Solutions IB Paper 6 1996

1) a) $$\bar{y}(s) = \frac{\frac{1}{s(s+2)}}{1+\frac{k_p}{s(s+2)}}\bar{d}(s) + \frac{\frac{k_p}{s(s+2)}}{1+\frac{k_p}{s(s+2)}}\bar{r}(s)$$

$$= \underbrace{\frac{1}{s^2+2s+k_p}}_{\text{TF } \bar{d}(s)\to\bar{y}(s)}\bar{d}(s) + \underbrace{\frac{k_p}{s^2+2s+k_p}}_{\text{TF } \bar{r}(s)\to\bar{y}(s)}\bar{r}(s)$$

b) $\omega_n^2 = k_p$, $2c\,\omega_n = 2$ $\Rightarrow$ $c = \frac{1}{\sqrt{k_p}}$

$\Rightarrow$ $k_p = 1$ for critical damping

$$\bar{y}(s) - \bar{r}(s) = \frac{1}{s^2+2s+k_p}\bar{d}(s) - \frac{s^2+2s}{s^2+2s+k_p}\bar{r}(s)$$

$$\lim_{t\to\infty} y(t) - r(t) = \lim_{s\to 0} s \times \text{(above)}$$

i) $\bar{r}(s) = \frac{1}{s}$, $\bar{d}(s) = 0$ $\Rightarrow s(\bar{y}(s) - \bar{r}(s)) = -\cancel{s}\,\frac{(s^2+2s)}{s^2+2s+k_p}\cdot\frac{1}{\cancel{s}}$

$\Rightarrow \lim_{t\to\infty} y(t) - r(t) = \underline{\underline{0}}$] $= 0$ at $s=0$

ii) $\bar{r}(s) = 0$, $\bar{d}(s) = \frac{1}{s+3}$ $\Rightarrow s(\bar{y}(s) - \bar{r}(s)) = \frac{s}{(s^2+2s+k_p)(s+3)}$

$= 0$ at $s = 0$

$\Rightarrow \lim_{t\to\infty} y(t) - r(t) = \underline{\underline{0}}$

iii) $\bar{r}(s) = 0$, $\bar{d}(s) = \frac{1}{s}$ $\Rightarrow s(\bar{y}(s) - \bar{r}(s)) = \cancel{s} \times \frac{1}{s^2+2s+k_p} \times \frac{1}{\cancel{s}}$

$= \frac{1}{k_p}$ at $s=0$

$\Rightarrow \lim_{t\to\infty} y(t) - r(t) = \underline{\underline{\frac{1}{k_p}}}$

iv) $\bar{r}(s) = \frac{1}{s^2}$, $\bar{d}(s) = 0$ $\Rightarrow s\left(\bar{y}(s) - \bar{r}(s)\right) = -\cancel{s} \times \frac{s^2+2s}{s^2+2s+K_p} \times \frac{1}{s^{\cancel{2}}}$

$$= - \frac{s+2}{s^2+2s+K_p}$$

$$= \frac{-2}{K_p} \text{ at } s=0$$

$\Rightarrow$ s.s. error $= -\frac{2}{K_p}$

c) Use PI control $K_p + \frac{K_I}{s} = \frac{K_p s + K_I}{s}$

In which case,

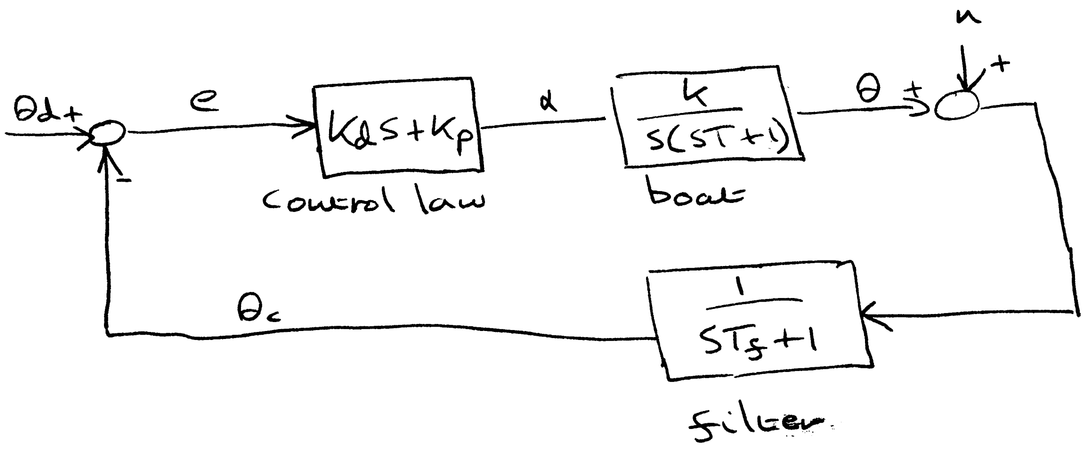
$$\text{TF } \bar{d}(s) \to \bar{y}(s) = \frac{s}{s^3 + 2s^2 + K_p s + K_I} = 0 \text{ at } s=0$$

