ENGINEERING TRIPOS PART IB

Monday 3 June 1996 2 to 4

Paper 2

STRUCTURES

Answer not more than **four** *questions.*

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1 (a) The formulae for Tresca's and von Mises' yield criteria are given in the Structures Data Book. State very briefly how the assumptions underlying the two criteria differ.

(b) An aluminium alloy mast of 4.2 m height has the cross-section shown in Fig. 1(a). It is fully fixed at the base, but is free to rotate at the top where it carries a light spar to which two loads of magnitude F will be simultaneously applied at the ends, as shown in Fig. 1(b). These loads remain in the horizontal plane, and their lines of action remain always perpendicular to the spar, even if the spar rotates. The aluminium alloy has a yield stress in simple uniaxial tension of 250 N/mm^2, and may be assumed to obey Tresca's yield hypothesis, such that inelastic behaviour will occur when the maximum shear stress reaches 125 N/mm^2.

(i) Using thin-walled elastic theory, determine the magnitude of the forces F (in kN) that will just cause inelastic behaviour.

(ii) Through what angle will the spar have rotated at the onset of inelastic behaviour?

(iii) A second 4.2 m mast is constructed, identical in every respect except that it has everywhere twice the wall thickness (i.e. the curved part of the wall is 4 mm thick and the straight parts are 2 mm thick). A torque is applied to this mast via loads at the ends of the spar, as in Fig. 1(b) again. What is the maximum angle through which the spar can be rotated without causing inelastic behaviour in the mast?

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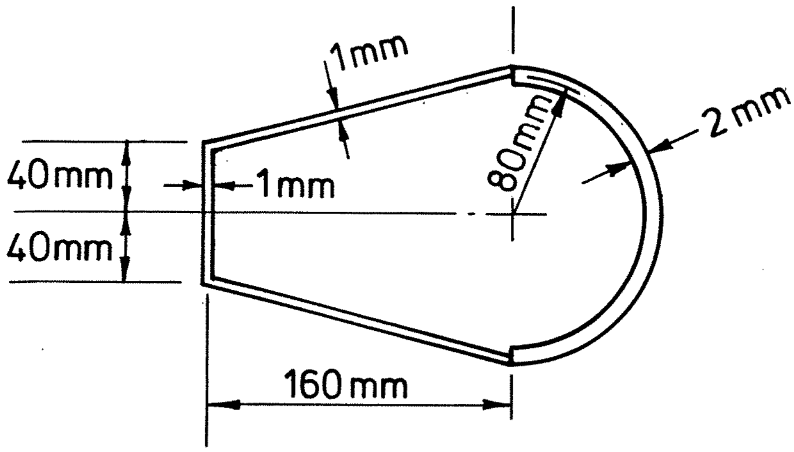

Fig. 1a

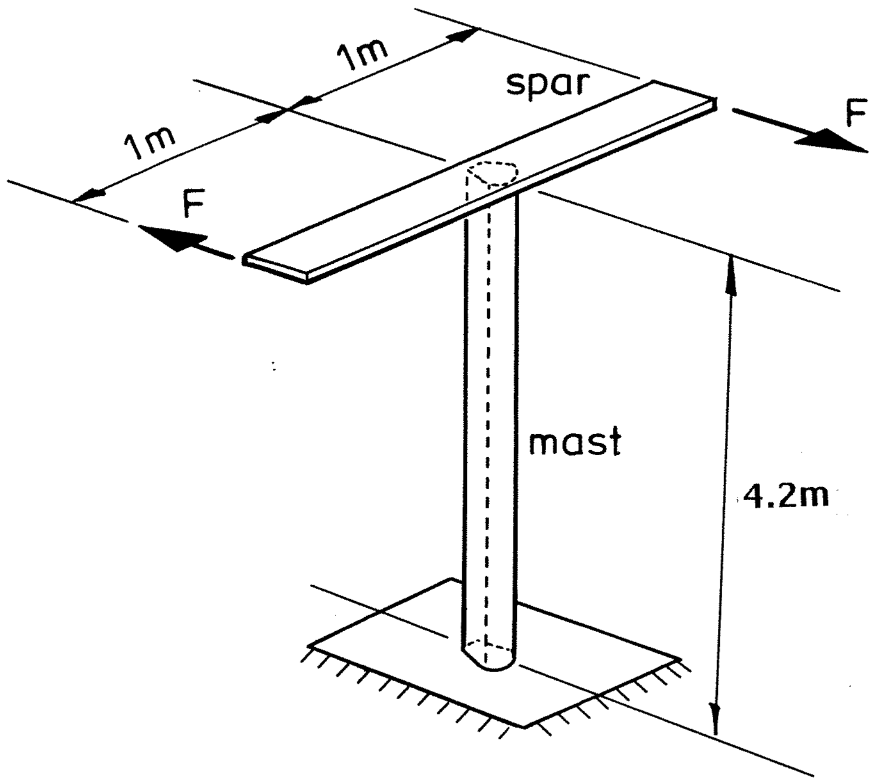

Fig. 1b

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2 A pipeline is made of steel of uniaxial yield stress 345 N/mm^2. The pipe has an external diameter of 2020 mm and a wall thickness of 20 mm. The pipe is subject to a large internal pressure. Over one section of the pipeline, the pipe experiences a substantial torque and an axial force. The readings from a rosette of strain gauges at a point on the pipe in this region are interpreted, and the corresponding state of stress is deduced to be that shown in Fig. 2.

