ENGINEERING TRIPOS PART IB

Thursday 6 June 1996 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than **four** *questions.*

Answer at least **one** *question from each section.*

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SECTION A

1 Figure 1 shows a feedback control system in which the output $y(t)$ is desired to follow the reference signal $r(t)$ in the presence of a disturbance $d(t)$ which acts at the input of the plant. The plant has the transfer function $1/s(s+2)$ and a proportional controller is used as shown.

(a) Assuming zero initial conditions, find an expression for $\bar{y}(s)$ in terms of $\bar{d}(s)$ and $\bar{r}(s)$. Hence write down the transfer functions from $\bar{d}(s)$ to $\bar{y}(s)$ and from $\bar{r}(s)$ to $\bar{y}(s)$ in their simplest forms.

(b) Derive an expression for the damping ratio of the closed-loop system in terms of K_p.

Choose K_p such that the closed-loop system is critically damped. What then is the steady-state error (that is, $\lim_{t\to\infty}\left(y(t)-r(t)\right)$) in each of the following cases:

(i) $r(t) = H(t)\,,\ d(t) = 0\,;$

(ii) $r(t) = 0\,,\ d(t) = e^{-3t}\,;$

(iii) $r(t) = 0\,,\ d(t) = H(t)\,;$

(iv) $r(t) = tH(t)\,,\ d(t) = 0\,;$

where $H(t)$ denotes the unit step function.

(c) How should the controller be modified if it is desired that the control system should reject the effect of constant disturbances, as in (b) part (iii)?

How would such a modification improve the tracking ability of the control system even in the absence of any disturbances?

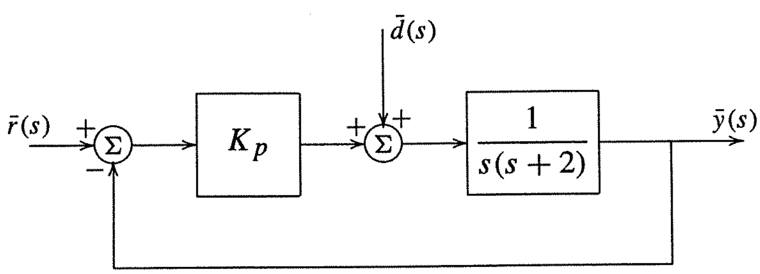

Fig. 1

2 Figure 2 shows a boat, whose heading angle θ is related to the rudder angle α by the differential equation

$$T\ddot{\theta} + \dot{\theta} = K\alpha$$

An automatic steering system is implemented which applies a 'proportional plus derivative' control law:

$$\alpha = K_p e + K_d \dot{e}$$

where e is the error between the desired heading θ_d and the heading indicated by the compass, θ_c. Waves induce motion of the boat, which causes a spurious 'noise' signal n to appear in the compass output. A filter is introduced to reduce this, so that θ_c is related to θ and n by:

$$\bar{\theta}_c(s) = \frac{1}{1 + sT_f}\left(\bar{\theta}(s) + \bar{n}(s)\right)$$

(a) Draw a block diagram of the automatic steering system, showing clearly the transfer functions of the boat, the control law and the filter.

(b) Find the closed-loop characteristic equation of the system.

If $K = 0.2\ \mathrm{s}^{-1}$, $T = 2\ \mathrm{s}$, and the three closed-loop poles are to be located at -0.4, $-0.3 \pm 0.3j$, find the required values of K_p, K_d and T_f.

(c) Assuming the values found in part (b), find the closed-loop transfer function from the noise n to the rudder angle α. If n is a sine wave of amplitude 5 degrees and frequency $6\ \mathrm{rad\ s}^{-1}$, and the desired heading θ_d is constant, what are the approximate amplitudes of oscillation of the rudder angle α and the heading angle θ?

