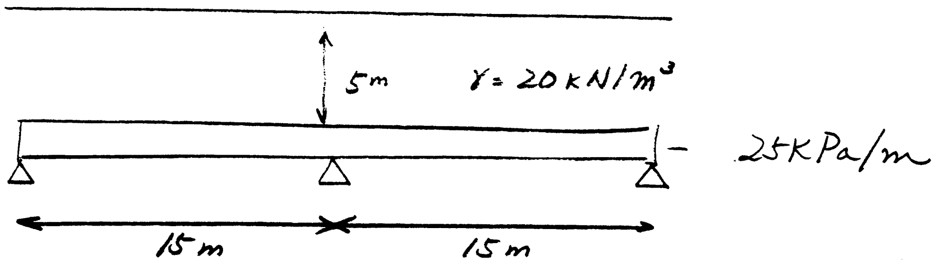
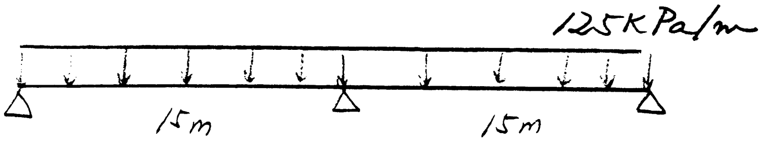
1B 1996

SELECTED TOPICS

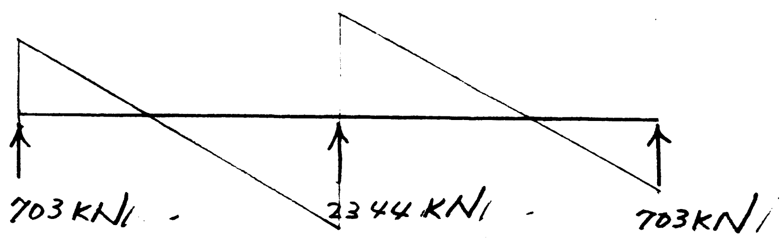
CRIBS

1.

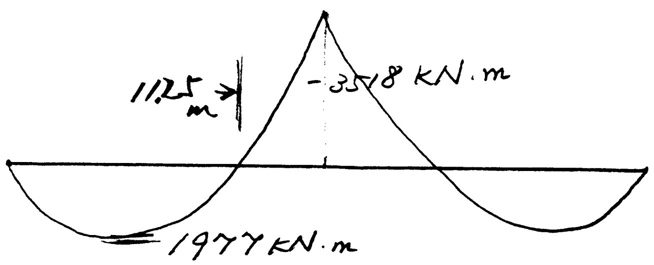




Shear Force Diagram per unit length



(a) Moment Diagram



(b) $M = 0.15 f_{cu} b d^2$

$$1977 \times 10^6 \leq 0.15 \times 40 \times 1000 \times d^2$$

$$d \geq 574 \text{ mm}$$

This is a deep section allow cover 50mm + 25 mm for bars ∴ Overall depth 650mm minimum

Better make overall depth 675 mm and d = 600 mm //

(C) To resist sagging bending

Guess $x = 0.5$

$M = 0.87\, f_y\, A_s\, d(1 - x/2)$

$1.977 \times 10^6 = 0.87 \times 460 \times 600 \times (1 - \frac{0.5}{2})\, A_s$

$A_s = 10978\ mm^2$

$\therefore$ $6 \times No\ 50\phi = 11778\ mm^2$

OR

$9 \times No\ 40\phi = 11304\ mm^2$ ← in Tension Face

Assuming $A_s = 11304\ mm^2$

$x = 2.175\left(\frac{460}{40}\right)\left(\frac{11304}{600 \times 1000}\right)$

$= 0.471$

0.7645

check $1977 \times 10^6 = 0.87 \times 460 \times 600 \times (1 - \frac{0.471}{2}) \cdot A_s$

$A_s = 10770\ mm^2$ OK

(d) To resist hogging $M = 3515\ kNm$

$d = 600\ mm$ $d' = 75$

$M = 0.15 f_{cu} b d^2 + 0.75 f_y A_s'(d - d')$

$3515 \times 10^6 = 0.15 \times 40 \times 1000 \times 600^2 + 0.75 \times 460 \times A_s' \times (600 - 75)$

$A_s' = 7481\ mm^2$

$\therefore$ $6 \times No\ 40\phi = 7536\ mm^2$ in Comp. face

$0.87 f_y A_s = 0.75 f_y A_s' + 0.2 f_{cu} b d$

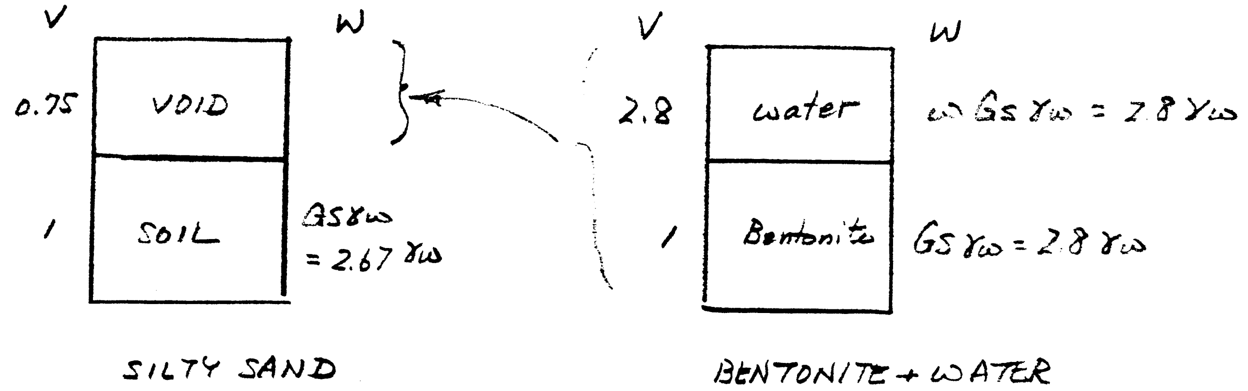
$0.87 \times 460 \times A_s = 0.75 \times 460 \times 7536 + 0.2 \times 40 \times 1000 \times 600$

$A_s = 18491\ mm^2$

$\therefore$ $15 \times No\ 40\phi = 18840\ mm^2$ in Tension face

2

(a) Plastic Limit = LL − PI = 550 − 450 = 100



weight of dry silty sand $= 2.67\gamma_w$

weight of dry Bentonite $= 0.75 \times \frac{1}{1+2.8} \times 2.8\gamma_w$

$= 0.55\gamma_w$

% of dry silty sand $= \frac{0.55\gamma_w}{2.67\gamma_w} = 0.207$

$= 20.7\%$ //

(b) weight of the soil (W_s)

$= 2.67\gamma_w + 0.55\gamma_w = 3.22\gamma_w$

weight of the water (W_w)

$= 0.55\gamma_w$

Water content $= \frac{W_w}{W_s} = \frac{0.55\gamma_w}{3.22\gamma_w} = 0.1708$

$= 17.1\%$ //

Void ratio $= V_w/V_s$

$= \frac{0.75 \times \frac{2.8}{1+2.8}}{1 + 0.75 \times \frac{1}{1+2.8}} = \frac{0.55}{1.197} = 0.46$ //

(c)

Assumptions :

- the linear is saturated
- contaminats move with the water flow (no diffusion)
- voids are all interconnected and act as flow paths
- flow is one-dimensional
- $k = 1.0 \times 10^{-10}$ m/sec

$$V_w = \text{Velocity of pore fluid} = \frac{\text{dary velocity}}{\text{porosity}}$$

$$= \frac{ki}{n}$$

$$250 \times 365 \times 24 \times 60 \times 60 = \frac{L}{V_w} = \frac{Ln}{ki}$$

$$= \frac{L \times \frac{0.46}{1+0.46}}{1 \times 10^{-10} \times \frac{2}{L}}$$

$L^2 = 5.00$ $\quad$ $\underline{L = 2.24 \text{ (m)}}$ //

(d) an essay.

