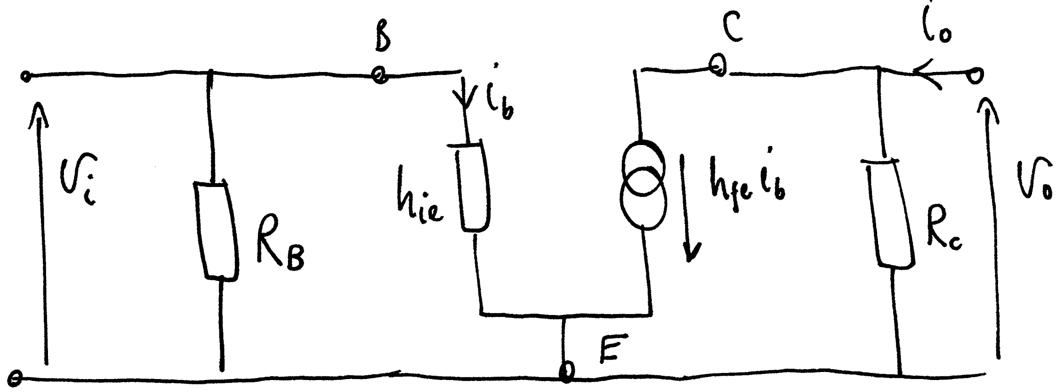
PART 1B PAPER 5 1996 CRIB.

1/ Class A amplifier – transistor is biased into its linear region, usually such that the collector-emitter voltage is half the supply voltage, allowing maximum voltage swings. Consequently, the quiescent collector current is large, which results in large quiescent power dissipation. This is reflected in the poor efficiency, typically < 25%.

Class B amplifier – both transistors are biased so that $V_{BE} = 0, I_B = 0, I_C = 0$, and V_{CE} = half the supply voltage. Consequently, quiescent power dissipation is zero, and the efficiency is far greater than Class A amplifiers, typically 78%.

i) Class A amplifier

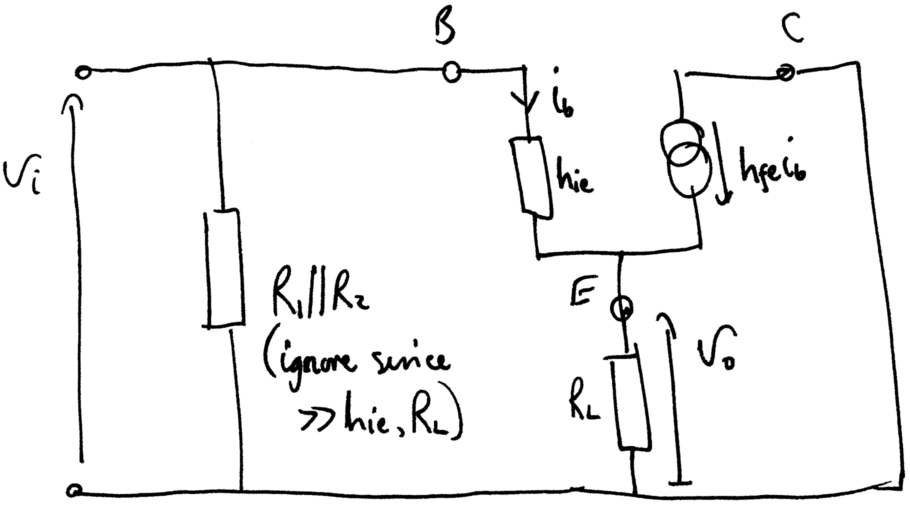


$$i_b = \frac{v_i}{h_{ie}} \qquad v_o = -h_{fe} i_b R_c = -h_{fe} \frac{v_i}{h_{ie}} R_c$$

$$\Rightarrow \quad \underline{\underline{\frac{v_o}{v_i} = -\frac{h_{fe} R_c}{h_{ie}}}} = \text{Gain}$$

$$\text{Output resistance} = \left.\frac{v_o}{i_o}\right|_{v_i = 0} = \underline{\underline{R_c}}$$

ii) In a Class B amplifier, only one transistor conducts at any one time, so choose Q_2

