CAMBRIDGE UNIVERSITY ENGINEERING DEPARTMENT

EXAMINATION PAPERS

1996

ENGINEERING TRIPOS: 1B

ENGINEERING TRIPOS PART IB

Monday 3 June 1996 9 to 11

Paper 1

MECHANICS

Answer not more than **four** *questions.*

(TURN OVER

1 (a) What is meant by the term *radius of gyration*? Calculate the radius of gyration about one edge of a thin, flat plate of uniform material with the shape of an equilateral triangle of side $2a$.

(b) The thin circular disc, centre Q and radius r shown in Fig. 1, rotates with constant angular speed ω_1 about the shaft OQ which is itself rotating with the shaft OP at a constant angular speed ω_2. The distance from O to Q is q. The point R lies on the rim of the disc and instantaneously QR is parallel to OP. Unit vectors $\underline{i}$, $\underline{j}$ and $\underline{k}$ rotate with OP and are such that $\underline{i}$ is parallel to OQ and $\underline{k}$ parallel to OP. Obtain expressions in terms of these unit vectors for the velocity and acceleration of the point R at the instant shown.

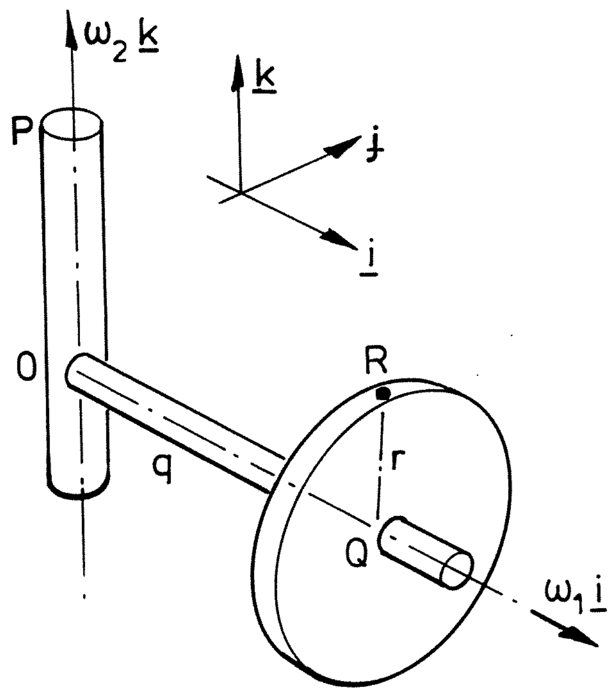

Fig. 1

2 (a) The shaft in Fig. 2(a) rotates at the rate of 5 rad s^{-1} and at the instant shown the point A has coordinates (1,3,1) referred to the set of Cartesian axes shown. Distances are in metres. An inextensible cord attached to A passes through a fixed eye at the point B which has coordinates (2,4,0). At the instant shown, with what speed does the cord pass through the eye?

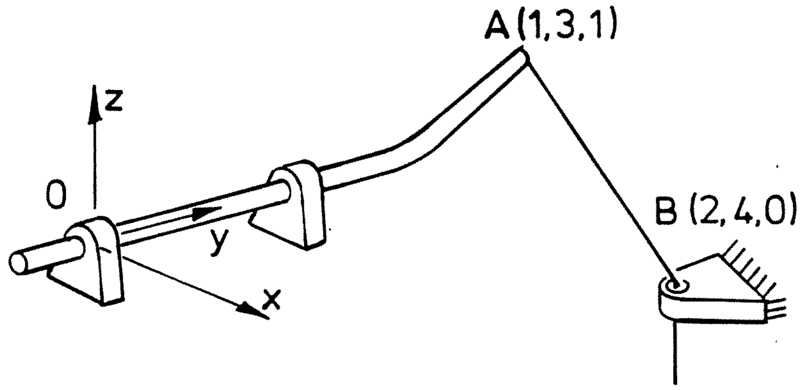

Fig. 2 (a)

(b) A uniform sphere of mass M and radius r is at rest on a horizontal surface. It is struck by a radial impulse of magnitude I, which lies in a vertical plane, at a point with an elevation of $1.5r$ as shown in Fig. 2(b). The coefficient of friction between the sphere and the surface is 0.2. Verify that in the subsequent motion there must be slip at the point of contact.