3

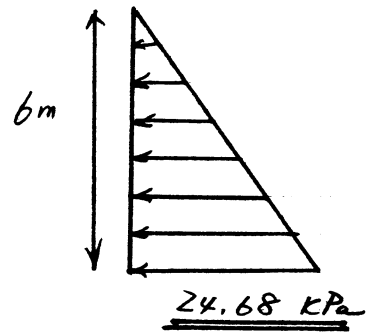
(a) $I_D = 60\% = \dfrac{e_{max} - e}{e_{max} - e_{min}} = \dfrac{0.88 - e}{0.88 - 0.49}$

$e = 0.586$

From the table $\phi_{max} \approx 36^\circ$

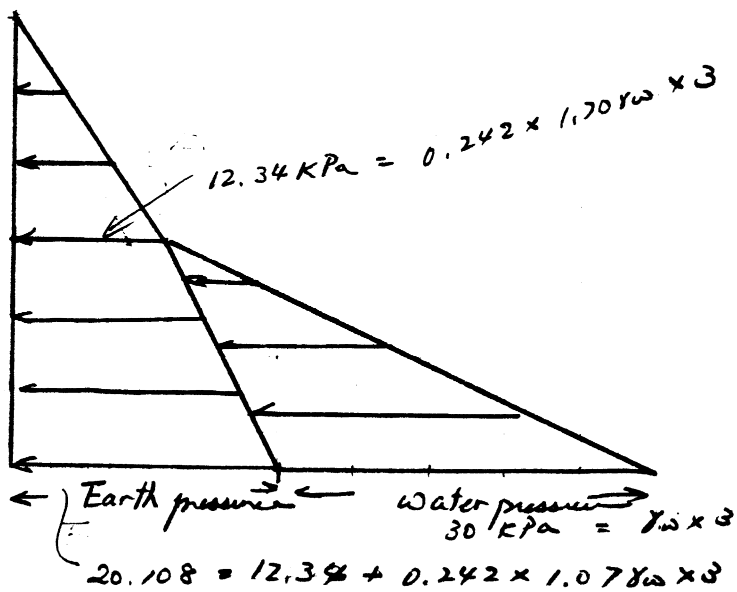
$K_a = \dfrac{1 - \sin 36^\circ}{1 + \sin 36^\circ} = 0.242$

$\gamma_{dry} = \dfrac{2.70}{1 + 0.586}\gamma_w = 1.70\,\gamma_w$



(b) $\gamma_{sat} = \dfrac{2.70 + 0.586}{1 + 0.586} = 2.07\,\gamma_w$

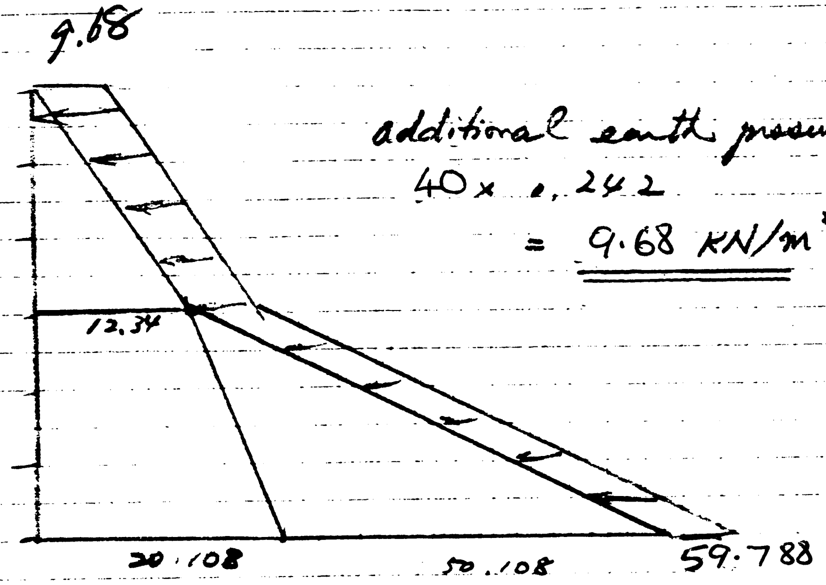
$\gamma'_{effective} = 2.70\,\gamma_w - \gamma_w = 1.07\,\gamma_w$



Total pressure at the bottom

$20.108 + 30 = \underline{50.108\ kPa}$

(C)

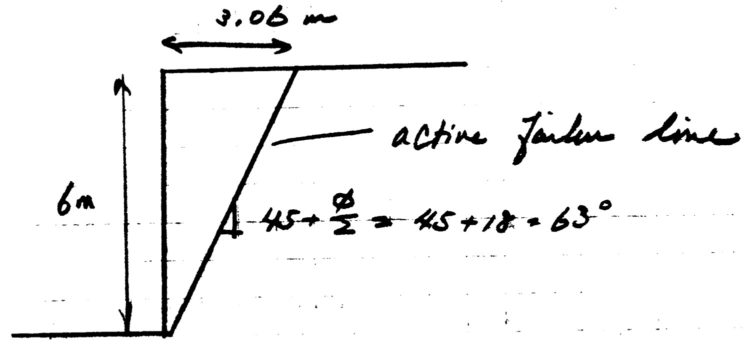


additional earth pressure by line load

40×0.242

$= \underline{9.68\ kN/m^2}$

Very rough estimate would be that the track to be constructed away from the active pressure failure line.



allows some factor of safety (say 1.3)

4m away from the wall

(d)

- Overturning
- sliding
- bearing failure at the foot
- excessive settlement by the clay layer
- stability failure through the weak clay layer
- failure of the wall itself.

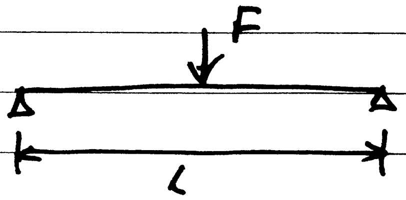
Q4 (a) The rotational speed of the eccentric mass is defined by the natural frequency of vibration for a 15m unsupported span.

This frequency may be estimated by assuming the pipe span to be either a clamped or unclamped beam, with a mid-span load representing the force applied by the vibrating pig.

The beam stiffness is given by:

$$k = \frac{F}{\delta l} = \frac{\text{applied force}}{\text{resulting deflection}}$$

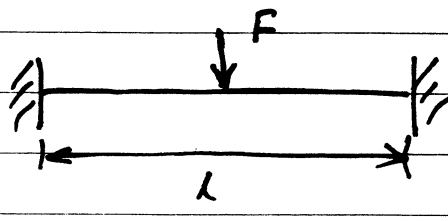
Unclamped beam:



use $l \approx 0.8L$ (L = span of pipe) to allow for additional freedom of movement at ends of span.

$$\text{then} \quad k = \frac{48EI}{(0.8L)^3} \approx \frac{94EI}{L^3} \qquad \text{(p6 - structures d.b)}$$

Clamped beam:

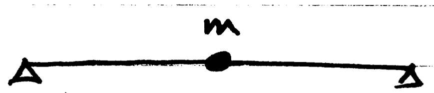


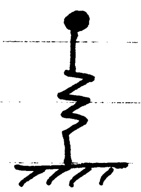
use $l \approx 1.25L$ to allow for additional constraints to movement at ends of span

$$\text{then} \quad k = \frac{192EI}{(1.25L)^3} \approx \frac{98EI}{L^3} \qquad \text{(p6 - structures d.b)}$$

Q4 (a) cont...

To estimate the natural frequency consider an equivalent system of a mass on a spring where the mass is half the span mass (approximates the moving mass).





$$k \approx \frac{100EI}{L^3}$$

then natural frequency $\omega_n = \sqrt{\frac{k}{m}}$

Half span mass $m = \frac{1}{2}\rho A l = \frac{1}{2}$.density × area × length

$$= \frac{1}{2} \times 7840 \times \pi \times 0.2 \times 0.01 \times 15 = 370\,kg$$

since $A \approx \pi R t = \pi \times$ radius × thickness for a thin wall tube.