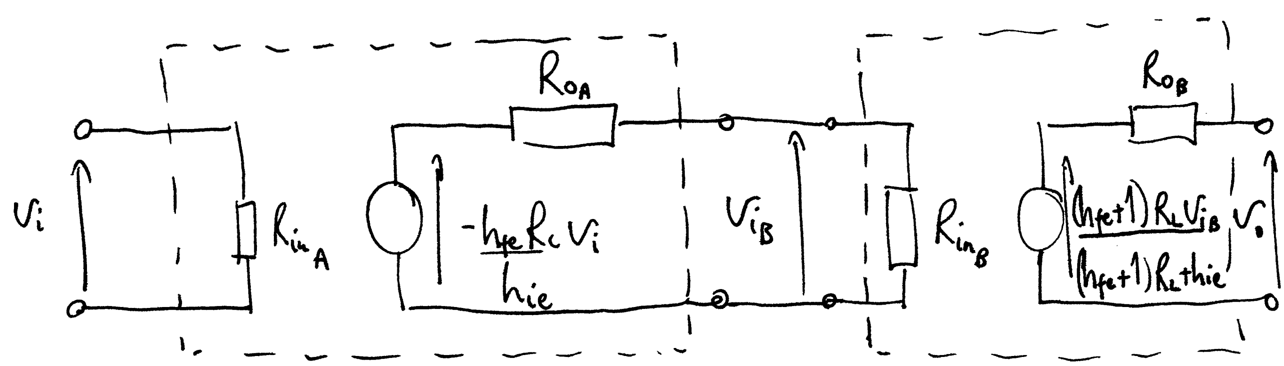


$$V_o = (h_{fe}+1)\, i_b R_L$$

$$\Rightarrow \quad V_i = (h_{fe}+1)\, i_b R_L + h_{ie} i_b$$

$$R_{in} = \frac{V_i}{i_b} = \underline{\underline{(h_{fe}+1) R_L + h_{ie}}}$$

$$\text{Gain} = \frac{V_o}{V_i} = \underline{\underline{\frac{(h_{fe}+1) R_L}{(h_{fe}+1) R_L + h_{ie}}}}$$



Class A amplifier Class B amplifier

$$V_{i_B} = \frac{R_{in_B}}{R_{in_B} + R_{O_A}} \times \left(- \frac{h_{fe} R_C V_i}{h_{ie}} \right) \qquad \text{(Potential divider)}$$

where $R_{in_B} = (h_{fe}+1) R_L + h_{ie}$, $R_{O_A} = R_C$

$$\Rightarrow V_{i_B} = - \frac{(h_{fe}+1) R_L + h_{ie}}{(h_{fe}+1) R_L + h_{ie} + R_C} \cdot \frac{h_{fe} R_C V_i}{h_{ie}}$$

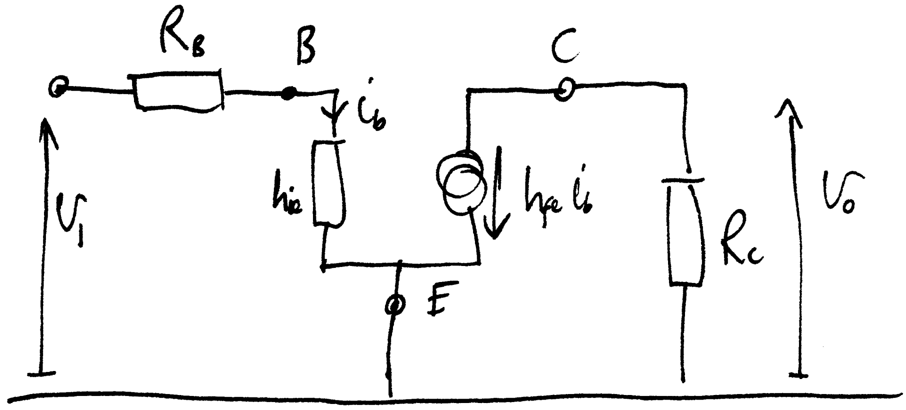
$$\therefore V_o = V_{i_B} \times \text{gain of class B amplifier}$$

$$= - \frac{(h_{fe}+1) R_L + h_{ie}}{(h_{fe}+1) R_L + h_{ie} + R_C} \cdot \frac{h_{fe} R_C V_i}{h_{ie}} \cdot \frac{(h_{fe}+1) R_L}{(h_{fe}+1) R_L + h_{ie}}$$

$$= - \frac{h_{fe} R_C (h_{fe}+1) R_L}{\left((h_{fe}+1) R_L + h_{ie} + R_C\right) h_{ie}}$$

2/ CMRR = ratio of differential gain : common mode gain. A large value means that common-mode signals (unwanted) are not amplified, and so do not appear at the o/p, as required.

i) Differential signal means $v_1 = -v_2 \Rightarrow$ no small-signal current in $R_T \Rightarrow$ no small signal voltage at A