(a) Assuming there are no other forces acting on the pipe at this section, determine the magnitude of the applied torque, the axial force carried longitudinally by the pipe walls and the difference between the internal and external pressure. (Give all answers in units of kN and m.)

(b) Calculate the magnitude and direction of the principal stresses and their corresponding strains.

(c) Assuming the applied torque is removed but the pressure difference remains unaltered, what range of values of longitudinal force (in tension and in compression) can the pipe walls carry without yield occurring, according to Tresca's hypothesis?

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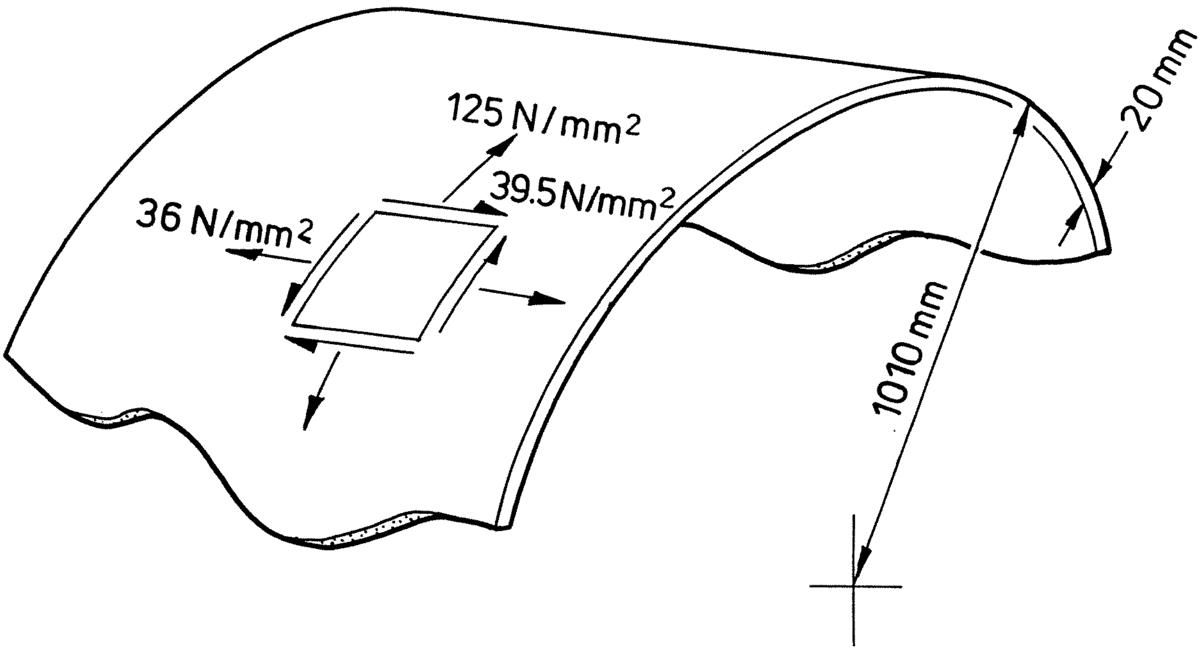

Fig. 2

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3 The beam shown in Fig. 3 is continuous from A to D, and the supports A, C and D provide no resistance to rotation. The fully plastic moment of the beam is M_p from A to C and αM_p from C to D, ($0 < \alpha \leq 1$). Span AC is of length L and span CD is of length S. The general point B is a distance βL from A. The beam carries a uniformly distributed load λw per unit length over span AC.

(a) By considering a mechanism with hinges at B and C, show that the beam must collapse if $\lambda > 2M_p/wX^2$, where $X = \beta_c L$, and β_c is a solution of

$$\alpha\beta_c^{\,2} + 2\beta_c - 1 = 0$$

[It may be of use to note that this quadratic equation can be rewritten as $\alpha\beta_c + 1 = (1 - \beta_c)/\beta_c$.]

(b) Sketch the bending moment diagram at collapse by this mechanism.