Moment of Inertia $I = \pi R^3 t = \pi \times (\text{radius})^3 \times$ thickness for an annulus (p 10 mechanics db)

$$\approx \frac{\pi R^4}{4} - \frac{\pi(R-t)^4}{4}$$

for difference between 2 circles (p 5 structures db)

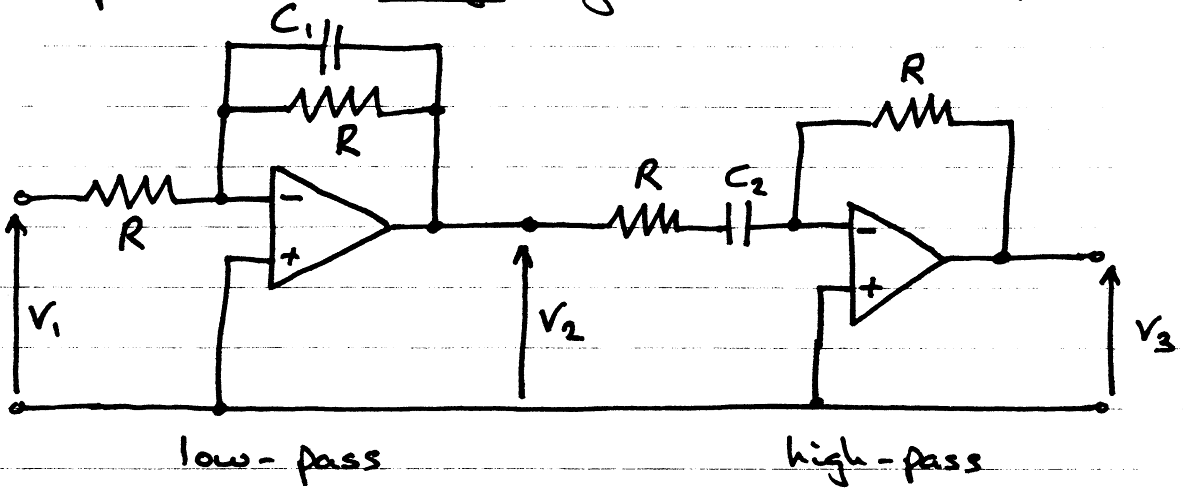
$$\therefore \quad I = \pi \times \left(\frac{0.2}{2}\right)^3 \times 0.01 = 3.14 \times 10^{-5}\ m^4$$

Hence $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \times 210 \times 10^9 \times 3.14 \times 10^{-5}}{15^3 \times 370}} = 23.0\ rads^{-1}$

or $f_n = \frac{23.0}{2\pi} \approx 4\ Hz$

$\therefore$ <u>rotational speed $\approx$ 220 r.p.m.</u>

Q4 (b) The band-pass filter may comprise two components: a low-pass and a high-pass filter. The input to the filter is a voltage signal, as is the output.



$$\frac{V_2}{V_1} = \frac{-1}{1 + j\omega C_1 R} \qquad \frac{V_3}{V_2} = \frac{-1}{1 + 1/j\omega C_2 R}$$

Frequency of interest = 23 rads^{-1} (4 Hz)

Set low-pass cut-off at 100 rads^{-1} (32 Hz)

Set high-pass cut-off at 5 rads^{-1} (0.8 Hz)

$$\therefore \quad \omega_l = 100 = \frac{1}{C_1 R}$$

$$\omega_h = 5 = \frac{1}{C_2 R}$$

Let $R = 10\,k\Omega$, $C_1 = 0.1\,\mu F$, $C_2 = 2.0\,\mu F$ (say)
R should be less than $1\,M\Omega$.

Check mid-band gain at 23 rads^{-1} → 0.95
Slight adjustment of some Rs will ensure unity gain at middle of pass band.

Q4 (c) Accelerometer noise typically arises from vibration of the pig moving along pipe (high frequency noise). Movement of spans due to ocean currents may cause low frequency noise. There may also be electrical and vibrational noise from with in the pig.

(d) Sampling rate must be at least twice the largest frequency of interest.

Hence sample at rate > 8 Hz for accelerometer

Say at 10 Hz.

Sample at say 1 Hz for the odometer.

Use 8-bit resolution for all data (1 byte per data).

∴ Total data = data rate × time

$$= \underbrace{(2 \times 10 + 1)}_{\text{data rate}} \times \underbrace{60}_{\text{sec}} \times \underbrace{60}_{\text{hours}} \times \underbrace{10}_{\text{total hours}} = 756 \text{ kbyte}$$

∴ <u>Allow for 1 Mbytes of data.</u>

Q5 Chief Designer's Report:

General: The answer should look like a Report and be generally persuasive in its tone. Try to sell the project to the Board.

Market: Approximate size, value, potential customers, competitor products, effect of legislation, value of service to the customers, fit with existing products;

Design: Review of current design, development required, resource/time required, mode of operation including sketches and typical frequencies of interest, data analysis requirement

Production: Volumes to be made, integration with existing components, definition of inspection service including training and validation requirements, type of production systems required;

Management: Management resource, project resource, risk assessment, concurrent engineering techniques.

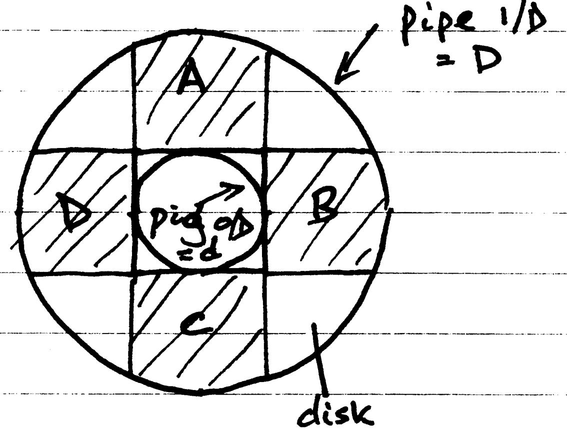
New Product Manager's Report:

Production: Safety, use of existing production facilities, new equipment requirements, volume requirements personnel training, raw materials, production type, production systems.

Q6 (a) The primary function of the support disk of a vibrating pig is to transmit the excitation force from the pig to the pipe. The disks also hold the pig in the centre of the pipe away from the walls and may provide some propulsion force from the pipe flow.

(b) The disks should be stiff enough to transmit the required force and have natural frequencies of vibration well above the operating range of the pig. The disks must be a good fit in the pipe and be of a size to allow the pig to move around the sharpest corners to be expected in a pipe.

(c) Consider the support disk to be made up of four components as shown below:



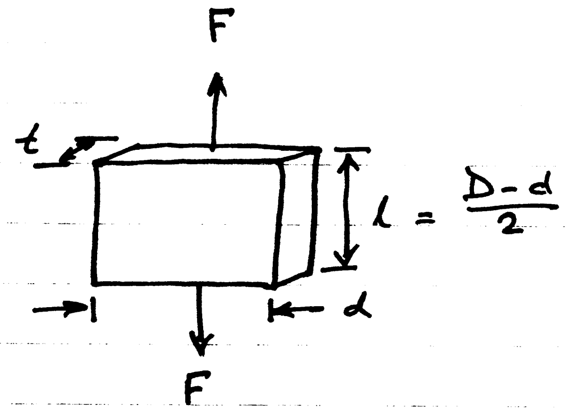
For vertical movement of the pig elements A and C act in tension/compression and elements B and D in shear.

In order to calculate the natural frequencies for the pig support disks we must first calculate a stiffness, k, for the system, where stiffness is given by:

$$k = \frac{F}{\delta l} = \frac{\text{force}}{\text{deflection}}$$

Q6 (c) cont...