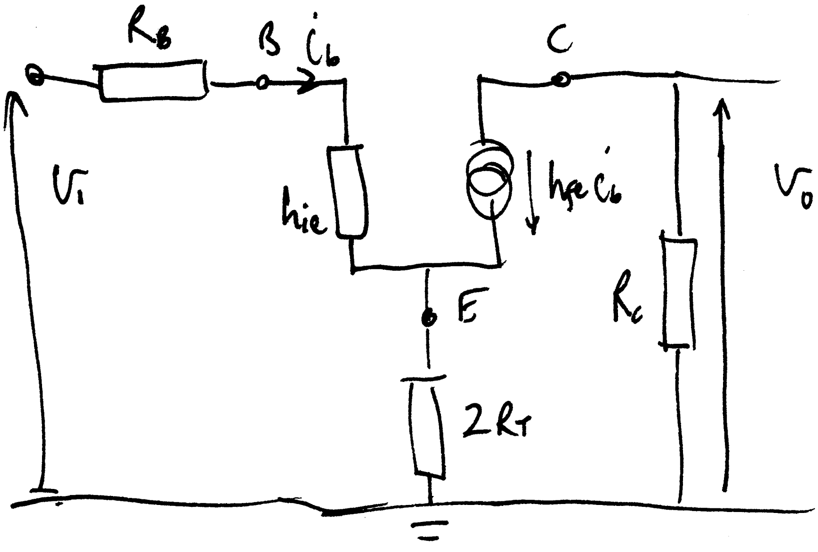
$$v_1 = (R_B + h_{ie})\, i_b$$

$$v_o = -h_{fe}\, i_b\, R_C$$

$$\text{so } v_o = -\frac{h_{fe} R_C}{R_B + h_{ie}}\, v_1$$

$$\Rightarrow \text{Diff gain} = -\frac{h_{fe} R_C}{R_B + h_{ie}}$$

For common-mode gain, $v_1 = v_2$, $i_{e_1} = i_{e_2}$ and so there is now a small signal voltage at A. ∴ To simplify analysis, regard R_T as $2R_T // 2R_T$.



$$V_i = (R_B + h_{ie}) i_b + (h_{fe}+1) i_b . 2 R_T$$

$$V_o = - h_{fe} i_b R_c$$

$$\Rightarrow V_o = \frac{-h_{fe} R_c}{R_B + h_{ie} + (h_{fe}+1) 2 R_T} V_i$$

$$\Rightarrow \text{Common-mode gain} = \frac{-h_{fe} R_c}{R_B + h_{ie} + (h_{fe}+1) 2 R_T}$$

$$\Rightarrow \quad CMRR = \frac{A_{diff}}{A_{com}} = \frac{R_B + h_{ie} + 2(h_{fe}+1) R_T}{R_B + h_{ie}}$$

$$A_{diff} = \frac{-h_{fe} R_c}{R_B + h_{ie}} = 100$$

$$R_c = 450, \; h_{fe} = 300, \; h_{ie} = 10^3$$

$$\Rightarrow \quad \frac{300 \times 450}{R_B + 10^3} = 100$$

$$R_B = 350\ \Omega.$$

$$CMRR \approx 10^4 = \frac{R_B + h_{ie} + 2(h_{fe}+1) R_T}{R_B + h_{ie}}$$

$$= \frac{350 + 10^3 + 2(300+1) R_T}{350 + 10^3}$$

$$\Rightarrow \underline{\underline{R_T = 22.4 \, k\Omega}}.$$

Assume $I_E = I_C = 20 \, mA$

$\therefore$ 40 mA flows in R_T, so $V_{EE} = -40 \times 22.4$

$$\approx \underline{\underline{-897 \, V.}}$$

$$V_{cc} = V_{CE} + I_C R_C = 10 + 20 \times 10^{-3} \times 450$$
$$= \underline{\underline{19 \, V.}}$$

$\therefore$ Total d.c. voltage supply required ≈ 916 V !

This is rather large.

Replacing R_T with a constant current source means no small signal current can flow from point A, so $i_{e_1} \simeq -i_{e_2}$. $\therefore$ No common-mode gain, but without requiring a huge d.c. voltage supply.

Constant current $= 2 \times 20 mA \approx \underline{40 mA.}$

If ideal constant current source, $i_{e_1} = i_{e_2}$, and CMRR is infinite.

3/ Alternating voltage mean that transformers can be used to increase transmission voltage, for greater efficiency.

3ϕ make efficient use of magnetic and electrical materials in the generator.

i) $$P_\Delta = \mathrm{Re}\left(\frac{3V^2}{R_1 + jX_1}\right) = \frac{3V^2}{Z_1^2} R_1 \,,\quad Q_\Delta = \mathrm{Im}\left(\frac{3V^2}{R_1 + jX_1}\right) = \frac{3V^2}{Z_1^2} X_1$$

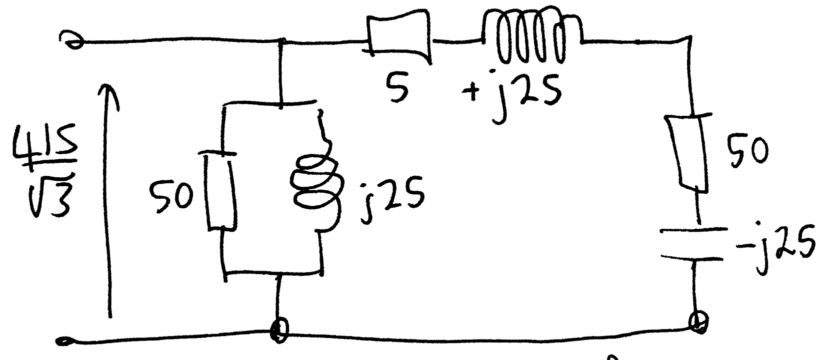
ii) $$P_\lambda = \mathrm{Re}\left(\frac{3(V/\sqrt{3})^2}{R_2 + jX_2}\right) = \frac{V^2}{Z_2^2} R_2 \,,\quad Q_\lambda = \mathrm{Im}\left(\frac{3(V/\sqrt{3})^2}{R_2 + jX_2}\right) = \frac{V^2}{Z_2^2} X_2$$

$P_\Delta = P_\lambda$, $Q_\Delta = Q_\lambda$ only if $\overline{Z}_2 = \frac{\overline{Z}_1}{3}$ (sub into P_Δ, P_λ and Q_Δ, Q_λ to prove it).

