ENGINEERING TRIPOS PART IB

Thursday 6 June 1996 9 to 11

Paper 5

ELECTRICAL ENGINEERING

*Answer not more than **four** questions.*

*Answer at least **one** question from each section.*

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

1 Figure 1(a) shows a class A amplifier, and Fig. 1(b) shows a class B amplifier. Explain the differences between the operation of these two types of amplifier with reference to the dc operating point, quiescent power dissipation and amplifier efficiency.

Draw small-signal circuits for both amplifiers, valid for mid-band frequencies, and derive expressions for:

(i) the small-signal output impedance and voltage gain of the class A amplifier;

(ii) the small-signal input impedance and voltage gain of the class B amplifier.

You may assume that the bias resistors R_1 and R_2 are equal, and are so large compared with h_{ie} and R_L that they may be neglected.

You may also ignore the transistor small-signal parameters h_{oe} and h_{re} .

The two amplifiers are connected together so that the points marked A, B and C in Fig. 1(a) are connected to their counterparts in Fig. 1(b). The small-signal parameters of all the transistors are identical. Using the results derived above, or otherwise, find an expression for the small-signal voltage at the input to the second stage of the amplifier. Hence show that the overall small-signal voltage gain of the amplifier is given by:

$$-\frac{h_{fe}\left(h_{fe}+1\right)R_L\,R_C}{\left(\left(h_{fe}+1\right)R_L+h_{ie}+R_C\right)h_{ie}}$$

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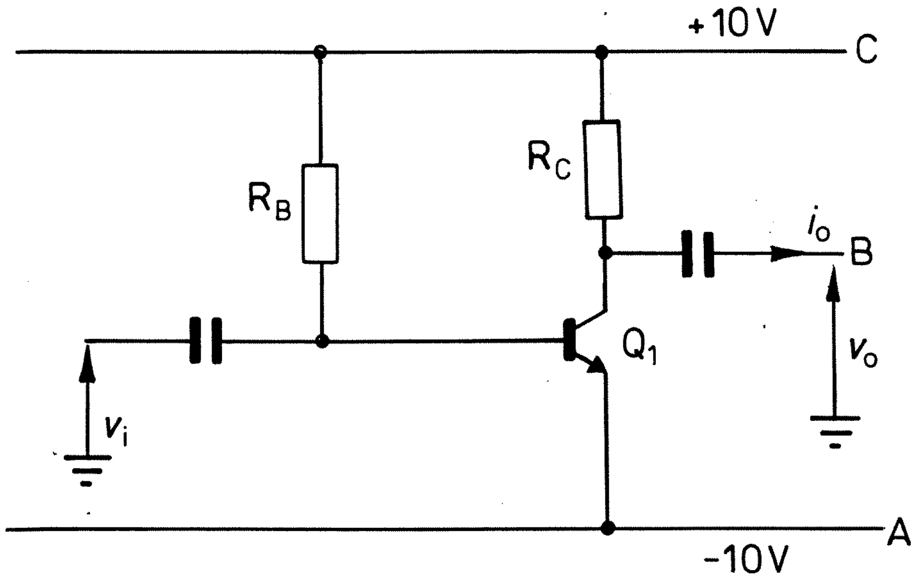

Fig. 1 (a)

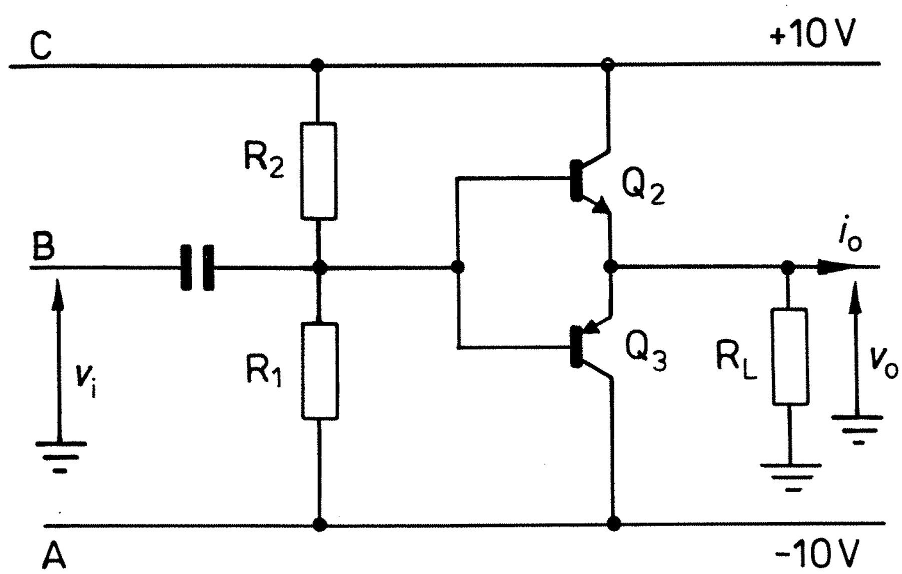

Fig. 1 (b)

