ie. symmetry

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{\partial}{\partial r}\left[(T_0 - T_w)\left(-\frac{18}{5}\frac{r^2}{R^2} + \frac{12}{5}\frac{r^3}{R^3}\right)\right]$$

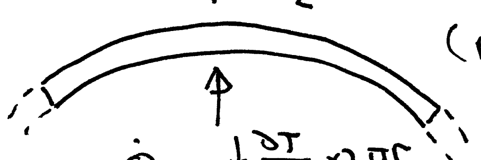
$$= (T_0 - T_w)\left(-\frac{36}{5}\frac{r}{R^2} + \frac{36}{5}\frac{r^2}{R^3}\right)$$

$$\Rightarrow \frac{\partial}{\partial r}\left(2\pi r\frac{\partial T}{\partial r}\right)_w = 0 \checkmark$$

This must hold true because of no-slip at the wall, so no heat is transported axially by fluid, so locally we have a heat balance due to conduction.

$$\dot{Q}_2 = -k\frac{\partial T}{\partial r} \times 2\pi r$$

(per unit length)



$$\dot{Q}_1 = -k\frac{\partial T}{\partial r} \times 2\pi r$$

For steady state, $\dot{Q}_1 = \dot{Q}_2$

$$\Rightarrow \quad \underline{\frac{\partial}{\partial r}\left(2\pi r\, k\frac{\partial T}{\partial r}\right) = 0}$$

(iii)

$$h = f(k,\ \rho c_p,\ \nu,\ D,\ \bar{V})$$

7 independent parameters ; 4 dimension (M, L, T, Θ)

$\Rightarrow$ 3 groups.

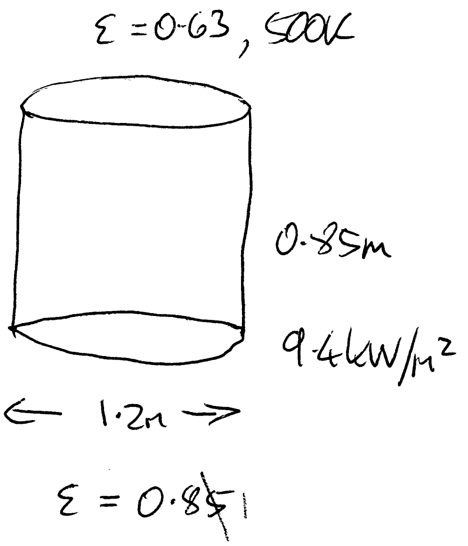
They are,

$$\begin{cases} Nu = \dfrac{hd}{k} & \text{(or Stanton no. will do)} \\[2ex] Re = \dfrac{\bar{V}D}{\nu} \\[2ex] Pr = \dfrac{(\rho c_p)\nu}{k} \end{cases}$$

1B Paper 4 Exam Question Crib 1996

Radiation Heat Transfer and Combustion

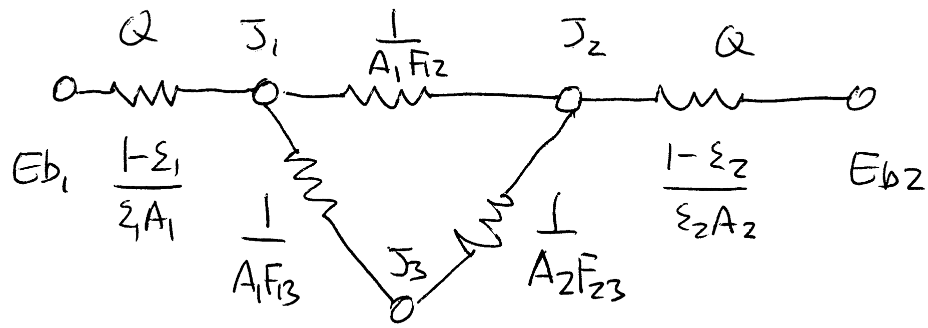
6.