Furthermore, if $\bar{d}(s) = 0$, $\bar{y}(s) - \bar{r}(s) = \frac{s^3+2s^2}{s^3+2s^2+K_p s+K_I}$

and $\lim_{s \to 0} s \times (\text{this}) \times \frac{1}{s^2} = 0$

So, control system can track a ramp (as in (b) part (iv)) without error.

2) a)



b) Characteristic equation is

$$1 + \frac{(K_d s + K_p)\,k}{s(sT+1)(sT_f+1)} = 0$$

ie $\quad s(sT+1)(sT_f+1) + (K_d s + K_p)k = 0$

or $\quad \underline{\underline{s^3 TT_f + (T+T_f)s^2 + (1+kK_d)s + kK_p = 0}}$

c.f. $\quad (s+0.4)(s+0.3-0.3j)(s+0.3+0.3j)$

$$= (s+0.4)(s^2+0.6s+0.18)$$

$$= s^3 + s^2 + 0.42s + 0.072$$

Comparing coefficients, we get

$$\frac{T+T_f}{TT_f} = 1\,, \quad \frac{1+kK_d}{TT_f} = 0.42\,, \quad \frac{kK_p}{TT_f} = 0.072$$

Solve, using $k = 0.2$, $T = 2$,
to get

$$\underline{\underline{T_f = 2\,, \quad K_d = 3.4\,, \quad K_p = 1.44}}$$

c) $$\bar{\alpha}(s) = \frac{-\dfrac{k_d s + k_p}{sT_f + 1}}{1 + \dfrac{k_d s + k_p}{sT_f+1} \cdot \dfrac{k}{s(sT+1)}} \bar{n}(s)$$

$$= \frac{-1}{\dfrac{sT_f+1}{k_d s + k_p} + \dfrac{k}{s(sT+1)}} \bar{n}(s)$$

We need the modulus of this at $s = 6j$,

ie $$1 \bigg/ \left| \frac{12j+1}{3.4 \times 6j + 1.44} + \frac{0.2}{6j(12j+1)} \right|$$

$$= 1 \bigg/ \left| 0.5861 \, e^{-j0.0131} \right| = 1.7063$$

So, if n has amplitude $5°$, then α has amplitude $1.7063 \times 5° \approx \underline{\underline{8.5°}}$

Now, $$\bar{\theta}(s) = \frac{k}{s(sT+1)} \bar{\alpha}(s)$$

at $s = 6j$, modulus $= \dfrac{0.2}{6\sqrt{1+12^2}}$

$\Rightarrow \theta$ has amplitude $\dfrac{0.2}{6\sqrt{1+12^2}} \times 8.5° = \underline{\underline{0.0235°}}$

3) a) require $\omega_c > 0.2$, in which case PM is largest if ω_c 0.8 (giving a PM of 50°)

$|G(j0.8)| \approx 12\,dB = 4 \Rightarrow$ require $K_p = \underline{\underline{\frac{1}{4}}}$

$\angle G(j\omega) = -180°$ at $\omega = 5$

$|G(j5)| \approx -11.5\,dB = 0.27$

$\Rightarrow |K_p G(j5)| \approx \frac{0.27}{4}$

So GM $\approx \frac{4}{0.27} \approx \underline{\underline{14.8}}$ $(= 11.5 + 12 = 23.5\,dB)$

b) require $\angle K(j\omega_c) \geq -5°$ at $\omega_c = 0.8$ as above

but $\angle K(j\omega_c) = \tan^{-1}\frac{0.8}{\alpha} - \tan^{-1} 40$

$\Rightarrow$ require $\tan^{-1}\frac{0.8}{\alpha} \geq \tan^{-1} 40 - 5°$

$\frac{0.8}{\alpha} \geq \tan(\tan^{-1} 40 - 5°)$

$\alpha \leq \frac{0.8}{\tan(\tan^{-1} 40 - 5°)}$

$= \underline{\underline{0.09}}$

(would then choose K s.t. $|K(j\omega_c)| = 4$)

4) a) As $G(s)$ itself is stable, and $K=1$, we have "The feedback system is stable (that is, $\frac{G(s)}{1+G(s)}$ has no RHP poles) if and only if

the complete Nyquist diagram of $G(s)$ (that is, the locus of $G(j\omega)$ for ω from $-\infty$ to ∞) does not encircle the -1 point in the complex plane."