For element A :



stress $\sigma = \frac{F}{A} = \frac{\text{force}}{\text{area}}$

strain $\epsilon = \frac{\delta l}{l} = \frac{\text{displacement}}{\text{length}}$

$$k = \frac{F}{\delta l} = \frac{\sigma A}{\epsilon l} = \frac{EA}{l}$$ where $E = \frac{\sigma}{\epsilon}$, young's mod

Similarly for shear in B and D, $k = \frac{GA}{l}$

Hence for all elements $k = \frac{2EA}{l} + \frac{2GA}{l}$

now $A = dt$, $l = \frac{D-d}{2}$ and $E = 3G$

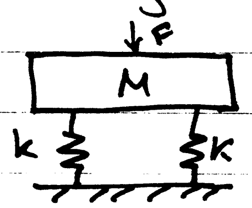
$$\therefore \quad k = \frac{16\, G.d.t}{D-d}$$

$$= \frac{16 \times 200 \times 10^6 \times 0.1 \times 0.02}{0.2 - 0.1} = 64 \times 10^6 \text{ N/m}$$

$\therefore$ <u>$k = 64 \times 10^6$ N/m</u> for one disk

Consider the bounce mode of vibration:

This may be modelled as a mass on two springs



The frequency of oscillation is given by

$$\omega_n = \sqrt{\frac{2k}{M}} = \sqrt{\frac{2 \times 64 \times 10^6}{15}}$$

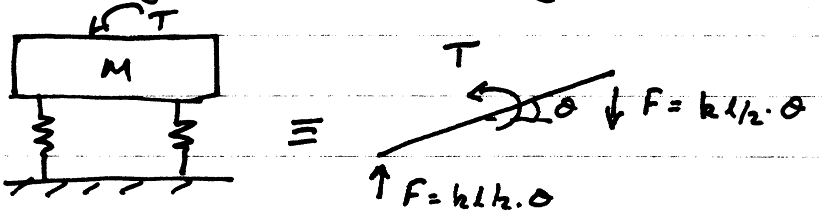
≈ 2900 rad s^{-1}

<u>Hence $f_n \approx 465$ Hz</u>

Q6 (c) cont...

Consider the pitch mode of vibration:

This may be modelled again as a mass on two springs



l = distance between disks.

torque $T = 2 \cdot (k l/2 \cdot \theta) \cdot \frac{l}{2} = \frac{k l^2}{2} \theta$

$\therefore$ torsional stiffness $K_T = \frac{l^2}{2} \cdot k = \frac{.3^2}{2} \cdot 64 \times 10^6$
$= 2.9 \times 10^6$ Nm/rad

The frequency of oscillation is given by:

$$\omega_n = \sqrt{\frac{k_T}{J}}$$

where J, the moment of inertia, is given as:

$$J = m \cdot \left(\frac{1}{4}\left(\frac{d}{2}\right)^2 + \frac{1}{12} \cdot L^2\right) \quad \text{(p 12 mechanics db)}$$

$$= 15 \cdot \left(\frac{1}{4}\left(\frac{0.1}{2}\right)^2 + \frac{1}{12} \cdot (0.4)^2\right) \approx 15 \cdot \frac{1}{12} \cdot (0.4)^2$$

$$\approx 0.21 \text{ kg m}^2$$

Hence $\omega_n = \sqrt{\frac{k_T}{J}} = \sqrt{\frac{2.9 \times 10^6}{0.21}} \approx 3700 \text{ rad s}^{-1}$

$\therefore$ $f_n \approx 590$ Hz

Q6 (d) Substitution in the equation yields $k = 84 \times 10^6$ N/m

This in turn gives $f_n \approx 530$ Hz for bounce

$f_n \approx 680$ Hz for pitch

(e) Both estimates of k rely upon assumptions of thin disks which remain flat and in good contact with the pipe wall, and good material data.

The simpler approximation is adequate since it is important only to ascertain if the natural frequencie of vibration are well removed from the operating frequency of the pig.

You would expect the estimate to be lower than the result from the equation since not all parts of the disk were included in the simpler model.

7(a) $M = 0.85 \quad T_a = 227\,k \quad P_a = 28.7\,kPa.$

$$T_{02} = T_a\left[1 + \tfrac{\gamma-1}{2}M^2\right] = \underline{259.8\,k}$$

$$P_{02} = P_a\left[1 + \tfrac{\gamma-1}{2}M^2\right]^{\gamma/\gamma-1} = \underline{46.5\,kPa}$$

(b)

$$\eta_c = \frac{T_{03s} - T_{024}}{T_{03} - T_{024}} \qquad \frac{T_{03s}}{T_{024}} = \left(\frac{P_{03}}{P_{024}}\right)^{\gamma-1/\gamma}$$

$$\therefore T_{03} = T_{024}\left[\frac{1}{\eta_c}\left[\left(\frac{P_{03}}{P_{024}}\right)^{\gamma-1/\gamma} - 1\right] + 1\right] = 301 \times 2.676 = \underline{805.5}$$

$$P_{03} = P_{02} \times 25 = \underline{1840\,kPa}$$

(c) Per kg mass flow, $W_T = W_c$

$$c_p(T_{03} - T_{024}) = c_p(T_{04} - T_{045})$$

$$\Rightarrow T_{045} = 1450 - (805.5 - 301) = \underline{945.5\,k}$$

$$\eta_T'' \overset{0.9}{=} \frac{T_{04} - T_{045}}{T_{04} - T_{045s}} \qquad \left(\frac{P_{04}}{P_{045}}\right)^{\gamma-1/\gamma} = \left(\frac{T_{04}}{T_{045s}}\right)$$

$$\therefore T_{045s} = 889.4\,k.$$

$$P_{045} = \underset{1840}{P_{04}}\left[\frac{889.4}{1450}\right]^{\gamma/\gamma-1} = \underline{332.6\,kPa}$$

Turbine PR less than compressor PR due to divergence of const pressure lines on T-s diag, reasonable efficiency of each component & equal works (ΔT_0's). This is equivalent to saying that although ΔT_0's are same, the T_0's are not & PR ≠ TR

7 (d) $T_{05} = T_{045} - 361\,K = 584.5\,K$.

$\eta_T = 0.92 \Rightarrow T_{05s} = 553.1$

$\therefore P_{05} = P_{045}\left(\frac{T_{05s}}{T_{045}}\right)^{\gamma/\gamma-1} = \underline{\underline{50.92\,kPa}}$

Nozzle is isentropic $\therefore c_p(T_{09} - T_9) = \frac{1}{2}V_9^2$; $T_{05} = T_{09}$

$$\Rightarrow V_9 = \left[2 c_p T_{05}\left(1 - \left(\frac{p_a}{P_{05}}\right)^{\frac{\gamma-1}{\gamma}}\right)\right]^{1/2}$$

$V_9 = \underline{\underline{422.4}}$ m/s $\quad (= V_{19})$

(e) Isentropic compression in bypass stream

$\therefore c_p(T_{013} - T_{02}) = \frac{1}{2}V_{19}^2 - \frac{1}{2}V_1^2 = W_f$

$V_1^2 = 0.85^2(1.4 \times 287.5 \times 227) = 256.93^2$

$\Rightarrow W_f = 112409 = c_p \times \frac{111.3}{2}$

Energy balance: $\dot{m}_b W_{f_b} + \dot{m}_c W_{f_{core}} = \dot{m}_c W_{LPT}$

$$B \underset{}{W_{f_b}} + W_{f_{core}} = W_{LPT}$$

$\left(W_{f_b} = \frac{111.3}{2}c_p,\quad W_{f_{core}} = c_p(301 - 259.8),\quad W_{LPT} = 361\,c_p\right)$

$$B = \frac{361 - 41.2}{\frac{111.3}{2}} = \underline{\underline{5.75}}$$

8 (a) Gross Thrust $= (\dot{m}_a + \dot{m}_f^{\approx 0}) V_j \equiv$ Thrust on test-bed

Net Thrust $=$ Gross Thrust $- \dot{m}_a V_1 =$ Thrust in flight on aircraft

(b) Information in a supersonic flow cannot travel upstream. Nozzle is choked ∴ condition at nozzle only depend on upstream values. For a given non-di operating point, there might be P_{02}, T_{02}, $\dot{m}$ air or $\dot{m}_f$ ie in non-dimensional terms

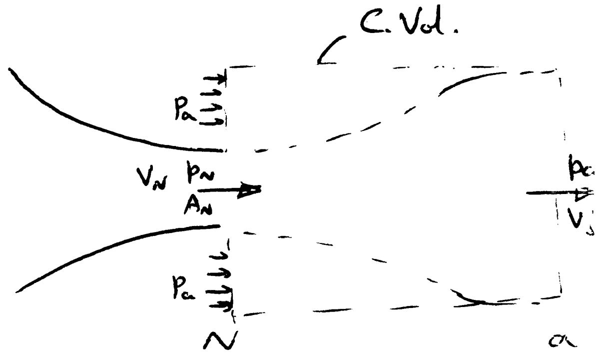
$$\frac{\dot{m}_{air}\sqrt{c_p T_{02}}}{D^2 P_{02}} \quad \text{or} \quad \frac{\dot{m}_f \, LCV}{\sqrt{c_p T_{02}}\, D^2 p_{02}}$$ could be used.