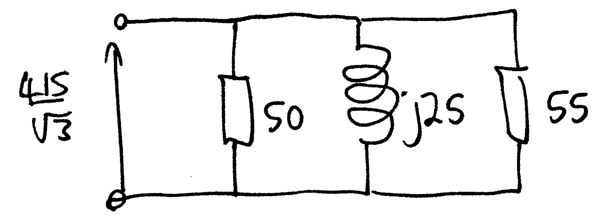
Delta-connected load = $(150 - j75)\,\Omega$ (Note -ve sign because it is a capacitive reactance).

$\therefore$ Star-connected equivalent load $= \frac{150 - j75}{3}$

$= (50 - j25)\,\Omega$.



$\therefore$ One phase of this 3ϕ load look like, which reduces to



$$\therefore\ P = \frac{3\left(\frac{415}{\sqrt{3}}\right)^2}{50} + \frac{3\left(\frac{415}{\sqrt{3}}\right)^2}{55} = (415)^2\left(\frac{1}{50}+\frac{1}{55}\right)$$

$$= \underline{\underline{6575\,W}}$$

$$Q = \frac{3\left(415/\sqrt{3}\right)^2}{25} = \frac{(415)^2}{25} = \underline{\underline{+6889\ VAR.}}$$

Reactive power consumed by Δ-connected capacitor $= -6889$ VAR to give unity power factor

$$\Rightarrow\ -\frac{3V^2}{X_c} = -6889 \ \Rightarrow\ X_c = 75\,\Omega = \frac{1}{\omega C}$$

$$\Rightarrow\ C = \frac{1}{2\pi\times50\times75} = \underline{\underline{42.4\,\mu F.}}$$

Current flowing in one phase of load 2 $= \dfrac{415/\sqrt{3}}{55}$

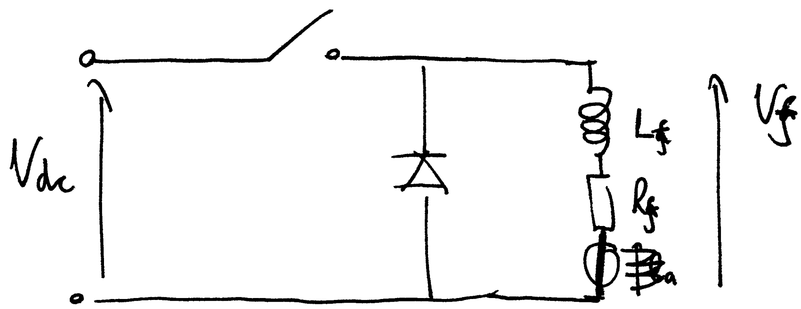
$$\Rightarrow\ \text{Power} = 3\left(\left(415/\sqrt{3}\right)/55\right)^2\times 50 = \underline{\underline{2847\ W}}$$

$$\text{Reactive power} = -3\left(415/\sqrt{3}/55\right)^2\times 25 = \underline{\underline{-1423\ VAR}}$$

Line voltage at load 2 given by $\sqrt{3}\,V_L I_L = S = 3183$ VA

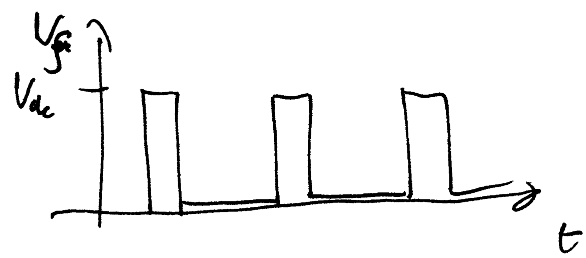
$$\Rightarrow\ \underline{\underline{V_L = 421.8\,V.}}$$

4a) $e_a = k\phi\omega$. Ignoring the voltage drop across armature resistance R_a, then $e_a \simeq V_a = k\phi\omega$. Flux ϕ is controlled by field current i_f, so $\omega = \dfrac{V_a}{k\phi(i_f)}$. Hence, controlling field current controls speed.



Diode gives free wheel path for ~~armature~~ field current to flow in when switch is open, clamping V_f to 0V. When switch is closed, $V_f = V_{dc}$.