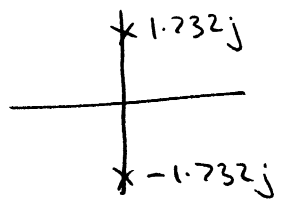
b) i) $G(j\omega) = -0.25$ at $\omega = 1.732$, and 2 at $\omega = 0$

$\Rightarrow$ need $-\frac{1}{K} < -0.25$ or $-\frac{1}{K} > 2$

ie $\underline{\underline{-0.5 < K < 4}}$ for the given TF to be stable.

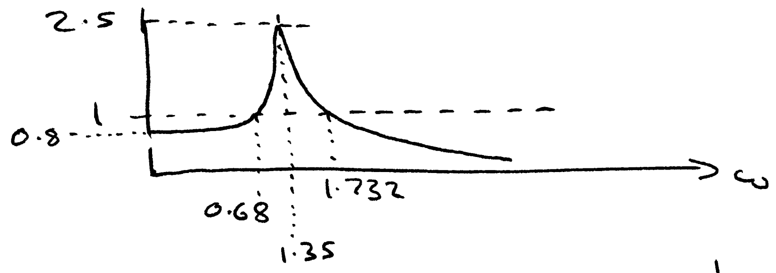
when $K=4$, $1 + KG(\pm j1.732) = 0$

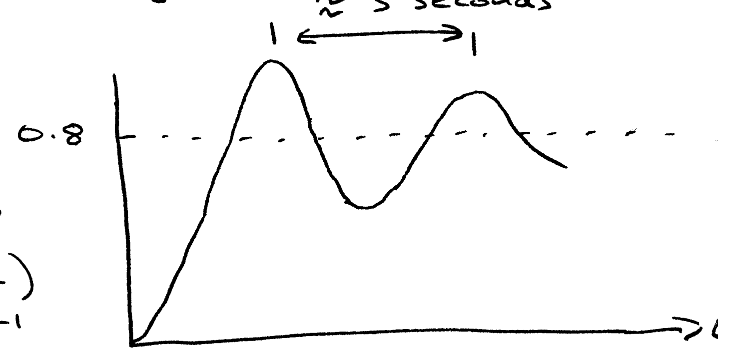
$\Rightarrow \frac{KG(s)}{1+KG(s)}$ has poles at $s = \pm j1.732$



ii) For GM $= 2$, need $K = 2$.

Then have





Expect response to be fairly oscillatory (due to large peak in freq response) at a freq of ≈ 1.35 rad s^{-1} $= 0.2$ Hz

5) a) Modulation is necessary when the frequency content of the signal doesn't match the pass band of the path. Modulation is then used to match the characteristics of the signal to those of the path.

Common Techniques

AM - Simple, bandwidth of modulated signal ≈ 2 bandwidth of original signal. Easy and cheap to demodulate using envelope detection. BUT low efficiency, no rejection of noise or interference.

Variations of AM/
suppressed carrier - greater efficiency
SSB - halves bandwidth
Yet both need a more complex receiver, ie demodulation more difficult.

FM - greater immunity to noise and interference at expense of greater bandwidth. Demodulation more difficult again.

Digitization Allows S/N to be made arbitrarily good (at the expense of bandwidth, of course). Allo[ws] Compression to be used.

b) i) Channel bandwidth ≈ 300Hz to 3.4kHz - adequate for carrying Voice without modulation.

ii) Signal bandwidth ≈10kHz. AM with transmitted bandwidth 2×10 kHz = 20kHz is suitable (room for many channels on MW band)

iii) Signal bandwidth ≈ 25kHz ⇒ bandwidth using ASK ≈ 50kH[z]. This is very small compared with available bandwidth at UH[F] frequencies - so could use FSK for greater noise immunity.

6) a) i) 1) bit rate = $12 \times 50k$ = $\underline{600\ k\ \text{bits}^{-1}}$

2) bit rate = $16 \times 37.5k$ = $\underline{600\ k\ \text{bits}^{-1}}$

ii) Time required to generate each bit $= \frac{1}{600}$ ms in each setting. This suggests successive approximation is being used - it taking $\frac{1}{600}$ ms for the D/A converter to settle and to then perform a comparison.