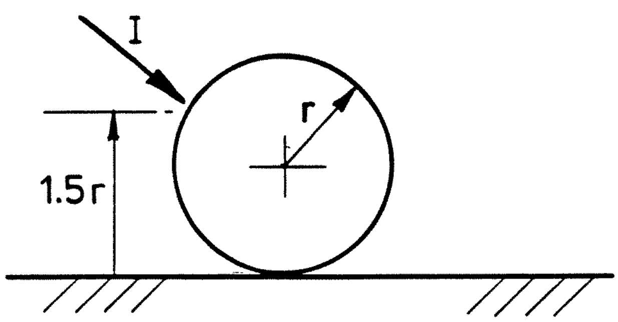

Fig. 2(b)

(TURN OVER

3 Figure 3 shows a plane mechanism operated by a pneumatic ram BG. The distance from the pivot at E to the point D is 0.2 m. When the pneumatic cylinder extends at a rate of 1 m s^{-1} show that the angular velocity of the link EF is approximately 7.6 rad s^{-1}.

Assuming that the ram extends at a constant rate, evaluate the angular acceleration of the link EF at this instant. Suitable scales for velocity and acceleration diagrams are 0.2 m s^{-1} per 10 mm and 10 m s^{-2} per 10 mm respectively.

The diagram is to scale and may be pricked through.

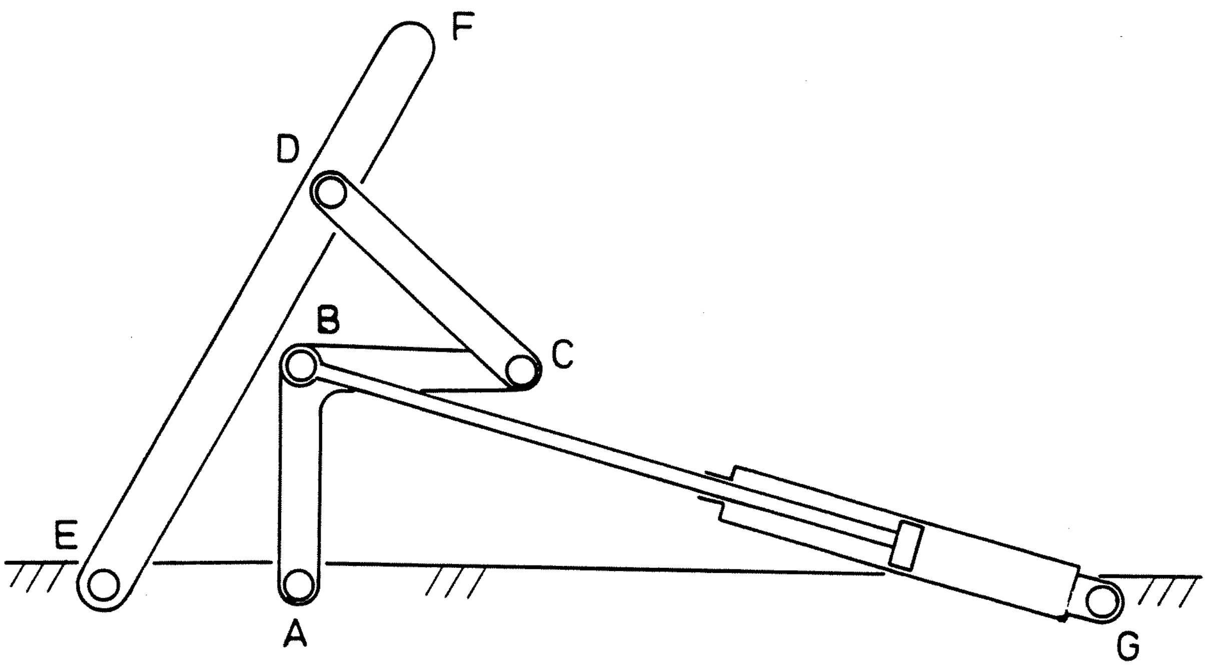

Fig. 3

4 Under what circumstances is the mechanical energy of a system conserved?

A thin hoop of mass m and radius r starts from rest at the highest point A and rolls down a circular cylindrical surface of radius a and centre O, as indicated in Fig. 4.