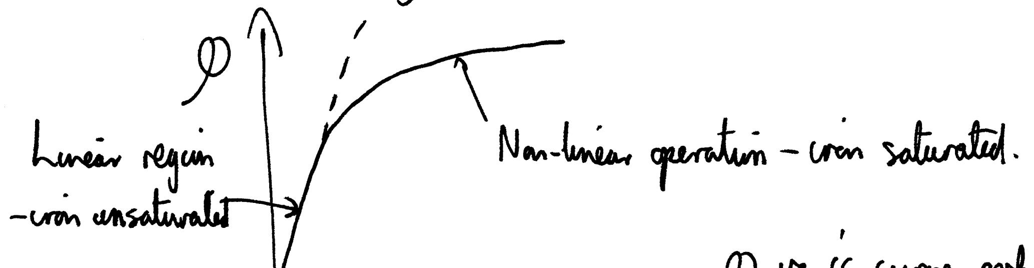
$\therefore$



By controlling the mark : space ratio of the chopper, the average ~~arm~~ field voltage is controlled, and hence the average field current.

b) Open-circuit test $\Rightarrow V_a = V_{oc} = e_a$

$\therefore$ $k\phi = e_a/\omega$, so if i_f is varied, and V_{oc} is measured, we know $k\phi$ vs. i_f.



ϕ vs i_f curve exhibits magnetic saturation above a certain field current.

c) $V_f = 240V, \quad i_f = \dfrac{V_f}{R_f} = \dfrac{240}{80} = 3A.$

From o.c. data, $k\phi\Big|_{i_f=3A} = \dfrac{210}{1000 \times 2\pi/60}$

$$T = k\phi i_a \Rightarrow 40 = \frac{210}{1000 \times 2\pi/60} i_a$$

$$\Rightarrow i_a = 19.95A, \text{ so } e_a = V_a - i_a R_a = 240 - 19.95 \times 1$$

$$= 220.05V$$

$$\omega = \frac{e_a}{k\phi} = \underline{\underline{1048 \text{ rpm.}}}$$

d) Mark : space ratio = 1:2

$\Rightarrow$ Average field voltage $= \dfrac{1}{3} \times 240 = 80V$

$\therefore$ Field current = 1A, so $k\phi = \dfrac{75}{1000 \times 2\pi/60}$ (o.c. test data).

$$T = k\phi i_a \Rightarrow i_a = 55.85A, \; e_a = V_a - i_a R_a$$
$$= 240 - 55.85$$
$$= 184.15V$$

$$\therefore \omega = \frac{e_a}{k\phi} = \underline{\underline{2455 \text{ rpm.}}}$$

Output power from d.c. chopper = $80 \times 1 = 80W$.

$\therefore$ Input power to dc chopper = $\dfrac{80}{0.8} = 100W$.

Input power ~~fr~~ to armature circuit = $240 \times 55.85 = 13404W$.

$\therefore$ Total input power = $13404 + 100 = 13504$ W.

Output power = $e_a i_a = 184.15 \times 55.85 = 10285$ W.

$\therefore$ Overall efficiency = $\dfrac{10285}{13504} = \underline{\underline{76.2\%}}$

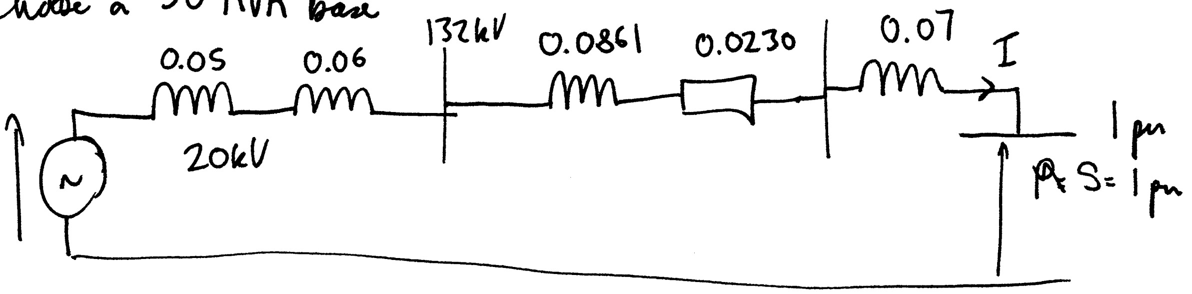
5/ The p.u. system simplifies power flow calculations in systems with differing voltage levels.

$$VA_b = \sqrt{3}\, V_b I_b \quad , \quad I_b = \frac{V_b}{\sqrt{3} Z_b}$$

$$\Rightarrow VA_b = \sqrt{3} V_b \, . \, \frac{V_b}{\sqrt{3} Z_b}$$

so $$\underline{\underline{Z_b = \frac{V_b^2}{VA_b}.}}$$

Choose a 50 MVA base



$P = 40\,MW, \cos\phi = 0.8$
$\Rightarrow S = 50\,MVA, \; Q = 30\,MVAr$

For the feeder, $Z_b = \frac{V_b^2}{VA_b} = \frac{(132 \times 10^3)^2}{50 \times 10^6} = 348.5\,\Omega$

$$P_{pu} = \frac{40}{50} = 0.8\,pu \; , \; Q = \frac{30}{50} = 0.6\,pu \; , \; S = \frac{50}{50} = 1\,pu \; ,$$

so $I = 1\,pu$

$$\therefore P_{gen_{pu}} = P_{load\,pu} + P_{feeder_{pu}} = 0.8 + (1)^2 \times 0.023$$

