ENGINEERING TRIPOS PART IB

Tuesday 4th June 1996 2 to 4

Paper 4

FLUID MECHANICS AND HEAT TRANSFER

Answer not more than **four** *questions.*

Answer at least **one** *question from each section.*

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

1 (a) The air in a deep valley has a uniform temperature of T_o. The pressure and density at the base of the valley are p_o and ρ_o. Derive an expression for the variation of pressure with altitude, y. (Take y to be zero at the valley floor). You may assume that the air obeys the ideal gas law

$$p = \rho RT .$$

A rough estimate of the pressure distribution may be obtained by assuming the density of the air is constant, so that p is given by

$$p = p_o - \rho g y \; ; \; \rho = \rho_o .$$

Show that this strategy provides a good estimate of p provided the depth of the valley is much less than $p_o / \rho_o g$.

(b) State Pascal's law of hydrostatics and give the technical definition of a fluid. Briefly indicate how you might construct a proof of the fact that, when a fluid is in motion, the pressure need not be the same in all directions.

2 (a) A vertical cylindrical vessel is connected via a pipe to a large reservoir as shown in Fig. 1. The diameter of the vessel is D and that of the pipe is d. The fluid level in the vessel is a height h above that of the reservoir, and the base of the tank is level with the reservoir surface. Assume that the pressure surrounding the pipe exit is hydrostatic, that the flow is quasi-steady, and that frictional losses may be neglected. Show that as the fluid in the tank drains into the reservoir the level h drops according to

$$\frac{dh}{dt} = -\left(\frac{d}{D}\right)^2 \left[\frac{2gh}{1-(d/D)^4}\right]^{\frac{1}{2}} .$$

If the initial level in the tank is h_o, derive an expression for the time it takes for the tank to empty.

(b) The pipe has length L and friction factor f. Derive a new expression for the time taken to drain the tank, but this time take the frictional losses in the pipe into account. By comparing this expression with that of part (a), determine the conditions under which it is reasonable to neglect the frictional losses in the pipe.

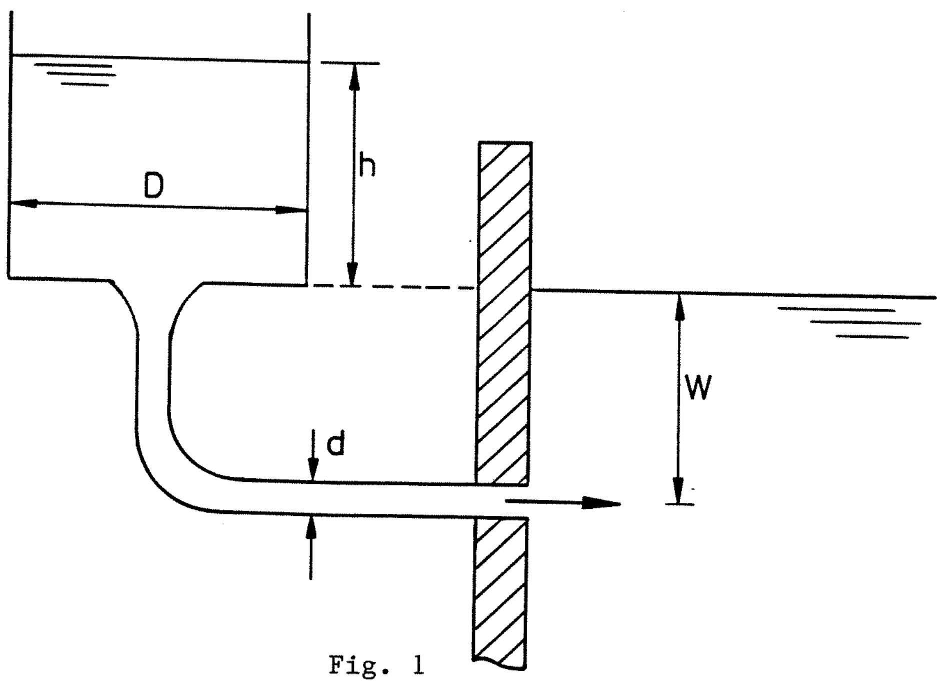

Fig. 1

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3 (a) State Newton's law of viscosity. Not all fluids are Newtonian. On a graph of shear stress versus strain rate indicate other common types of behaviour. Name one non-Newtonian fluid and give a simple physical explanation for its non-Newtonian behaviour.