Other alternatives are $\frac{T_{04}}{T_{02}}$ & $\frac{ND}{\sqrt{\gamma R T_{02}}}$ but these do not satisfy the requirements of question

(c) Consider mm equation from throat to far downstream

$$p_N A_N + \dot{m} V_N = p_a A_N + \dot{m} V_j$$

$$X_g = \dot{m} V_j = \dot{m} V_n + A_N (P_N - P_a)$$



(d) $\frac{\dot{m}\sqrt{T_{02}}}{p_{02}} =$ const ∴ $\dot{m}_{cruise} = 50.0 \left[\frac{\sqrt{255}}{45} \frac{101}{\sqrt{288}}\right]^{-1} = 236.7$ kg/s

For same op. point $\frac{T_{04}}{T_{02}} =$ const ∴ $T_{ET} = 1610 \times \frac{255}{288}$

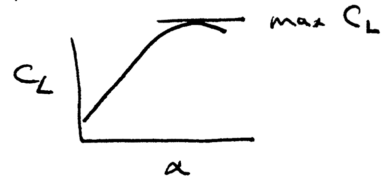
$= 1425$ K

(e) $\frac{X_g + p_a A_N}{P_{02} A_N} =$ const $= \frac{50 + 101.0.5}{101.0.5} = 1.99$

$\Rightarrow X_{g_{flight}} = 44.78 - p_a A_N = 44.78 - 27 \times 0.5 = 31.28$ k

9 (a) Lift coefficient $C_L = \frac{Lift}{\frac{1}{2}\rho V^2 A}$

At take off speeds,



Take off speed typ ~ 90 m/s (limited by runway length & overheating of tyres)

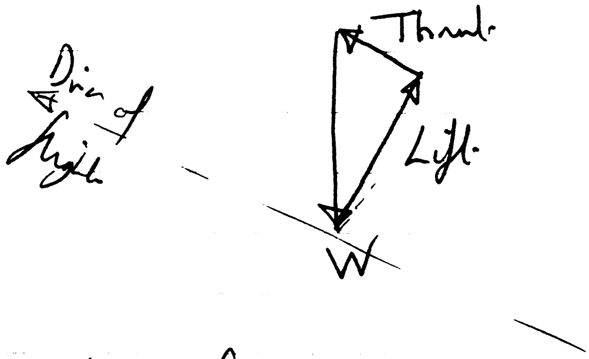
C_L limited by safety margin away from stall ($C_{L max}$)

Lift = weight

Hence A is determined.

(b) Flight speed V much greater (~260 m/s) than at takeoff. To support aircraft during cruise, lift remain similar to that at takeoff $\therefore \rho C_L$ must be reduced. $C_{L opt} \approx 0.5$ but the reduction is not sufficient to achieve opt L/D [max weight for given thrust] hence ρ must be reduced by flying at altitude.

(c) Thrust balances drag in normal cruise conditions. Drag specified because aircraft cruises near max VL/D. L ~~is~~ fixed by weight, M (or V) approx constant ∴ D is specified for cruise. However, 'top of climb' is more important because some thrust is used to 'lift' the aircraft



Note that for twin-engined aircraft, the loss of an engine at takeoff is most important and engines are sized for this condition on this type of aircraft.

(d) Proof (not required)

$W = \text{Lift} = L$

$D = \text{Drag} = \text{Thrust} = \dot{m}_{fuel}/sfc$

$$\frac{dW}{dt} = -\dot{m}_{fuel}\cdot g = -D.\,sfc\cdot g$$

$D = W/(L/D)$ $\Rightarrow$ $$\frac{dW}{dt} = -g\, sfc\, \frac{W}{(L/D)}$$

$ds = V dt$ $$ds = \frac{-1}{g.sfc}\,\frac{VL}{D}\,\frac{dW}{W}$$

Neglecting fuel + distance during take-off & landing gives

$$s = -\frac{VL}{D}\,\frac{1}{g.sfc}\,\ln\frac{W_{end}}{W_{start}}$$

if $\frac{VL}{D}\cdot\frac{1}{sfc}$ is const.

Max range occurs when $\frac{VL}{D}\cdot\frac{1}{sfc}$ maximised

[At high altitude T = const $\therefore$ V = const $\Leftrightarrow$ M = const]

In practice, aircraft fly at/near opt $\frac{ML}{D}$

Maintaining opt. value would require a slow climb as burnt fuel reduces weight. In practice, 4000' "jumps" are required

(e) $$\eta_p = \frac{\text{Power to Aircraft}}{\Delta KE \text{ to jet}}$$

$$= \frac{\text{flight speed} \times \text{net thrust}}{\frac{1}{2}\left((\dot{m}_a + \dot{m}_f)^2 V_j - \dot{m}_a V^2 \right)}$$

$$= \frac{V \left[(\dot{m}_a + \dot{m}_f) V_j - \dot{m}_a V \right]}{\frac{1}{2}\left[(\dot{m}_a + \dot{m}_f)^2 V_j - \dot{m}_a V \right]}$$

Neglecting $\dot{m}_f$

$$\eta_p = \frac{\dot{m}_a V (V_j - V)}{\frac{1}{2}\dot{m}_a (V_j^2 - V^2)} = \frac{2V}{V + V_j}$$

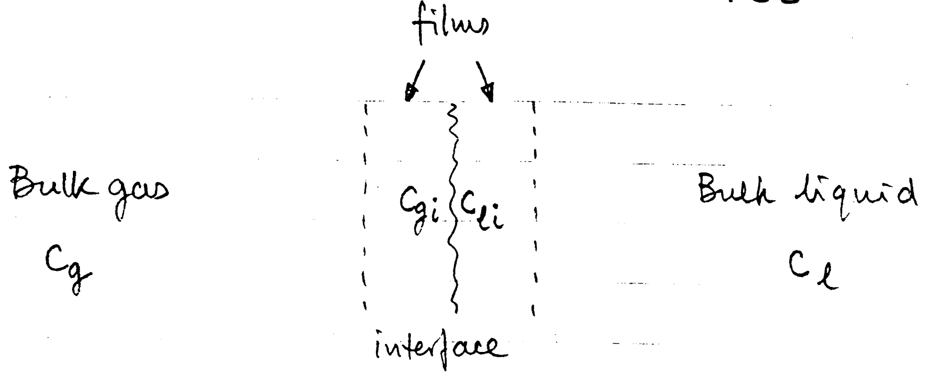
If $V_j = V$, $\eta_p = 1$ but $X_N = 0$

(f) Overall efficiency $\eta_o = \eta_{cy} \cdot \eta_p$

$\therefore$ High η_{cy} & η_p required

High η_{cy} $\Rightarrow$ high core temps & pressure ratios but these conditions would create a very high velocity core jet & slow η_p. Instead, some of enthalpy drop available for exit of core is needed to power LP turbine and drive fan. Hence core jet velocity is reduced & fan produces large mass of slower moving air ($V_{j\,fan} > V$) & high η_p also achieved doubly.