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2 Explain the term *common-mode rejection ratio*, and state briefly why a large value is desirable in a differential amplifier.

Figure 2 shows the circuit of a differential amplifier. Draw small-signal equivalent circuits of the differential amplifier for:

(i) differential input signals ;

(ii) common-mode input signals .

You may assume that the small-signal parameters of Q_1 and Q_2 are identical and you may ignore the transistor small-signal parameters h_{oe} and h_{re} . Derive expressions for the differential and common-mode gains of the amplifier and show that the common-mode rejection ratio is given by:

$$\frac{R_B + h_{ie} + 2\left(h_{fe} + 1\right) R_T}{R_B + h_{ie}}$$

The differential amplifier of Fig. 2 is to be designed so that the magnitude of the differential gain is 100 , and the common-mode rejection ratio is 10^4 . Both transistors are to operate with a quiescent collector-emitter voltage of 10 V and a quiescent collector current of 20 mA. At this operating point the transistor small-signal parameters are $h_{ie} = 1\ \text{k}\Omega$ and $h_{fe} = 300$. The quiescent voltage at the point marked A in Fig. 2 is to be 0 V . Resistor R_C is chosen to be 450 Ω . Determine values for resistors R_B and R_T , and power supply voltages V_{CC} and V_{EE} to meet this specification.

Comment on the value of the dc voltage supply required, and explain why it is desirable to replace resistor R_T with a constant current source to achieve a high common-mode rejection ratio. What constant current would such a source have to supply, and what would the theoretical common-mode rejection ratio be if this current source was ideal?

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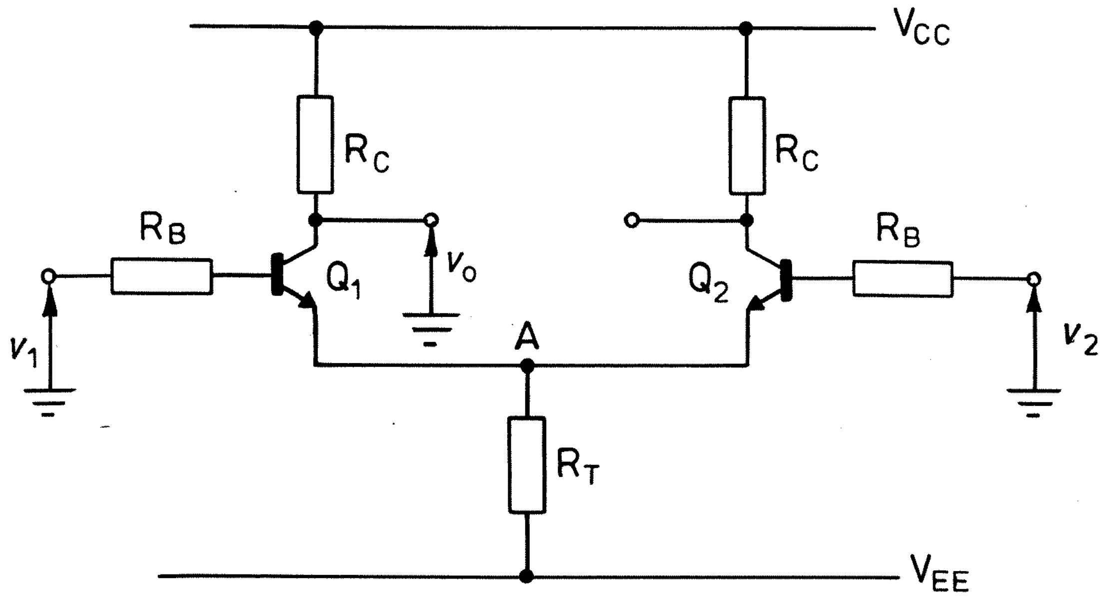

Fig. 2

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SECTION B

3 Give two reasons why electrical power is mainly generated as three-phase alternating voltages.

Working from first principles, derive expressions for the real and reactive power consumed by:

(i) a delta-connected three-phase load of impedance $\overline{Z}_1 = R_1 + jX_1$ per phase;

and

(ii) a star-connected three-phase load of impedance $\overline{Z}_2 = R_2 + jX_2$ per phase,

when connected to a three-phase voltage supply of line voltage V.