(b) Water from an overflowing sink escapes down an inclined draining board as shown in Fig. 2. The water forms a thin film on the board which has an angle of inclination α to the horizontal. The flow is steady and laminar. It is also uniform in the sense that the fluid pressure p, velocity u, and film-thickness T are independent of the streamwise coordinate x. The width of the draining board into the page is W.

By considering the equilibrium of the small element of fluid shown in Figure 2, show that the shear stress distribution in the water film is governed by,

$$\frac{d\tau}{dy} = -\rho g \sin\alpha \quad .$$

(The sign convention for shear stress is shown in Fig. 2). Hence show that the film thickness is related to the volumetric flow rate, Q, by

$$T = \left(\frac{3\mu Q}{W\rho g \sin\alpha}\right)^{1/3}$$

All symbols have their usual meaning.

(c) Calculate the film thickness T for the case of $Q = 45 \times 10^{-6}\ m^3/s$, $W = 0.3\,m$ and $\alpha = 6°$. (You may take g as $9.81\ m/s^2$ and the kinematic viscosity of water as $10^{-6}\ m^2/s$.)

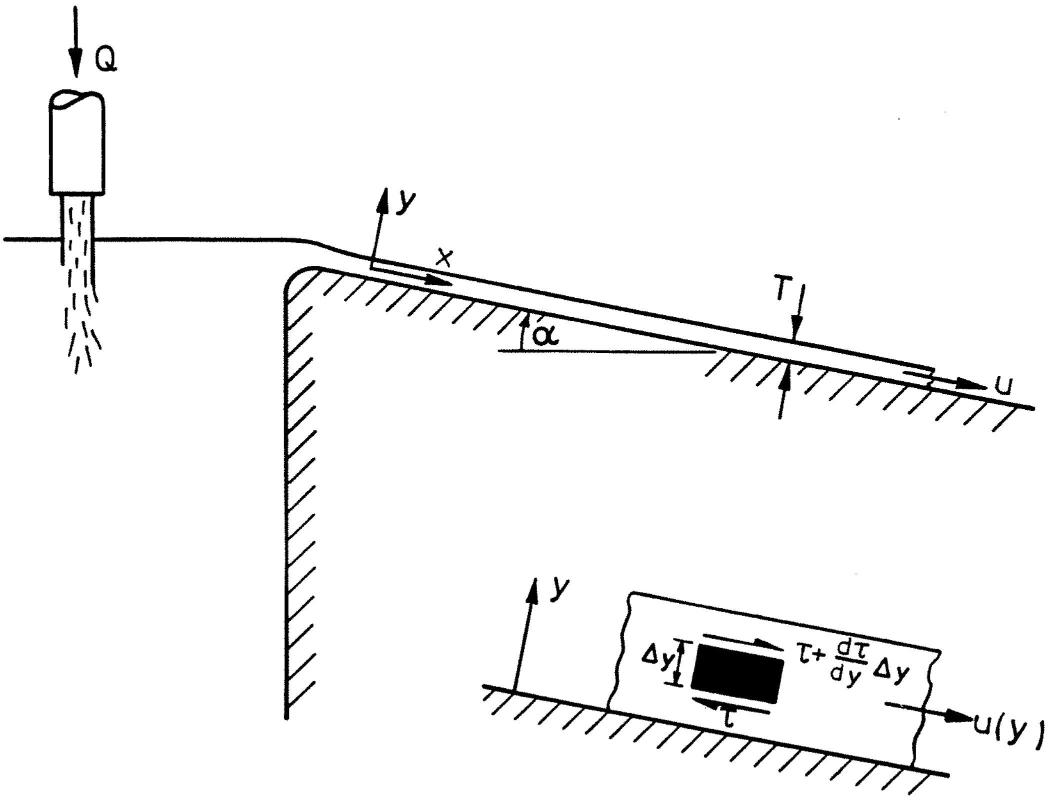

Fig. 2

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4 (a) An hydraulic jump occurs in a channel as shown in Fig. 3. The flow is uniform well upstream and downstream of the jump at sections 1 and 2. The location of the jump is fixed in space and the flow is steady-on-average. The speed of fluid at any section is V and the depth of the flow is h. Explain why the pressure distribution in the fluid at sections 1 and 2 may be taken as hydrostatic. Now use the force-momentum equation to show that:

$$(g/2)\left(h_1^2 - h_2^2\right) = h_1 V_1 (V_2 - V_1)$$

You may ignore friction on the channel bed.

(b) Show that the depth ratio h_2/h_1 is related to the upstream Froude number Fr by the quadratic equation:

$$\left(\frac{h_2}{h_1}\right)\left(1 + \frac{h_2}{h_1}\right) = 2Fr^2 = 2\frac{V_1^2}{gh_1}$$

Find h_2 and V_2 for the case where $h_1 = 1.5$ m and $V_1 = 9.6$ m/s.

(c) If you wished to reproduce this jump in the laboratory, scaled down by a factor of 3, what value of V_1 should you use to give dynamic similarity?

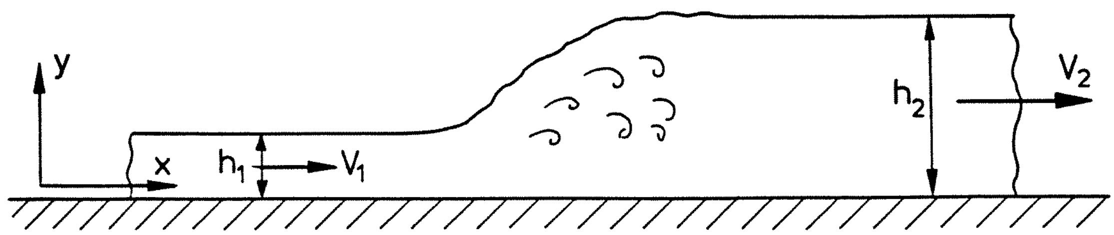

Fig. 3

SECTION B

5 (a) Hot fluid passes along a cylindrical tube of radius R. The flow is laminar and the wall temperature is T_w. A good approximation to the radial variation in temperature is

$$T - T_w = (T_o - T_w)\left(1 - (9/5)(r/R)^2 + (4/5)(r/R)^3\right)$$

What does T_o represent?

Show that the Nusselt number, based on the difference between the wall and centre-line temperatures, is equal to 2.4.

(b) Confirm that the temperature profile given in part (a) satisfies:

$$\frac{\partial T}{\partial r} = 0 \quad \text{at } r = 0$$

and,

$$\frac{\partial}{\partial r}\left[2\pi r\left(\lambda\frac{\partial T}{\partial r}\right)\right] = 0 \quad \text{at } r = R$$

Why are both of these conditions a necessary feature of the temperature profile?

(c) When the flow in a tube is turbulent, the convective heat transfer coefficient depends, amongst other things, on the four material properties, λ, ρ, c_p and $\nu = \mu/\rho$. (All symbols have their usual meaning). Show that there are three dimensionless groups which characterize the dependency of the heat transfer coefficient on this flow and give expressions for them.

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6 (a) A cylindrical furnace has diameter 1.2 m and height 0.85 m. An electric heating element produces a uniform heat flux of 9.4 kW/m^2 over the entire bottom surface, which has emissivity 0.81. The top surface has emissivity 0.63 and is kept at a constant temperature of 500 K. The cylindrical surface is insulated. Taking the view factor between the top and bottom surfaces to be 0.25 calculate the temperature of the bottom surface.

(b) A certain type of burner is fuelled with a stoichiometric methane air mixture. The fuel flow rate is 85 kg/hour and the initial temperature of both fuel and air is 25°C. Heat losses of 600 kW are incurred in the exhaust pipe. Calculate the final temperature of the exhaust gas.

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