(c) A design for a two-span bridge includes a single row of steel beams which is continuous over the full length of the bridge. This row is made of the following beams:

over span AC :	762 x 267 UB 173 (length L = 27 400 mm)
over span CD :	762 x 267 UB 147 (length S = 21 200 mm)

The yield stress of the steel is 345 N/mm^2. The beams will be fully welded together at the central pier C, and simply supported at each abutment (A & D), such that Fig. 3 is a reasonable diagram showing the elevation of these two beams.

(i) Determine the load per unit length that, when applied to the beam over span AC only, will cause collapse.

(ii) How would this collapse load change if, due to foundation settlement, the abutment A moved downwards by a small amount?

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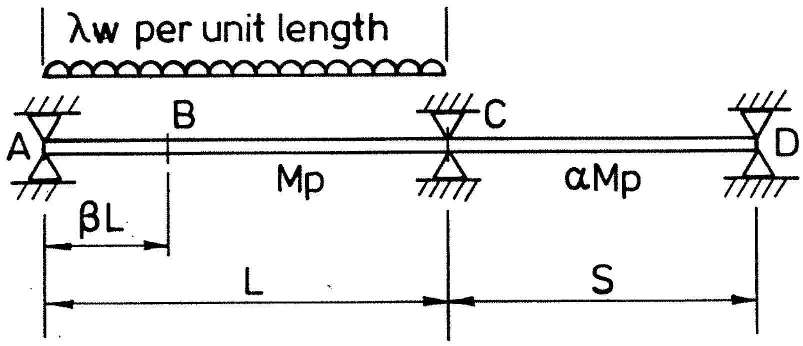

Fig. 3

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4 The rectangular steel plate ABCD shown in Fig. 4 is built-in along the three edges AB, BC and CD, but is unsupported along the free edge AD. The plate carries a total transverse load λW uniformly distributed over the full area of the plate.

(a) Based on the yield-line pattern defined by the parameter α shown in Fig. 4, show that the collapse value of the load factor λ is given by

$$\lambda = \frac{24m}{W}\left\{\frac{8\alpha+1}{\alpha(6-\alpha)}\right\}$$

where m is the moment of resistance per unit length for all yield lines.

(b) Numerically evaluate $(8\alpha + 1) / [\alpha(6 - \alpha)]$ at a number of values of α to determine its minimum value to two significant figures, and hence give a least upper bound for λ.

(c) Changing only the details of the pattern near the corners B and C, sketch a possible yield-line pattern that might collapse at a lower load.

(d) Assume that the true collapse load factor, based on the worst possible yield line pattern, equals $40m/W$, and consider such a plate to be made of steel of yield stress $Y = 355$ N/mm^2.

Determine the thickness of plate such that collapse just occurs when the load reaches 12 kN.

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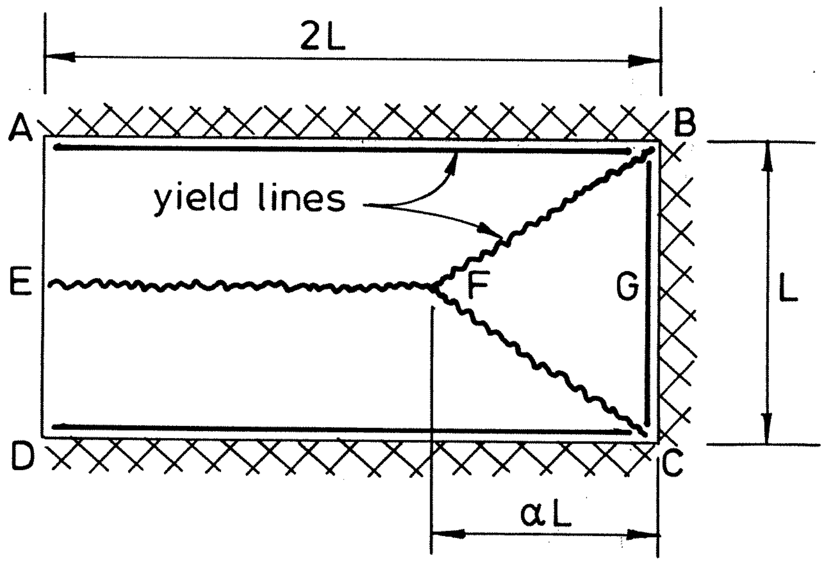

Fig. 4

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5 Figure 5 shows a uniform pin-ended strut of length L, relevant flexural rigidity EI and cross-sectional area A. The relevant radius of gyration of the section $r = \sqrt{I/A}$. The unloaded strut has a small initial bow v_0 given by

$$v_0 = a_0 \sin(\pi x/L)$$

where a_0 is the initial displacement of the midpoint and x measures distance along the strut from one end. It is well-known (THUS DO NOT PROVE) that under the action of an axial force P, the mid-point displacement of the strut increases from a_0 to the value a given by

$$a = \frac{a_0}{\left(1 - \sigma / \sigma_E\right)}$$

where $\sigma = P / A$ is the mean stress, and $\sigma_E = \pi^2 E/(L/r)^2$ is the so-called "Euler stress".