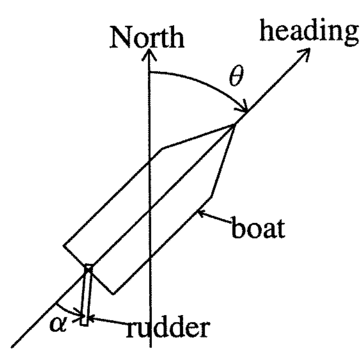

Fig. 2

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3 In the feedback system of Fig. 3, the plant $G(s)$ has the transfer function:

$$G(s) = \frac{30(3s+1)}{(s^2+0.02s+0.01)(s+3)(s+10)}$$

The Bode diagram of $G(s)$ is given as Fig. 4.

(a) Consider first the use of a proportional controller

$$K(s) = K_p.$$

It is required that $|K_pG(j\omega)| > 1$ for all frequencies $\omega < 0.2$. Subject to this constraint, estimate the value of K_p for which the phase margin is greatest. What is the gain margin for this value of K_p ?

(b) In order to improve the low frequency performance, a phase-lag compensator

$$K(s) = K_1 \frac{s+\alpha}{s+0.02}$$

is to be used instead. How large can α be, and what is the corresponding value of K_1 , if it is required that the gain crossover frequency (that is, the frequency for which $|K(j\omega)G(j\omega)| = 1$) is unchanged from part (a), and that the phase margin is not decreased by more than 5° ?