$$= 0.823 \; p.u.$$

$$\therefore P_{gen} = 0.823 \times 50\,MW = \underline{41.15\,MW}$$

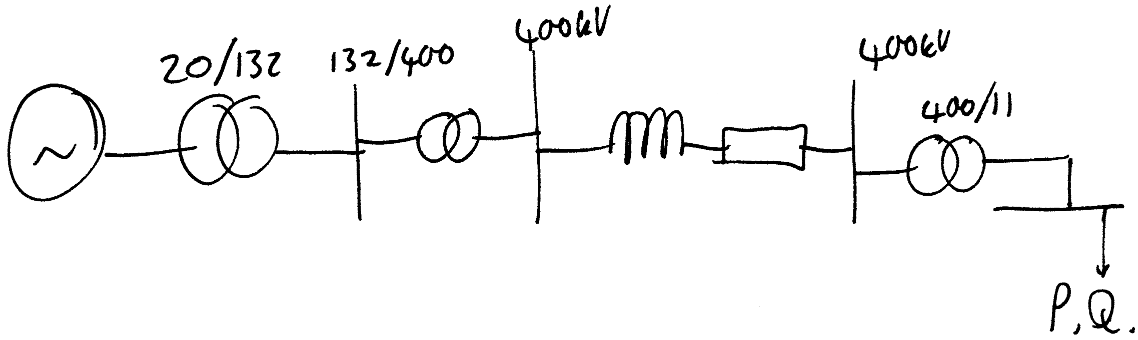
$$Q_{gen_{pu}} = Q_{load\,pu} + (1)^2 (0.07 + 0.0861 + 0.06 + 0.05)$$

$$= 0.6 + 0.2661 = 0.8661 \text{ pu}$$

$$\Rightarrow Q_{gen} = 0.8661 \times 50 \text{ MVAR} = \underline{\underline{43.3 \text{ MVAR}}}.$$

$$P_{feeder_{pu}} = (1)^2 \times 0.023 = 0.023 \text{ pu} = 0.023 \times 50 \text{ MW}$$

$$= \underline{\underline{1.15 \text{ MW}}}.$$

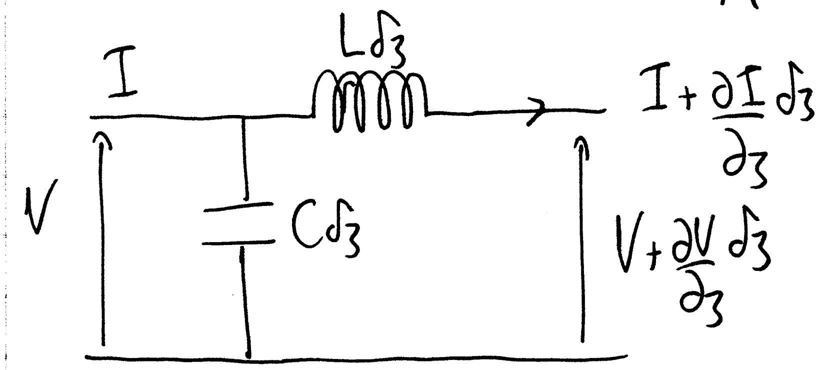


Since the load voltage, power and reactive power remain the same, so will the load current.

The main feeder current will be reduced by the factor $\frac{132}{400}$, and the losses are now $\left(\frac{132}{400}\right)^2 \times$ old losses

ie reduced by the factor $\left(\frac{400}{132}\right)^2 = \underline{\underline{9.2}}$.

6/



Kirchoff's Voltage Law : $V = L\delta z \frac{\partial}{\partial t}\left(I + \frac{\partial I}{\partial z}\delta z\right) + V + \frac{\partial V}{\partial z}\delta z$

$$\Rightarrow \frac{\partial V}{\partial z}\delta z = -L\delta z\left(\frac{\partial I}{\partial t} + \frac{\partial^2 I}{\partial t \partial z}\delta z\right) = -L\delta z\frac{\partial I}{\partial t} + O(\delta z)^2$$

$$\Rightarrow \underline{\underline{\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t}}} \qquad (1)$$

Kirchoff's current law : $I = C\delta z\frac{\partial V}{\partial t} + I + \frac{\partial I}{\partial z}\delta z$

$$\Rightarrow \underline{\underline{\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t}}} \qquad (2)$$

Differentiate (1) wrt z, (2) wrt t to eliminate I

$$\frac{\partial^2 V}{\partial z^2} = -L\frac{\partial^2 I}{\partial z \partial t}, \quad \frac{\partial^2 I}{\partial t \partial z} = -C\frac{\partial^2 V}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} \qquad (3).$$

Put $V = f_+(z - vt) + f_-(z+vt)$ into ③

$$\frac{\partial^2 V}{\partial z^2} = f_+''(z-vt) + f_-''(z+vt)$$

$$\frac{\partial^2 V}{\partial t^2} = v^2 f_+''(z-vt) + v^2 f_-''(z+vt)$$

$$\Rightarrow f_+''(z-vt) + f_-''(z+vt) = LC\left(v^2 f_+''(z-vt) + v^2 f_-''(z+vt)\right)$$