Hence show that a delta-connected load of impedance $\overline{Z}_1$ per phase may be transformed to a star-connected load of impedance $\overline{Z}_2$ per phase where

$$\overline{Z}_2 = \frac{\overline{Z}_1}{3}$$

Figure 3 shows a star-connected load (load 1) consisting of a 50 Ω resistor in parallel with a 25 Ω inductive reactance, connected via lines of impedance (5 + j25) Ω to a second load (load 2). Load 2 is delta-connected, and consists of a 150 Ω resistor in series with a 75 Ω capacitive reactance. A 415 V three-phase supply is connected to load 1. By transforming load 2 into a star-connected load, or otherwise, determine the real and reactive power which is supplied by the 415 V supply.

Determine the value of the delta-connected power factor correction capacitors which, when connected in parallel with load 1, would correct the overall load power factor to unity. Determine the real and reactive power consumed by load 2 and hence, or otherwise, calculate the line voltage at load 2.

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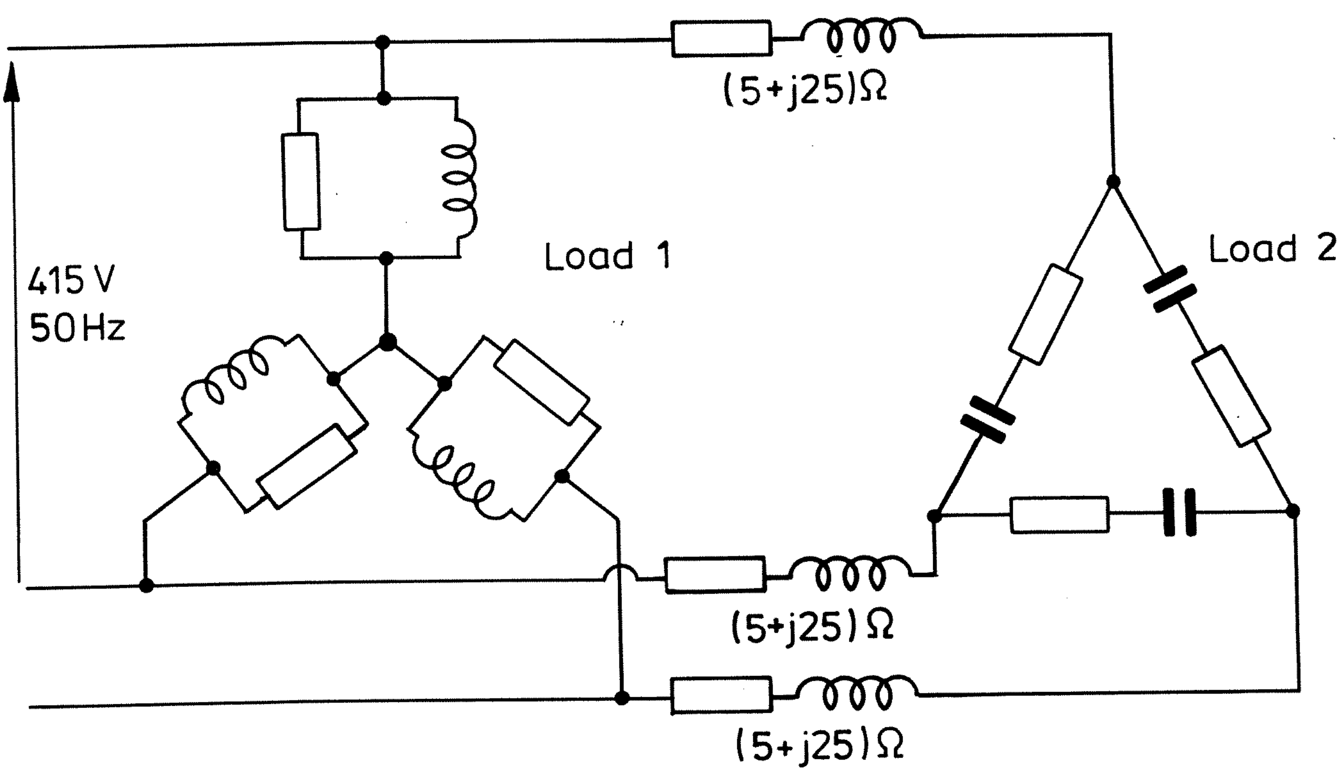