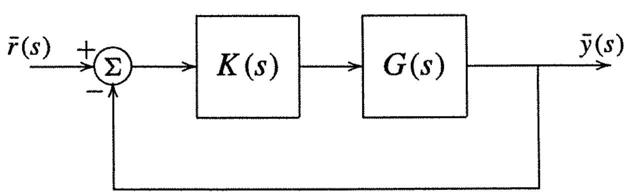

Fig. 3

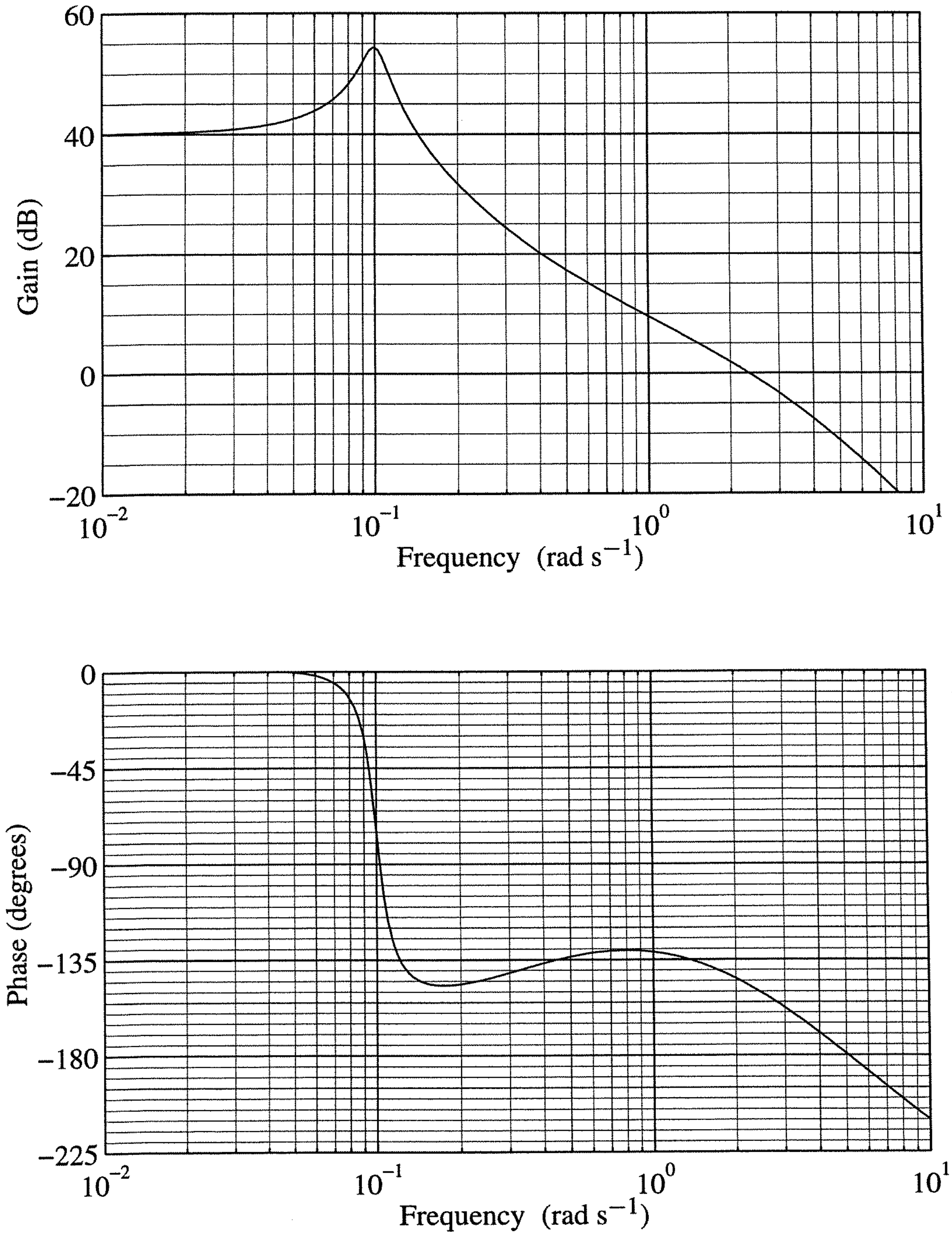

Fig. 4 Bode Diagram for Question 3

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4 (a) State carefully the Nyquist stability criterion as it applies to the specific case of the feedback system of Fig. 5 if $K = 1$ and all the poles of the transfer function $G(s)$ lie in the left half plane.

(b) Figure 6 shows the Nyquist diagram of a stable system $G(s)$, which is to be controlled as in Fig. 5.

(i) For what range of values of K is the transfer function

$$\frac{KG(s)}{1+KG(s)}$$

stable?

Show, on an Argand diagram, the location of two of the poles of this transfer function when K is at the upper end of this range.

(ii) Sketch, against frequency, the magnitude of the frequency response of the transfer function

$$\frac{KG(s)}{1+KG(s)}$$

when K is chosen such that the gain margin of the closed-loop system is 2. Pay particular attention to estimating the frequencies for which this transfer function has modulus greater than 1, and the frequency at which it attains its maximum modulus.

Draw a sketch of the estimated closed-loop step response to illustrate its most important features.

An extra copy of Fig. 6 is provided, and should be handed in with your answer if constructions are made on it.