11/



Assume

- equilibrium at the interface $c_{gi} = H c_{li}$
- molar flux across films given by

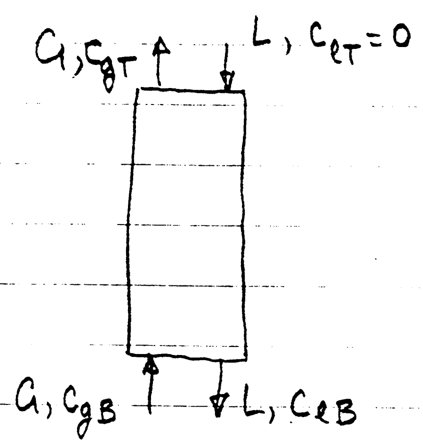
$$N = k_g (c_g - c_{gi}) = k_l (c_{li} - c_l)$$

$$= K_g (c_g - H c_l) \quad \text{definition}$$

$$\left.\begin{aligned} \frac{N}{k_g} &= c_g - c_{gi} \\ \frac{HN}{k_l} &= H c_{li} - H c_l \end{aligned}\right\} \quad N\left(\frac{1}{k_g} + \frac{H}{k_l}\right) = c_g - H c_l$$

$$\frac{1}{K_g} = \frac{1}{k_g} + \frac{H}{k_l}$$

(b)



Dilute — assume G and L are constant along the column

Overall material balance

$$G(c_{gB} - c_{gT}) = L c_{lB}$$

$$L \to \infty \Rightarrow c_{lB} \to 0$$

11/ If $c_l \simeq 0$ throughout the column, the driving force for mass transfer $(c_g - c_l)$ will be greatest $\Rightarrow$ c_{gT} will be least

i.e. $f = c_{gT}/c_{gB}$ will be minimum.

With $c_{lB} = 0$,

Rate of mass transfer $= G(c_{gB} - c_{gT})$ material balance

$$= k_g \pi d h \frac{c_{gB} - c_{gT}}{\ln(c_{gB}/c_{gT})}$$ rate formula

$$\rightarrow \ln(c_{gB}/c_{gT}) = \frac{k_g \pi d h}{G}$$

i.e. $$f_{min} = \exp\left[-\frac{k_g \pi d h}{G}\right]$$

(c) $$Re = \frac{1.19 \times 1.5 \times 0.03}{1.81 \times 10^{-5}} = 2959$$

$$Re^{0.83} = 760$$

$$Sc = \frac{1.81 \times 10^{-5}}{1.19 \times 1.46 \times 10^{-5}} = 1.042$$

$$Sc^{0.44} = 1.02$$

$$\rightarrow Sh = 17.8$$

$$k_g = 17.8 \times \frac{1.46 \times 10^{-5}}{0.03} = 8.66 \times 10^{-3} \text{ m/s}$$

$$\frac{1}{K_g} = \frac{1}{8.66 \times 10^{-3}} + \frac{9.03 \times 10^{-3}}{2.1 \times 10^{-4}}$$

$$= 115.5 + 43.0 = 158.5 \text{ s/m}$$

11/ $k_g = 6.31 \times 10^{-3}$ m/s

$$G = \frac{\pi d^2}{4} \times v = \frac{\pi}{4} \times 0.03^2 \times 1.5 = 1.06 \times 10^{-3} \text{ m}^3/\text{s}$$

$$\frac{k_g \pi d h}{G} = \frac{6.31 \times 10^{-3} \times \pi \times 0.03 \times 1}{1.06 \times 10^{-3}} = 0.561$$

$$f_{min} = \exp\left[-\frac{k_g \pi d h}{G}\right] = 0.571 .$$

$f = 1.2 f_{min}$ — now $c_{lB} \neq 0$; from the material balance

$$c_{lB} = \frac{G}{L}\left(c_{gB} - c_{gT}\right) \quad \leftarrow f c_{gB}$$

Then $$G(c_{gB} - c_{gT}) = k_g \pi d h \frac{(c_{gB} - H c_{lB}) - c_{gT}}{\ln\left[(c_{gB} - H c_{lB})/c_{gT}\right]}$$

is a nonlinear equation for L [with solution (not required) $\frac{HG}{L} \simeq 0.9$, $L \simeq 1 \times 10^{-5}$ m³/s]

? Reduce f_{min} ?

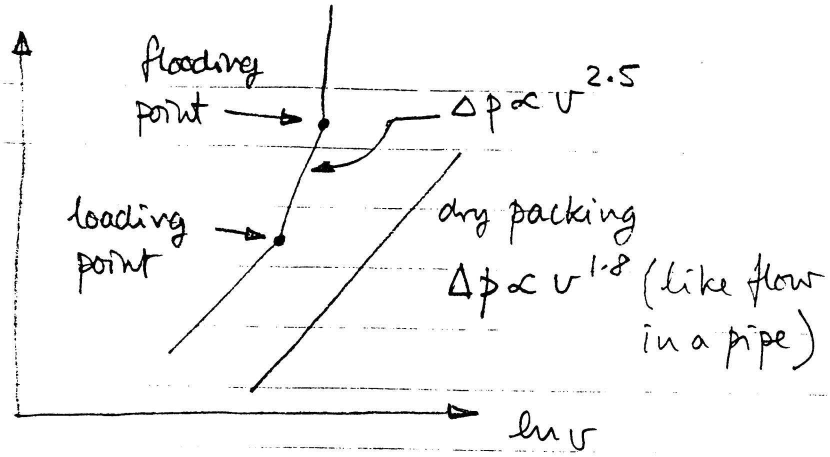
Need to increase $k_g \pi d h / G$

- a longer or wider column
- 2 (or more) columns

$k_g \propto v^{0.83}$, so k_g/G is almost independent of v

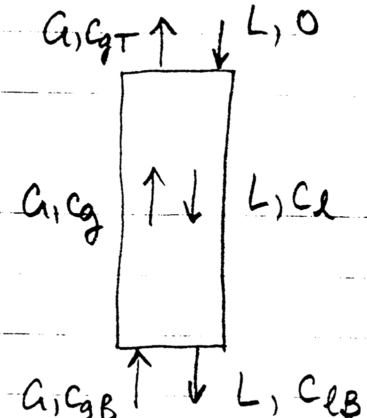
- increase surface area by packing column

12 (a)



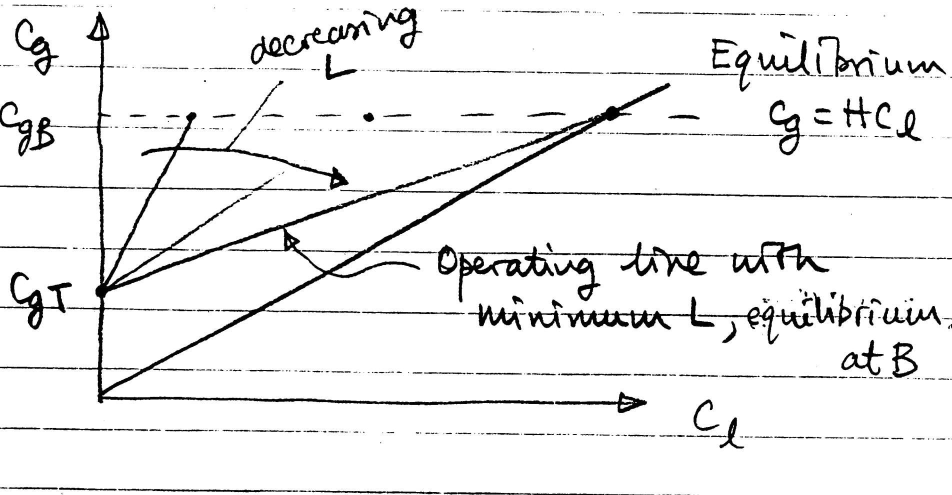
Operate between the loading and flooding points where there is a significant hold-up of liquid. After the flooding point, regions of continuous liquid phase exist — normal operation breaks down.

(b)



Mass balance $G\left(c_g - c_{gT}\right) = L\, c_l$

$$c_g = c_{gT} + \frac{L}{G} c_l \qquad \text{operating line}$$



12

$$C_{gT} + \frac{L_{min}}{G}\frac{C_{gB}}{H} = C_{gB}$$

$$L_{min} = HG\left(1 - \frac{C_{gT}}{C_{gB}}\right)$$

Can't go below L_{min}, or you'd get 'concentration crossover' — not allowed.

Operation at L_{min} would necessitate an infinitely high column — costly.