Fig. 3

4 (a) Explain briefly the principle of speed control of dc motors using field current control. Draw the circuit for a dc chopper, and describe briefly how it operates to control the field current of a dc machine.

(b) Explain how an open-circuit test at constant speed on a separately-excited dc machine may be used to obtain the relationship between magnetic flux and field current. Sketch a typical flux vs. field current characteristic, and comment on its shape.

(c) A separately-excited dc motor with armature resistance of $1\ \Omega$ and field resistance of $80\ \Omega$ is tested at 1000 rpm on open-circuit. The field current is varied, and the resulting armature voltage is tabulated below:

i_f	(A)	0	1	2	3	4	5	6
V_a	(V)	0	75	150	210	255	285	300

The motor is then used to drive a load which requires a constant torque of 40 Nm. If the armature and field windings are connected to separate 240 V supplies, determine the speed of rotation of the motor.

(d) A variable-speed drive is required for this load, and it is proposed to use field current control. Accordingly, a dc chopper is introduced between the 240 V field supply and the field winding, whilst the armature voltage supply remains at 240 V. Determine the speed of rotation of the motor if the mark to space ratio of the dc chopper is 1 : 2. If the dc chopper has an efficiency of 80 %, determine the overall drive efficiency.

5 State one advantage of using the per-unit system in the analysis of power supply networks, and derive an expression for base impedance in terms of base voltage V_b and base volt-amps VA_b .

Figure 4 shows a power distribution network, in which a 100 MVA , 20 kV synchronous generator of reactance 0.1 pu supplies a remote load via a 132 kV feeder of impedance (8 + j30) Ω . The generator is connected to the feeder through a 100 MVA , 20 kV/132 kV transformer of reactance 0.12 pu , and the feeder is connected to the load through a 50 MVA , 132 kV/11 kV transformer of reactance 0.07 pu . The load consumes 40 MW at a lagging power factor of 0.8 when connected to an 11 kV supply. The voltage at the load is maintained at exactly 11 kV by controlling the generator voltage. Determine the real and reactive power supplied by the generator, the power dissipated in the feeder and the voltage at the generator.

It is decided to upgrade the feeder from 132 kV to 400 kV operation to reduce power losses in the lines. Two transformers are installed with the following specifications:

(i) 132 kV/400 kV , 100 MVA , 0.1 pu ;
(ii) 400 kV/11 kV , 50 MVA , 0.05 pu .

Draw a diagram of the power supply network showing the locations of the new transformers, and determine the factor by which the feeder losses are reduced. Assume that the load voltage remains constant at 11 kV , and that the impedance of the feeder is unaltered.