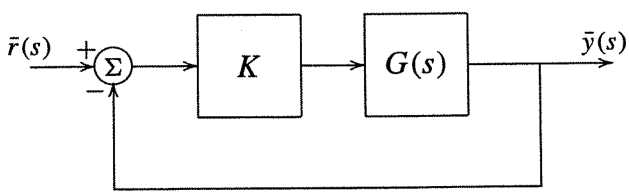

Fig. 5

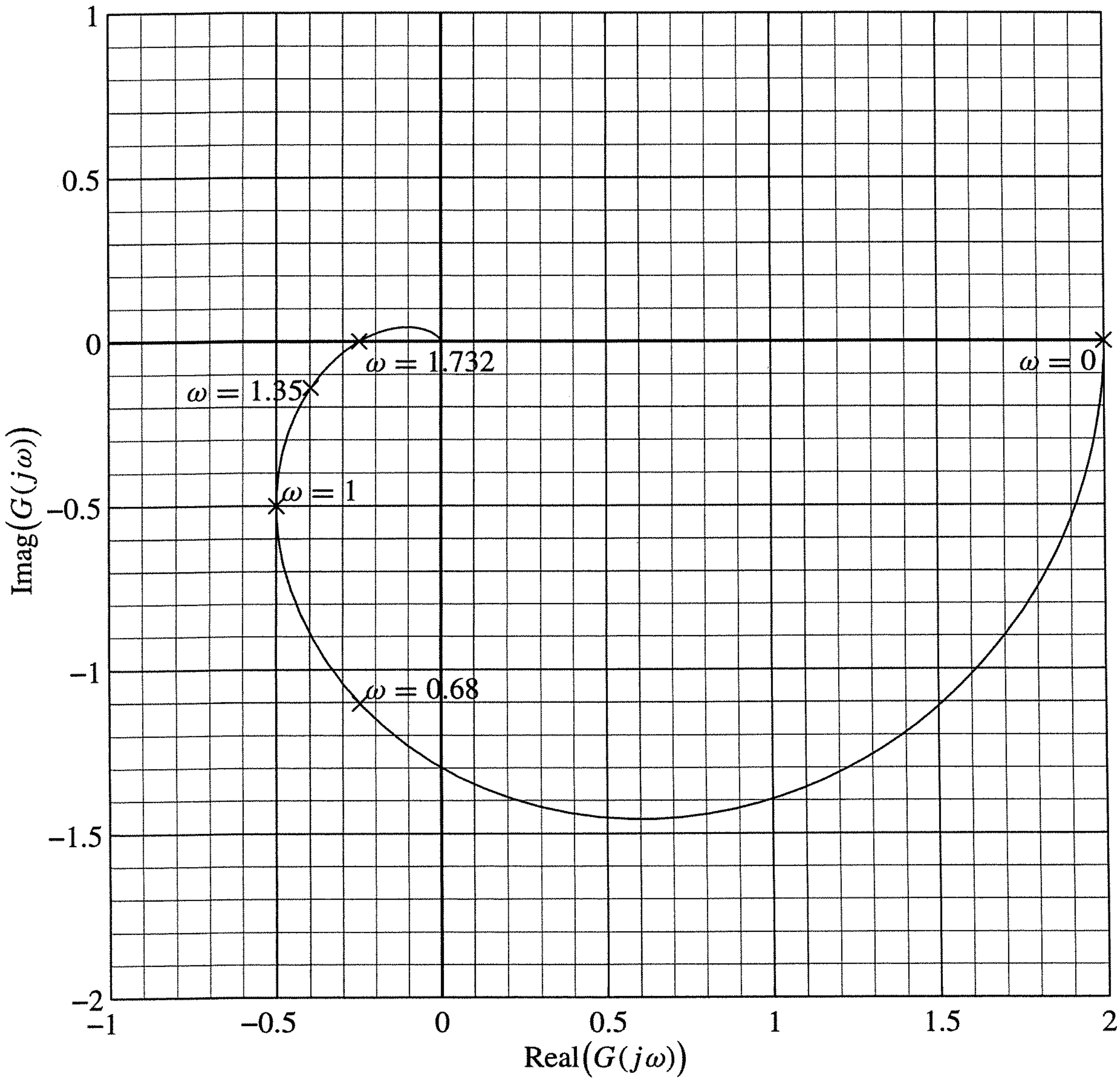

Fig. 6 Nyquist Diagram for Question 4

SECTION B

5 (a) Explain why it is often necessary to use modulation before an analogue signal can be sent along a transmission path. Discuss the benefits and drawbacks of common modulation techniques, including requirements placed on the receiving device. What benefits can discretization of the signal prior to modulation bring?

(b) Select, and justify, an appropriate transmission method in each of the following cases:

(i) transmission of a voice signal along a standard telephone line;

(ii) transmission of a medium quality audio signal over a medium wave radio channel;

(iii) transmission of computer data at $50,000$ bits/s over a VHF radio channel.

Back up your argument with calculation where necessary.

6 (a) A particular A/D converter is capable of sampling at 12 bit resolution at a sampling frequency of 50kHz, or at 16 bit resolution at a sampling frequency of 37.5kHz.

(i) What bit rate is required to transmit the output of the A/D converter on each setting.

(ii) What method of conversion do you think it is using, and what characteristic of the device is responsible for these limitations?

(b) The A/D converter of part (a) is to be used to discretize an audio signal, in conjunction with an anti-aliasing filter whose transition band is 25% of the width of its pass band and whose cut-off frequency is variable. For each setting of the A/D converter, calculate:

(i) the maximum achievable signal to noise ratio (in dB) when the input is a sinusoid;

(ii) the maximum possible signal bandwidth that can be converted without loss of information due to aliasing.

END OF PAPER