(c) $L_{min} = 9.03 \times 10^{-3} \times 1.4 \times 0.95 = 1.2 \times 10^{-2}\ m^3/s$

$L = 2.4 \times 10^{-2}\ m^3/s$, $\dot{m}_L = 24\ kg/s$

$G = 1.4\ m^3/s$, $\dot{m}_G = 1.68\ kg/s$

$$F_{LV} = \frac{24}{1.68}\sqrt{\frac{1.2}{1000}} = 0.49$$

$K_{flood} = 1.2$

Use $K = \frac{1}{2} K_{flood} = 0.6$

$$A^2 = \frac{42.9}{0.6}\ \frac{1.68^2\ 52\ (10^{-3}/1000)^{0.1}}{1.2\,(1000 - 1.2)} = 2.20\ m^4$$

$$A = 1.48\ m^2 = \frac{\pi d^2}{4}$$

$d = 1.37\ m.$

12 (d) check out a larger packing size
find mass transfer coeffts (from correlations, or expt)
find height of column
design column internals

cost estimate

vary L and try again

maybe vary column pressure, and try again (higher pressure → smaller plant, lower capital cost, but running a pump → higher running costs).

Q. 13

(a) Assuming full ionisation of the donors, conductivity, σ, is given by:

$$\sigma = eN_D\mu .$$

Thus, $N_D = \sigma/e\mu = 80/(1.6\text{x}10^{-19} \text{ x } 0.1) = \mathbf{5x10^{21}\ m^{-3}}$.

(b) The electric field in the channel is V/d , where d is the channel length. Thus the drift velocity of electrons, $\langle v \rangle = \mu V/d$ and the transit time, τ, is $d/\langle v \rangle = d^2/\mu V$. Thus,

$$d = [\mu V \tau]^{0.5} = [0.1 \text{ x } 4 \text{ x } 40 \text{ x} 10^{-12}]^{0.5} = \mathbf{4\ \mu m} .$$

Comment: this value of d gives a field in the channel of 10^6 Vm^{-1} which is close to the value at which the electron velocity saturates at 10^5 ms^{-1}. This is also the velocity predicted by the product of field and mobility and an equally valid approach is to assume that $\langle v \rangle$ is indeed 10^5 ms^{-1} and calculate d from that. with this method it is necessary to confirm that the field is high enough to give saturation.

(c) The cross-sectional area of the channel, A, is 40 x 1 x 10^{-12} m^2 . The channel conductance is $\sigma A/d$ and the current, I, is $V\sigma A/d$. Thus:

$$I = 4 \text{ x } 80 \text{ x } 4 \text{ x } 10^{-11}/4 \text{ x } 10^{-6} \text{ A} = \mathbf{3.2\ mA}.$$

The surface area of the device is 4 x 40 x 10^{-12} m^2 and the power dissipated is 3.2 x 4 mW. Thus the power density is $\mathbf{8 \text{ x } 10^7\ Wm^{-2}}$ and is just inside the limit.

(d) The transistor will be cut off when the depletion region under the gate reaches the substrate at the source, i.e. $V_{CO} = - eN_Da^2/2\varepsilon$. Taking ε_r for silicon as 12 gives:

$$V_{CO} = - 1.6 \text{ x } 10^{-19} \text{ x } 5 \text{ x } 10^{21} \text{ x } 1 \text{ x } 10^{-12} \text{ x } 36\pi \text{ x } 10^9/(2 \text{ x } 12) = \mathbf{- 3.77\ V}.$$

This neglects the built-in potential difference of the Schottky barrier so the actual answer will be slightly greater, i.e. less negative.

Q. 14

(a) Photoelectric effect. Energy of emitted electrons depends on the frequency of the radiation and not its intensity, a result which classical physics cannot explain.

(b) Standing waves: like those on a piano wire where waves are confined to a particular region. Mathematically:

$$\Psi = \Psi_0 \cos(kx) \,.\, \cos(\omega t)$$

Note that any similar combination of sine or cosine functions will do and that the displacement is zero at certain points (nodes) and oscillates with maximum amplitude at other points (antinodes).

Travelling waves: waves which can carry energy like light or any form of e.m. radiation. Mathematically:

$$\Psi = \Psi_0 \cos(kx - \omega t),$$

or more generally

$$\Psi = \Psi_0 \exp\ j(kx - \omega t).$$

In either formulation, a point of constant argument $\phi = kx - \omega t$ moves such that $d\phi/dt = 0 = k\ dx/dt - \omega$ so that the velocity of propagation, $dx/dt = \omega/k$.

Also, a superposition of two *travelling* waves moving in opposite directions,

$$\exp j(kx - \omega t) + \exp j(-kx - \omega t) = 2 \cos(kx) \,.\, \exp(-j\omega t),$$

gives a *standing* wave.

(c) In the equation, E stands for total energy, $T = hk^2/2m$ is kinetic energy and V is potential energy. The momentum of a particle, p, is equal to $hk = h/\lambda$, de Broglie's hypothesis, and writing p as mv gives T as $\frac{1}{2}mv^2$. Also, p is equivalent to the differential operator -jh d/dx and multiplying Schrodinger's equation by Y gives

$$d^2\Psi/dx^2 = 2mE/h^2 \text{ with solution } \Psi \propto \exp(j\sqrt{2mE}\,x/h).$$

When the electron is confined to the "infinitely" deep one-dimensional potential well of length a, its wave function must vanish at $x = 0$ and $x = a$. The spatial dependence of the resulting standing wave must therefore have the form $\sin(\sqrt{2mE}\,x/h)$ to ensure that it vanishes at $x = 0$ and in order for it to vanish at $x = a$, $\sqrt{2mE}\,a/h$ must be an integral multiple of π.

Thus E is quantised to the values $n^2 . (\pi h/a)^2/2m = n^2 . 6.56 \times 10^{-18}$ J = **41 n^2 eV**.

Comment: although a is similar to the diameter of a hydrogen atom, the energy of the ground state, $n = 1$, is far larger than the ionisation energy of a hydrogen atom (~13.5 eV).The reason for this is the fact that the potential well is one-dimensional. If we were dealing with a cubical well with side 0.1 nm, the greatest allowable wavelength would be $\sqrt{3} \times 0.1$ nm corresponding to an energy of 41/3 eV = 13.7 eV; close enough!

Q. 15

(a) While the choice of a positive or negative resist will depend to some extent on the area to be covered, such matters can be accommodated by the design of the mask. Usually, negative resists will be chosen because lower exposures are required, so giving greater through-put. Unfortunately, the resists tend to swell after development which reduces definition of fine structures. This effect can be reduced by using thinner coatings but with complex multi-layer structures there is a limit to how far this may be taken.

(b) Resolution $\approx$ λ/NA so that $10^{-6} = 0.2 \times 10^{-6}$/NA giving NA = 0.2. Depth of field $\approx$ l/(NA)^2 = **5 μm**. Note: some sources quote resolution as one half of the above giving a depth of field of **20 μm**. Either answer is acceptable.

(c) i. *Contact printing*: cheap and cheerful method which does not suffer from diffraction effects but by its nature is liable to cause damage to the mask and to the chips. High through-put because the whole wafer is exposed at once but cannot be used for long runs.
ii. *Proximity printing*: more expensive than contact printing because registration is more difficult. The mask is held just above the substrate so there is less risk of damage but diffraction effects have to be taken into account when deciding on the separation between mask and substrate. Again, high through-put.
iii. *Projection printing*: The most expensive method using two compound lenses with the mask located between them. Thus there is no possibility of damage to mask or substrate. In addition, the mask may be made over-sized and optically reduced which eases mask production. Modern lenses are nearly perfect and the limits to resolution are governed by diffraction.

(d) The power density of 0.05 Wcm^{-2} is equal to 50 mWcm^{-2} and so the exposure time is

$$140/50 \text{ s} = \mathbf{2.8\ s}.$$

(e) From the diagram there are 350 particles m^{-3} 0.5 μm or larger. If V ms^{-1} is the air flow rate, the volume of air flowing over a wafer in 2.8 s is:

$$V \times \pi \times [0.125/2]^2 \times 2.8 \ \text{m}^3.$$

Since the sticking coefficient is 0.6 and no more than 100 chips are to be contaminated, the maximum number of particles allowed in this volume is 100/0.6 = 167. Thus:

$$V \times \pi \times [0.125/2]^2 \times 2.8 \times 350 = 167 \text{ and } V = \mathbf{13.8\ ms^{-1}}.$$

Comment: in practice, the wafer will be exposed to atmosphere for a time far greater than 2.8 s and the air speed will need to be lower than a brisk 50 kmph!