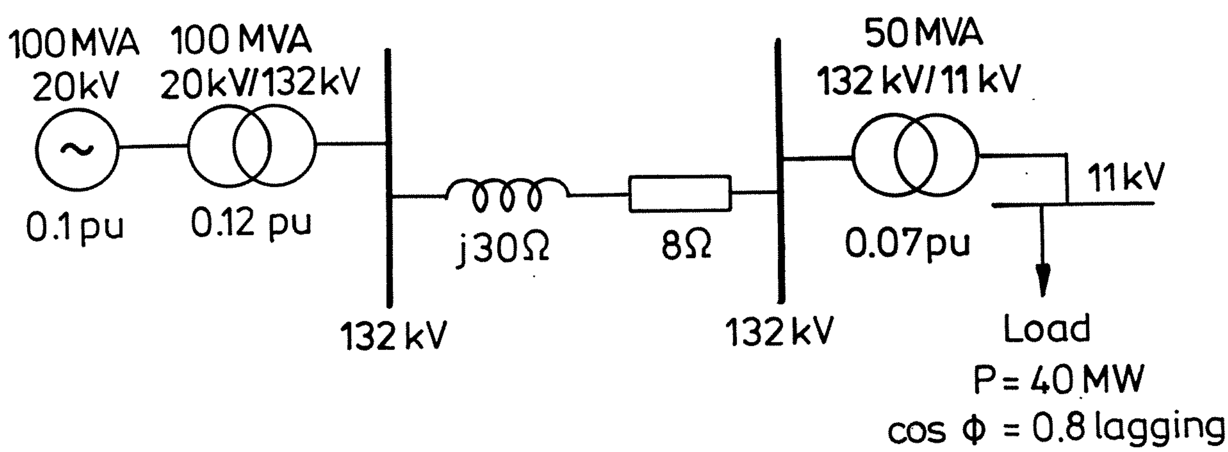

Fig. 4

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SECTION C

6 (a) Figure 5(a) shows the equivalent circuit for an infinitesimally small length, δz, of a lossless transmission line which has inductance and capacitance per unit length of L and C respectively. By applying Kirchoff's current and voltage laws to the circuit, derive two partial differential equations for $V(z, t)$ and $I(z, t)$. By eliminating $I(z, t)$ find a second order partial differential equation for $V(z, t)$ and show that this equation is satisfied by solutions of the form:

$$V(z, t) = f_+(z - vt) + f_-(z + vt)$$

Explain the physical significance of this result and determine v in terms of L and C.

(b) Figure 5(b) shows a dc voltage source of open-circuit emf V_S and source resistance R_S connected via a switch S to a lossless transmission line, which is short-circuited at its far end. The transmission line is of length l, characteristic impedance Z_O, and signals propagate along it with velocity v. Initially switch S is open and the transmission line voltage and current is everywhere zero. At time $t = 0$ the switch S is closed for a short period of time $\tau << l/v$, and then re-opened.

(i) Using the Electrical data book, determine the voltage reflection coefficients at both ends of the transmission line when the switch S is open, and show that when switch S is closed, a voltage pulse propagates along the transmission line of magnitude

$$V_+ = \frac{Z_O}{R_S + Z_O} V_S$$

(ii) If $V_S = 12\ V$, $Z_O = 50\ \Omega$, $R_S = 10\ \Omega$, $l = 6\ km$, $v = 3 \times 10^8\ ms^{-1}$ and the switch S is closed for a period of time $\tau = 1\ \mu s$, draw a graph, *to scale*, of the voltage variation with time at a point midway along the line, for a period of $80\ \mu s$.

Determine also the total energy supplied by the 12 V voltage source.

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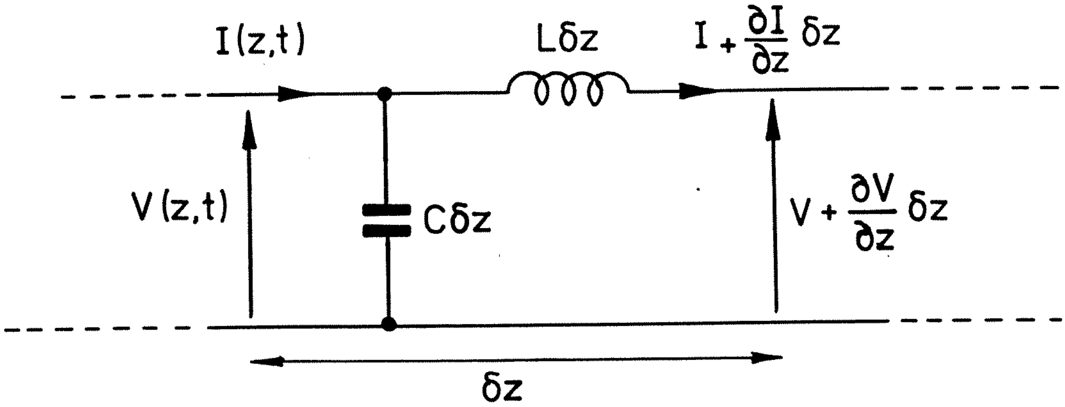
I(z,t)
Lδz
I + ∂I/∂z δz
V(z,t)
Cδz
V + ∂V/∂z δz
δz

Fig. 5 (a)