Let 1 ≡ bottom

2 ≡ top

3 ≡ sides



$$Q = \frac{Eb_1 - Eb_2}{R_T} \quad ; \quad R_T = \frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}$$

$$+ \frac{\frac{1}{A_1F_{12}}\left(\frac{1}{A_1F_{13}} + \frac{1}{A_2F_{23}}\right)}{\frac{1}{A_1F_{12}} + \frac{1}{A_1F_{13}} + \frac{1}{A_2F_{23}}}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{parallel}$$

$$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2}$$

Areas : $A_1 = A_2 = \frac{\pi}{4} \times (1.2)^2 = 1.131\,m^2$

$\varepsilon_1 = 0.81, \quad \varepsilon_2 = 0.63$ given.

View factor $F_{12} = 0.25$ given.

Thus $F_{13} = 1 - F_{12} = 0.75 = F_{23}$ since $A_1 = A_2$

Then
$$R_T = \frac{1-0.81}{0.81 \times 1.131} + \frac{1-0.63}{0.63 \times 1.131} + \frac{\frac{1}{1.131 \times 0.25}\left(\frac{1}{1.131 \times 0.75} + \frac{1}{1.131 \times 0.75}\right)}{\frac{1}{1.131 \times 0.25} + \frac{1}{1.131 \times 0.75} + \frac{1}{1.131 \times 0.75}}$$

$$= 0.2074 + 0.5193 + \frac{3.537\,(\overbrace{1.179 + 1.179}^{2.358})}{3.537 + 1.179 + 1.179)}$$

$$= 0.7267 + 1.415 = 2.1415\,m^{-2}.$$

$$Q = 9400 \times 1.131 = 10631.4\,W$$

$$E_{b1} = E_{b2} + QR_T$$

$$\sigma T_1^4 = \sigma T_2^4 + QR_T$$

$$T_1^4 = T_2^4 + QR_T/\sigma$$

$$T_1^4 = 500^4 + \frac{\overbrace{10631.4 \times 2.1415}^{22767.1}}{\underbrace{5.67 \times 10^{-8}}_{4.015 \times 10^{11}}}$$

$$\boxed{T_1 = 825K.}$$

b) $CH_4 + 2\times(O_2 + \frac{79}{21} N_2^*)$

$\Rightarrow CO_2 + 2H_2O + 2\frac{79}{21} N_2^*$

SFEE $\dot{Q} = H_{PT} - H_{R298}$ ← { 25° reactants

$= H_{PT} - H_{P298} - \Delta H_{298} - (\cancel{H_{R298}} - \cancel{H_{R298}})$ 0

$\Delta H_{298} \equiv -(LCV)$ assuming H_2O in products is vapour.

LCV (Haywood) $= 50\,010\ kJ/kg_{CH_4}$

Convert to moles $= 800160\ kJ/kmol_{CH_4}$

Heat lost $= 600kW$ (given)

Mass of CH_4 supplied $= 85kg/hour = 0.0236kg/s$

Heat lost/mole of CH_4 $= \frac{600}{0.0236} \times 16 = 406780\ kJ/kmol_{CH_4}$

$\therefore H_{PT} - H_{P298} = 800160 - 406780$

$= 393380\ kJ/kmol_{CH_4}$

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Products	CO_2	H_2O	N_2^*	
moles	1	2	7.524	
H_{mol} 298	9.37	9.90	8.67	
total	9.37	19.80	65.23	94.4 MJ
H_{mol} 1500	71.13	58.05	47.09	
total	71.13	116.1	354.3	541.53 MJ
				447.13 MJ too large
H_{mol} 1400	65.31	53.39	43.62	
total	65.31	106.28	328.20	500.29 MJ
				405.89 MJ close, but high
H_{mol} 1300	59.55	48.84	40.19	
total	59.55	97.68	302.39	459.62 MJ
				365.22 MJ too small

Interpolate: $T = \left[\frac{1400-1300}{405.89-365.22}\right] \times (393.38 - 365.22) + 1300$

$$\boxed{T = 1369°C}$$