If this is to be a solution to ③, then $LCv^2 = 1$

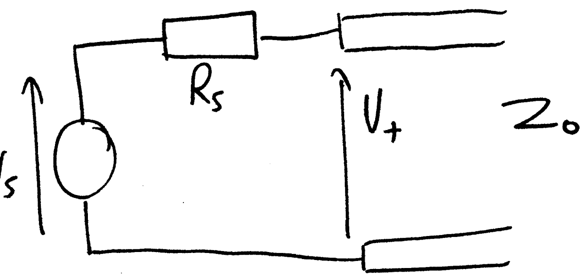
$$\Rightarrow \underline{\underline{v = 1/\sqrt{LC}}}$$

Physical significance: A lossless transmission line supports forwards and backwards travelling voltage (and current) signals, which maintain their spatial distribution as they travel at speed $1/\sqrt{LC}$.

b) i) $\rho_V = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

At far end there is a short circuit, so $Z_L = 0$, so $\underline{\underline{\rho_V = -1}}$

For switch end, there is an open circuit, so $Z_L = \infty$, and $\underline{\underline{\rho_V = +1}}$



By current continuity at start of line

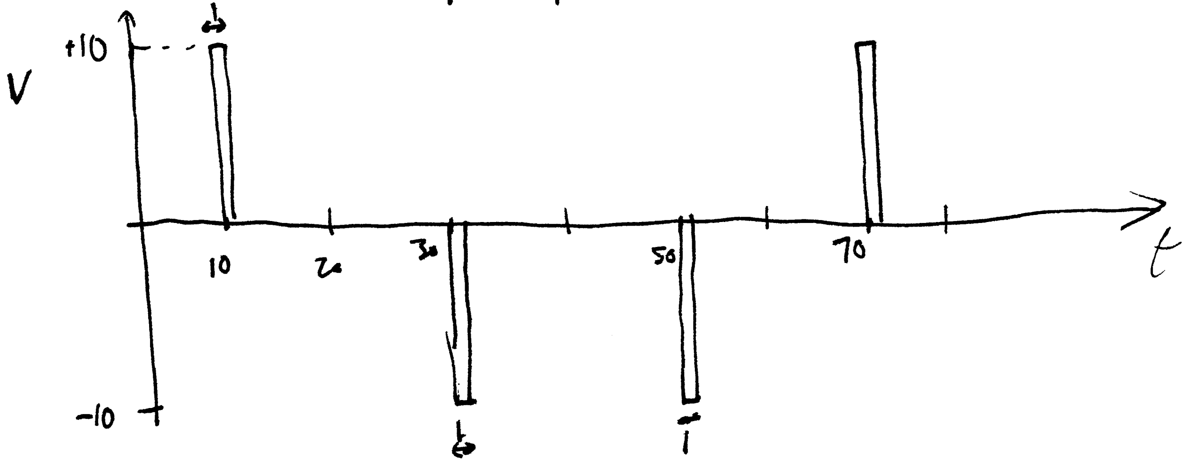
$$\frac{V_s - V_+}{R_s} = \frac{V_+}{Z_0} \Rightarrow \underline{\underline{V_+ = \frac{Z_0}{Z_0 + R_s} V_s}}$$

4) Magnitude of voltage pulse $= \frac{50}{50+10} \times 12 = 10V$.

It takes a time of $\frac{6\times10^3 \text{ m}}{3\times10^8 \text{ m/s}} = 20\mu s$ for pulse to travel from one end of the line to the other.

$\therefore$ Takes $10\mu s$ to reach midpoint of line

$\therefore$ Voltage is 0 for $t < 10\mu s$, then $+10V$ to $10 < t < 11 \mu s$.
On reaching the s.c. end, since $\rho_V = -1$, the voltage pulse will be $-10V$, for $30 < t < 31 \mu s$. At the o.c. end, $\rho_V = +1$, so pulse will remain as $-10V$ for $50 < t < 51 \mu s$. For $70 < t < 71 \mu s$, pulse is back to $+10V$ (reflected from s.c. end, so $-1 \times -10 = +10$)



Current supplied for $1\mu s$ by 12V source $= 2/R_s = 0.2A$
(or $10/Z_0 = 0.2A$)

$\therefore$ Power $= 12 \times 0.2$ $(V \times I)$

$\therefore$ Energy $= 12 \times 0.2 \times 1\times10^{-6} = \underline{\underline{2.4\times10^{-6} \text{ Joules}}}$.

7/ Impedance of free space $= \dfrac{|\bar{E}|}{|\bar{H}|}$ (analogous to electrical impedance, V/I)

$$\bar{E} = \underline{u}_x E_0 e^{j(\omega t - \beta z)}$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$, for sinusoidal time variation $\frac{\partial}{\partial t} \rightarrow j\omega$.