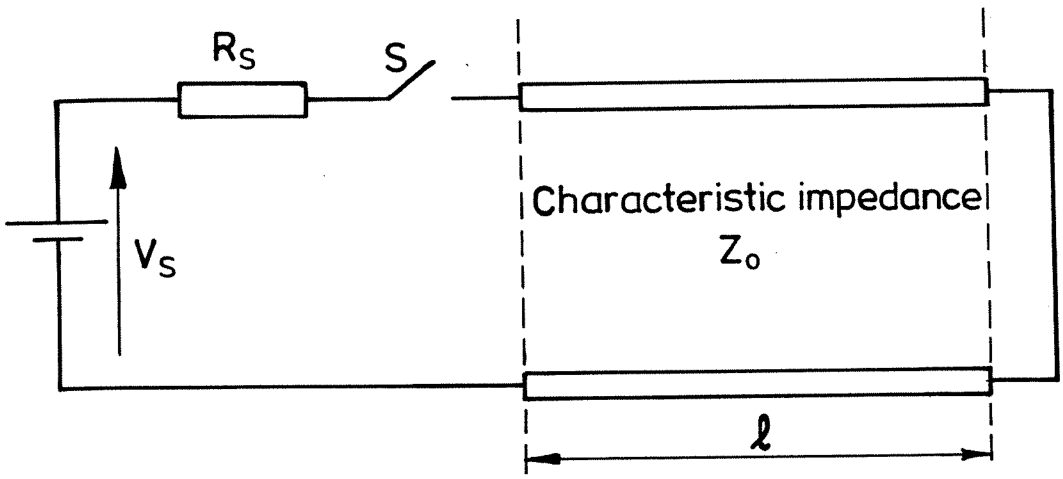
Rs
S
Vs
Characteristic impedance
Zo
ℓ

Fig. 5 (b)

7 Explain the term *impedance of free space* , given by $\eta_0 = \sqrt{\mu_0 / \varepsilon_0}$, as applied to plane electromagnetic waves propagating through free space. The electric field of a plane electromagnetic wave propagating through free space is given by:

$$\bar{E} = u_x E_o \exp j(\omega t - \beta z)$$

Using the complex form of Maxwell's equations, or otherwise, derive an expression for its magnetic field intensity, $\bar{H}$.

Define and explain the significance of the complex Poynting vector and find an expression for it, valid for plane electromagnetic waves, in terms of E_o .

Figure 6 shows a plane electromagnetic wave, of peak electric field strength E_i , propagating through a region of air (*region 1*) and then impinging normally on a flat slab of material (*region 2*) . On the far side of the slab is a further region of air (*region 3*) . The flat slab of material is of finite dimension in the direction of propagation of the wave, but is infinite in extent in the other two coordinate directions. The material has a relative permeability of μ_r , a relative permittivity of *1* , and is lossless .

(i) By analogy with transmission line theory, or otherwise, derive expressions for the reflected and transmitted electric field at the boundary between *regions 1* and *2* , and show that the electric field which propagates into *region 3* is given by

$$E = \frac{4\sqrt{\mu_r}}{\left(1+\sqrt{\mu_r}\right)^2} E_i$$

(ii) Hence show that the power per unit area which propagates through the slab and into *region 3* is

$$\frac{8\,\mu_r}{\left(1+\sqrt{\mu_r}\right)^4 \eta_0} E_i^2$$

(iii) If $\mu_r = 100$, find the proportion of the incident power per unit area in *region 1* which propagates into *region 3* .

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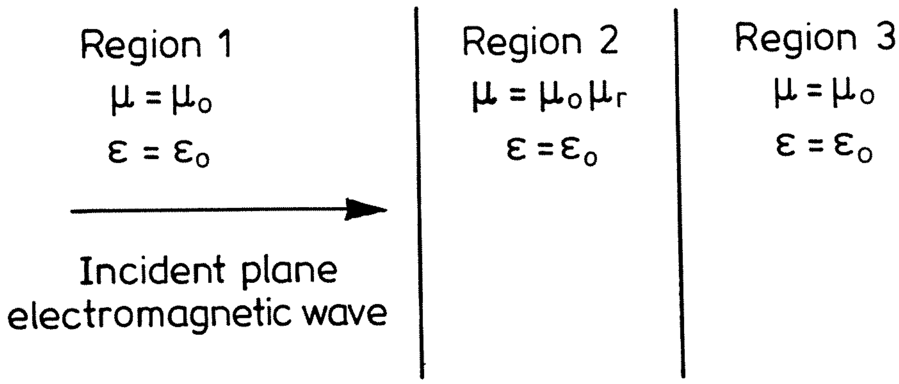

Fig. 6

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