b) i) if p-p is V then signal rms $= \frac{V}{2\sqrt{2}}$

n bits, resolution is $\delta v = \frac{V}{2^n}$

noise rms $= \frac{\delta v}{\sqrt{12}}$

$$\Rightarrow S/N = \frac{V/2\sqrt{2}}{\frac{V}{2^n}/\sqrt{12}} = \sqrt{\tfrac{3}{2}}\ 2^n$$

in dB $= 20 \log_{10}(2^n\sqrt{3/2})$

So, $n = 12 \Rightarrow S/N = \underline{74\text{dB}}$. $n = 16 \Rightarrow S/N = \underline{98\text{dB}}$

ii) require $1.25 f_c < \frac{\text{Sampling rate}}{2}$

$\Rightarrow$ allowable bandwidth $= 0.4 \times$ sampling rate

$n = 12$, $\underline{20\ \text{kHz}}$ $n = 16$ $\underline{15\text{kHz}}$

(Notice trade off between S/N and signal bandwidth)

Solutions IB Paper 6 1996

1) a) $$\bar{y}(s) = \frac{\frac{1}{s(s+2)}}{1+\frac{k_p}{s(s+2)}}\bar{d}(s) + \frac{\frac{k_p}{s(s+2)}}{1+\frac{k_p}{s(s+2)}}\bar{r}(s)$$

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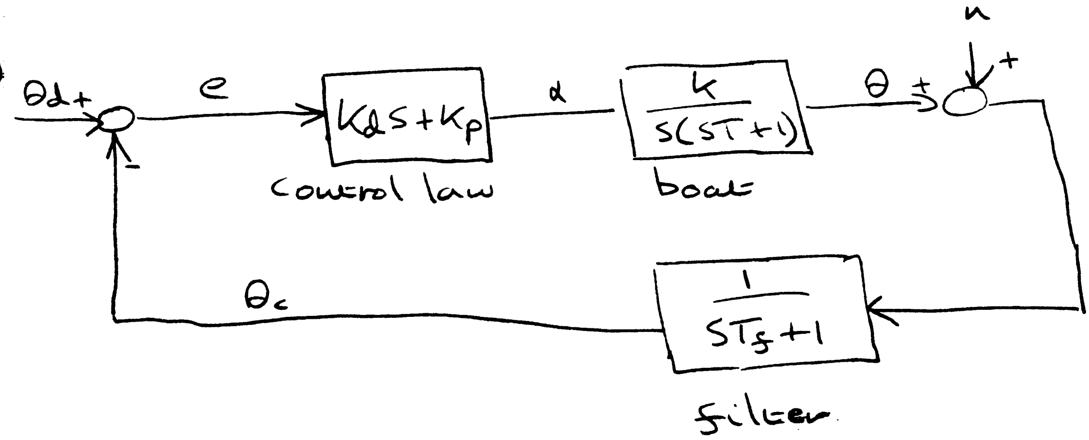

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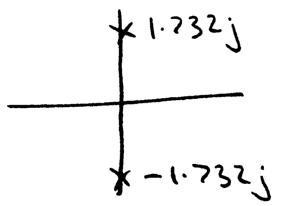

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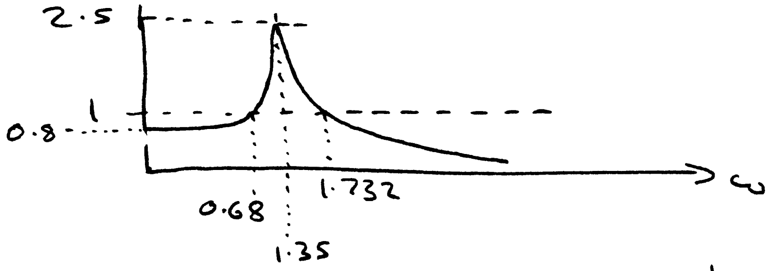

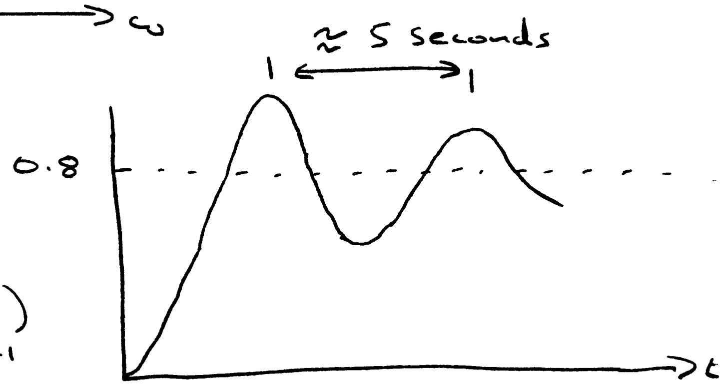

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in dB $= 20 \log_{10} (2^n \sqrt{3/2})$

So, n = 12 ⇒ S/N = 74 dB. n = 16 ⇒ S/N = 98 dB

ii) require $1.25 f_c < \frac{\text{Sampling rate}}{2}$

⇒ allowable bandwidth = 0.4 x sampling rate

n = 12, 20 kHz n = 16 15 kHz

(Notice trade off between S/N and signal bandwidth)