(a) Show that if the material behaviour remains linear elastic, the maximum compressive stress σ_{max} in the strut is given by

$$\sigma_{max} = \sigma \left\{1 + \frac{\eta}{1 - \sigma/\sigma_E}\right\}$$

where the Perry imperfection factor $\eta = a_0 h / r^2$. Here, h is the distance from the centroid of the section to the extreme fibre on the concave side of the strut.

(b) Thus derive, stating any assumptions being made, the Perry-Robertson formula, in the form

$$\left(\sigma_y - \sigma_{cr}\right)\left(\sigma_E - \sigma_{cr}\right) = \eta\, \sigma_E\, \sigma_{cr}$$

where σ_y is the yield stress and the critical stress σ_{cr} is the lower root of the resulting quadratic.

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(c) The main columns supporting the Inglis Building are 305 × 305 UC 240. Determine the maximum axial compressive load (in kN) that may be carried by such a column according to the Perry-Robertson formula, assuming that: the column is 4.0 m high; each end is completely free to rotate about any transverse axis but both ends are fully restrained against sway in all directions; the steel has a yield stress of 345 N/mm^2; the Perry imperfection factor η = 0.0055 (L/r); and there are no other external forces or moments to consider, nor are there any artificial eccentricities to be introduced at the ends.

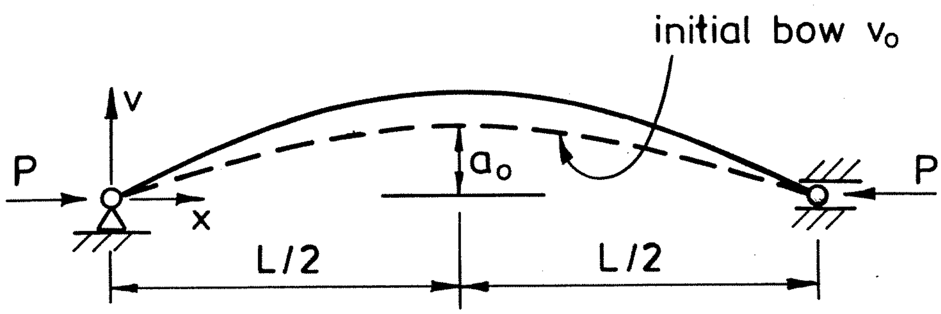

Fig. 5

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6 The pin-footed portal frame ABCD shown in Fig. 6 is initially stress-free. It is made of a material of Young's modulus E and coefficient of thermal expansion α. The relevant flexural rigidity of the beam BC is $2EI$, this being twice the flexural rigidity of each of the columns. A concentrated load W is applied at the centreline as shown and the temperature of the beam BC is increased by ΔT.

Assuming that the structure remains everywhere elastic, that there is no buckling or sway instability, and that axial compressibility and shear deformation may be neglected:

(i) show that the sagging moment at the centre of the beam under the combined action of the applied load and temperature rise is

$$\frac{3WL}{4} - \frac{EI\,\alpha\,\Delta T}{3L}$$

and sketch the bending moment diagram for the frame;

(ii) determine the corresponding vertical displacement at the centre of the beam.

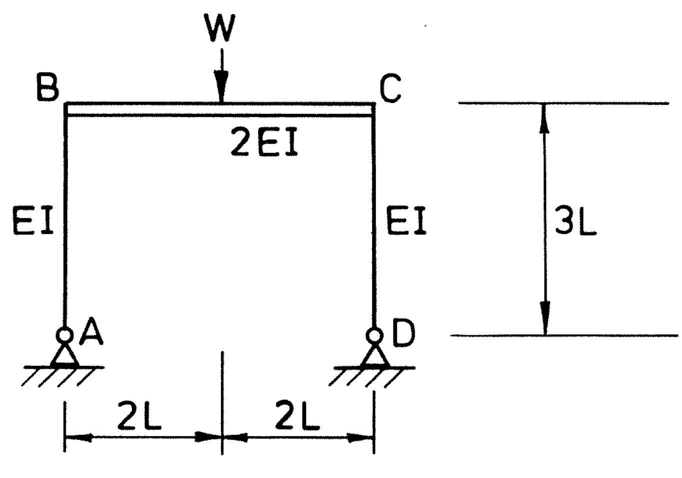

Fig. 6

END OF PAPER

Notice to Invigilators.

Part IB Paper 2 (Structures), 3 June 1996.

Minor typographical inconsistency in the paper.

If any student should query it, the symbol

a_0

in the equation on line 4, page 10, (Question 5) is the same variable as the symbol

a_0

which appears on the line immediately below the equation.