Also, $\underline{B} = \mu_0 \underline{H}$

$$\Rightarrow \nabla \times \underline{E} = -j\omega\mu_0 \underline{H}$$

$$\nabla \times \underline{E} = \begin{vmatrix} \underline{u}_x & \underline{u}_y & \underline{u}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_0 e^{j(\omega t - \beta z)} & 0 & 0 \end{vmatrix} = \underline{u}_y (-j\beta) E_0 e^{j(\omega t - \beta z)}$$

$$\Rightarrow \underline{u}_y (-j\beta) E_0 e^{j(\omega t - \beta z)} = -j\omega\mu_0 \underline{H}$$

so $$\underline{H} = \underline{u}_y \frac{\beta}{\omega\mu_0} E_0 e^{j(\omega t - \beta z)}$$ $$\frac{\omega}{\beta} = c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

$$= \underline{u}_y \frac{\sqrt{\epsilon_0\mu_0}}{\mu_0} E_0 e^{j(\omega t - \beta z)}$$

$$= \underline{u}_y \frac{E_0}{\eta_0} e^{j(\omega t - \beta z)}$$

N.B. Acceptable to say $|\underline{H}| = \dfrac{E_0}{\eta_0}$, $\underline{E}$ and $\underline{H}$ are in phase in time and space, and $\underline{E}$ and $\underline{H}$ are both orthogonal to each other and the direction of propagation, so that $\underline{E} \times \underline{H}$ is parallel to $\underline{u}_z$. $\therefore \underline{H}$ must be parallel to $\underline{u}_y$

$$\Rightarrow \underline{H} = \underline{u}_y \frac{E_0}{\eta_0} e^{j(\omega t - \beta z)}$$

Poynting vector $= \frac{1}{2} \underset{\sim}{E} \times \underset{\sim}{H}^*$, and represents the power per unit area being transmitted by the electromagnetic wave.

For a plane e-m wave

$$\underset{\sim}{P} = \frac{1}{2} \underset{\sim}{u}_x E_0 e^{j(\omega t - \beta z)} \times \left(\underset{\sim}{u}_z \frac{E_0}{\eta_0} e^{j(\omega t - \beta z)} \right)^*$$

$$= \frac{1}{2} \frac{E_0^2}{\eta_0} \underset{\sim}{u}_z .$$

i) Analogous quantities between e-m fields and transmission lines :-

$$\cancel{E \to V} \qquad E \to V$$

$$\cancel{I \to H} \qquad H \to I$$

$$\cancel{Z \to \eta} \qquad \eta \to Z$$

so reflection, transmission coefficients for e-m waves ≡ voltage reflection and transmission coefficients in transmission lines.

$$\Rightarrow \quad \text{Reflection coefficient} = \frac{\eta_L - \eta_0}{\eta_L + \eta_0} = \frac{\sqrt{\frac{\mu_0 \mu_r}{\epsilon_0}} - \sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{\frac{\mu_0 \mu_r}{\epsilon_0}} + \sqrt{\frac{\mu_0}{\epsilon_0}}}$$

$$= \frac{\sqrt{\mu_r} - 1}{\sqrt{\mu_r} + 1}$$

$$\text{Transmission coefficient} = \frac{2\eta_L}{\eta_L + \eta_0} = \frac{2\sqrt{\frac{\mu_0 \mu_r}{\epsilon_0}}}{\sqrt{\frac{\mu_0 \mu_r}{\epsilon_0}} + \sqrt{\frac{\mu_0}{\epsilon_0}}} = \frac{2\sqrt{\mu_r}}{\sqrt{\mu_r} + 1}$$

$$\therefore \text{Reflected electric field} = \frac{\sqrt{\mu_r} - 1}{\sqrt{\mu_r} + 1} E_i ,$$

Transmitted electric field = $\dfrac{2\sqrt{\mu_r}}{\sqrt{\mu_r}+1} E_i$.

Transmission coefficient from region 2 to 3

$$\rho_{T_{2-3}} = \frac{2\sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{\frac{\mu_0}{\epsilon_0}} + \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0}}} = \frac{2}{1+\sqrt{\mu_r}}.$$

$\Rightarrow$ Electric field propagating into region 3 = $E_i \, \rho_{T_{1-2}} \, \rho_{T_{2-3}}$

$$= \frac{2\sqrt{\mu_r}}{\sqrt{\mu_r}+1} \cdot \frac{2}{\sqrt{\mu_r}+1} E_i = \frac{4\sqrt{\mu_r}}{(1+\sqrt{\mu_r})^2} E_i$$

ii) Using Poynting vector, power p.u. area propagating into region 3 is:-

$$P = \frac{1}{2}\frac{E^2}{\eta_0} = \frac{1}{2\eta_0}\left(\frac{4\sqrt{\mu_r}}{(1+\sqrt{\mu_r})^2} E_i\right)^2$$

$$= \frac{8\mu_r}{(1+\sqrt{\mu_r})^4}\frac{E_i^2}{\eta_0}$$

iii) Proportion of incident power p.u. area entering region 3 is

$$\frac{8\mu_r}{(1+\sqrt{\mu_r})^4}\frac{E_i^2}{\eta_0} \Big/ \frac{1}{2}\frac{E_i^2}{\eta_0}$$

$$= \frac{16\mu_r}{(1+\sqrt{\mu_r})^2} = \frac{16\times 100}{(1+10)^4} = 10.9\%$$

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June 1996.