If, at some instant, the line joining O and the point of contact between the cylinder and the hoop makes an angle θ with OA, through what angle has the hoop rotated? Show that the rate of change of the angle θ is given by

$$\dot{\theta} = \sqrt{\frac{(1 - \cos\theta)g}{a + r}} .$$

It is observed that the hoop starts to slip when the angle θ is 30°: determine the coefficient of friction between the hoop and the cylindrical surface.

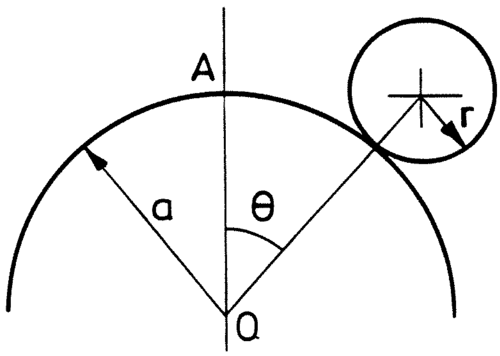

Fig. 4

5 Explain briefly what is meant by D'Alembert's principle.

A point mass of magnitude m is attached to the end B of a slender uniform rod AB of length L whose mass is also m. The mass and the rod are supported by a smooth horizontal table and the end A of the rod is to be accelerated from rest at a uniform rate a in the direction shown in the plan view of Fig. 5. Show that the magnitude of the initial angular acceleration of AB is

$$\frac{9a}{8L}\cos\theta$$

where θ is the angle indicated.

Obtain an expression for the initial magnitude of the force required at A when the angle θ is 30°.

Also find the location of the maximum bending moment in AB at this instant due to the imposed acceleration.

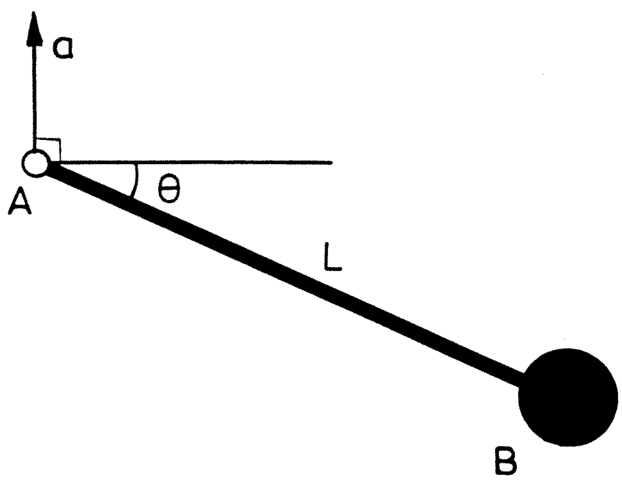

Fig. 5

6 Distinguish briefly between statically and dynamically balanced rotating shafts.

A shaft is supported in two bearings 1 m apart and projects 0.15 m from each of them. The shaft carries three pulleys, one at each end and one midway between the bearings. The end pulleys, A and C, are out of balance to the extent of 0.012 kg m and 0.020 kg m respectively, and the middle pulley B is out of balance by 0.016 kg m. The pulleys are fixed to the shaft so as to provide static balance. Find:

(i) the angular relationships between the out of balance masses of the three pulleys;

(ii) the dynamic loads on each of the bearings when the shaft rotates at 420 rpm;

(iii) the magnitude and position of the balance weights which must be added to pulleys A and C at a radius of 0.2 m to give dynamic balance while maintaining conditions of static balance.

END OF PAPER