(f) Although a case could be made for projection printing, I would go for proximity printing - certainly not contact if the run is of any length. Otherwise, the system should contain all the elements described in the question. The calculated NA gives an adequate depth of field, the class of the clean room should give an adequate yield (a 0.5 μm particle on a 1 μm track could wreak havoc) and the wavelength of 0.2 μm is satisfactory for 1 μm structures.

Only if the wafer were a one-off prototype would it be worth considering an electron beam system because such a process takes far longer than optical methods. While capable of greater precision and resolution, the current densities and current presently attainable in an electron beam limit the through-put to about 5 wafers per hour.

Solution Q.16

a)

ARP is used to obtain the MAC (or Datalink) address of a system given its IP (or Network layer) address. It is a separate protocol used alongside IP (or potentially other network layer protocols) to provide this mapping. The MAC address is needed in order to select which other interface on the Ethernet cable receives the subsequent IP packets containing data. The MAC address is in effect the hardware address of the interface and is used by the interface hardware to select packets intended for it. It is however the IP address of a system which is publicised since this can remain invariant even if the hardware has to be changed and it has the same format and value irrespective of the physical networks to which the end systems are attached. ARP need only be used when two systems first communicate since each may subsequently keep a record of the others MAC address.

Assume that system A (with MAC address MAC_A and IP address IP_A) wants to communicate with system B for which it knows the IP address (IP_B) but not the MAC address (MAC_B).

A broadcasts an ARP request packet containing:

MAC source address	MAC_A
MAC destination address	broadcast
ARP sender datalink address	MAC_A
ARP sender network address	IP_A
ARP target datalink address	(blank)
ARP target network address	IP_B

All systems on the local Ethernet receive this, the ARP software in each inspects the ARP target network address and only the machine whose IP address matches responds. In this case this is B which sends an ARP response packet containing:

MAC source address	MAC_B
MAC destination address	MAC_A
ARP sender datalink address	MAC_B (as required)
ARP sender network address	IP_B
ARP target datalink address	MAC_A
ARP target network address	IP_A

A (and only A) receives this and stores B's IP address for future use.

b)

In the case of two systems communicating via a gateway, ARP will be required by each system to map from the IP address of the gateway's interface on its network to the corresponding MAC address. Routing protocols will provide the appropriate IP address for the gateway.

Alternatively for simple networks with only a single gateway, a scheme called proxy-ARP can be employed in which the gateway responds to ARP requests for systems reachable through it. This means that the end system does not need to know about routes (or the existence of the gateway) but will involve it caching several MAC addresses (all the same) if it is communicating with several systems via the gateway.

c)
i) The type field in the MAC header identifies the rest of the packet as being an ARP packet and may thus be used to select ARP and only ARP packets. The monitor cannot do any selection based on the MAC header address fields and for most normal hardware will therefore have to receive all packets and filter in software based on the type field.

ii) In addition to the filtering described in (i), the monitor will need to inspect the ARP sender and target network addresses and see if either matches that of the system being monitored. It could also check the ARP sender and target datalink addresses and additionally include any which matched the system's MAC address; this would only be necessary if it were suspected that the problem might involve incorrect use of the network address fields.

Solution Q17

A CSMA/CD network is one in which all communicating devices share the same transmission medium. Normally a station wishing to transmit first checks to see if the medium is free. When a station transmits a packet, it simultaneously reads it back to check for a collision. If a collision is detected, then the station typically delays for a random period and tries again to send the packet.

The device registers may be described as follows

```
TYPE StatusReg = RECORD
                      inputReady : Boolean;
                      outputBusy : Boolean;
                      rxError    : Boolean;
                      unused     : 0..31
                   END;

     ControlReg = RECORD
                      enableDelay : Boolean;
                      delayCount  : 0..31;
                      unused      : 0..3
                   END;

VAR  data : Byte @ 0FC000H;              {read/write}
   status : StatusReg @ 0FC001H;         {read only}
  control : ControlReg @ 0FC001H;        {write only}
```

The SendPacket routine must send a packet byte by byte, simultaneously reading it back. If the data is different or an error occurs, the packet must be retransmitted with a transmit delay set for the first byte only. To simplify the problem, first assume procedures to send and receive a byte, then SendPacket is as follows

```
FUNCTION SendPacket(p : Packet):Integer;
    VAR i: 1..32;
        retries:Integer;
        ok : Boolean;
BEGIN
    retries := 0;
         i:= 1;
    REPEAT
      SendByte(p[1],retries>0);            {send first byte}
      ok := (GetByte = p[1]) AND NOT status.rxError;
      WHILE (i<32) AND ok DO BEGIN
            i := i+1;
            SendByte(p[i],false);    {send remaining bytes without delay}
            ok := (GetByte = p[i]) AND NOT status.rxError;
       END;
       IF NOT ok THEN retries := retries + 1
    UNTIL ok;
    SendPacket := retries
END;
```

The procedure SendByte sends the specified byte and if the second argument is true, it prepends a random delay

```
PROCEDURE SendByte(b:Byte; delay:Boolean);
VAR creg : ControlReg;
BEGIN
  WHILE status.outputBusy DO {wait};
        IF delay THEN BEGIN
          creg.enableDelay := true;
          creg.delayCount := Random * 32;
        END ELSE
          creg.enableDelay := false;
        control := creg;
        data := b
END;
```

The procedure GetByte waits for the next input byte and returns it

```
FUNCTION GetByte:Byte;
BEGIN
  WHILE NOT status.inputReady DO {wait};
  GetByte := data
END;
```

Limitations

- inefficient to transmit blindly without any mechanism for first checking if bus is quiet
- polled mode impractical since all stations must listen to network all the time
- byte by byte transfer very inefficient. DMA on a packet by packet basis would be better.

Solution Q18

A process describes a single thread of control. It is represented on a uniprocessor by a process record which holds information on the state of the process and when it is not actually executing, a copy of the CPU registers. The process record provides a place to store the total system state relating to that process. This allows the physical CPU to be shared amongst many processes by repeatedly saving the system state in one process record and loading a new system state from another. The latter operation is known as a context switch.

The four main process states are

```
running - process is executing
ready   - process is ready to run as soon as the CPU is available
waiting - process has been suspended waiting for some specific
           event to occur
killed  - process is no longer active and any memory used by it
          can be reclaimed
```

In Pascal these states can be represented by a simple enumerated type thus,

```
TYPE  Status        = (running, ready, waiting, killed);
```

A circular process queue can be constructed by adding a pointer to each record to enable each to point to the next record in the circle. The currently running process can be conveniently identified by ensuring that it is always at the 'head' of the queue.

```
TYPE  ProcId  = ^PrRec;

      PrRec   = RECORD
                   next : ProcId;       {next proc in circle}
                   state: Status;
                   prio : Priority;
                   ev   : Event;
                   regs : RegSet
                END;

VAR   head : ProcId;          {Pointer to current process in queue}
```

A design for the Sleep procedure is as follows

```
PROCEDURE Sleep(ev:Event);
VAR maxPrio : Priority;
      maxp, p : ProcId;
BEGIN
  {Suspend current process}
        head^.state := waiting;
        head^.ev := ev;

        {Search for highest priority ready process}
        maxp := head^.next;
        maxPrio := maxp^.prio;
        p := maxp^.next;
        WHILE (p<>head) DO BEGIN
                IF ( (p^.prio > maxPrio) AND (p^.state = ready) THEN
                BEGIN
                        maxp := p;
                        maxPrio := maxp^.prio
                END;
                p := p^.next
        END;

        {Finally make maxp^ the current process}
        maxp^.state := running;
        p := head;
        head := maxp;
        ContextSwitch(p,head)
END;
```

Note that the above assumes that there is at least one ready process in the queue and mutual exclusion wrt to interrupt processes is assumed!

ContextSwitch(oldp,newp) has the following effect

a) the current CPU registers are copied into oldp^.regs
b) the contents of newp^.regs are copied into the CPU regs

Note that call to ContextSwitch must be the last action performed by Sleep so that when the current process is eventually resumed it does nothing to disturb the new state of the process queue.