**1997 Part IB Paper 1**

**Answers**

1 a) $m\frac{a^2+b^2}{4}$ $m\frac{a^2+b^2}{4}$

b) 12.5 Nm east

2 a) 0.0349 mN acting left to right

b) angle BA is 90°
angle AC is 112.6°

1562 Nm

3 (i) $\frac{V^2}{R}$ towards centre of path

(ii) $\left(-\frac{V^2}{R} - 2a\omega\,\dot{\theta}\cos\theta\right)\mathbf{e}_r - a\omega^2\sin\theta\mathbf{e}_1 - a\dot{\theta}^2\mathbf{e}_2$

(iii) $(-1 - 0.4)\,\mathbf{e}_r - 10\mathbf{e}_2$ m/s$^2$ (the "0.4" is coriolis)

$-\mathbf{e}_r - 0.004\mathbf{e}_1 - 10\mathbf{e}_n$ m/s$^2$ (no coriolis)

(iv) Coriolis is 4% of weight
Centripetal is 100% of weight

4 (i) $a\,\mathbf{e}_1 + a\,\mathbf{e}_2$

$a\omega\mathbf{e}_1^* + a(\omega+\dot{\theta})\mathbf{e}_2^*$

$a\dot{\omega}\mathbf{e}_1^* - a\omega^2\mathbf{e}_1 + a(\dot{\omega}+\ddot{\theta})\mathbf{e}_2^* - a(\omega+\dot{\theta})^2\mathbf{e}_2$

(ii) $\ddot{\theta} = \dot{\omega}(\cos\theta - 1) + \omega^2\sin\theta$

5 $\frac{\omega}{2\sqrt{3}}$ clockwise

$\frac{mg}{2} - m\omega^2 a\left(\frac{9\sqrt{3}+4}{54}\right)$

3.68 rad/s

6 (i) 10 Ns

(ii) 10 rad/s clockwise

(iii) 2 m/s left

(iv) 50 Ns

(v) Friction has to be a small effect during the impulse $I_1$.

**1997 Part IB Paper 2**

**Structures**

**Answers**

1. a) 831 875 $mm^4$

   c) ii) 0.11 degrees

2. a) i) Cylinder: $\sigma_h = 200\text{MPa}$, $\sigma_l = 100\text{MPa}$, $\sigma_r = 0$

   Domes: $\sigma_{m1} = \sigma_{m2} = 250\text{MPa}$, $\sigma_r = 0$

   ii) Cylinder: $\varepsilon_h = 486 \times 10^{-6}$, $\varepsilon_l = 2386 \times 10^{-6}$, $\varepsilon_r = -1414 \times 10^{-6}$

   Domes: $\varepsilon_{m1} = \varepsilon_{m2} = 2393 \times 10^{-6}$, $\varepsilon_r = -2357 \times 10^{-6}$

   b) i) 126 kNm

   ii) 164 kNm

3. a) $PL/M_p = 5.5$, (1)

   $QL/M_p = 6.8571$ (2)

   $0.6\,(PL/M_p) + 0.35\,(QL/M_p) = 4.1$ (3)

   b) W = 511 kN ((3) governs)

4. b) 225 kPa

5. a) 6246 kN

   c) $\delta$=12mm at 6000 kN

6. b) 0.74 mm

**1997 Part IB Paper 3**

**Materials**

**Answers**

1. (d) (i) Guaranteed lifetime is about 20 years and about 10% would fail the proof test.

   (ii) Reduce the working stress to 25 MPa (for R=2.5)

4 (a) $P_{ave} = \sigma_y \left[ 1 + \frac{mw}{4h} \right]$

  (b) $F = 13.8 \sigma_y h$

  (c) (ii) increasing w/h by reducing h increases force F.

5. (c) (i) 285 C; 30% martensite + retained austenite
   (ii) 375 C; 100% lower bainite
   (iii) 465 C; 75% upper bainite + retained austenite
   (iv) 645 C; primary ferrite + 100% fine pearlite
   (v) 690 C; primary ferrite + 75% pearlite + retained austenite
   (vi) 825 C; all austenite
   (vii) 915 C; all austenite.

   A water quench after 2 hours would result in any austenite phase transforming to martensite, the remaining phases unaffected.

6. (a) $M_1 = \left[ \frac{1}{A\alpha R} \right] \left[ \frac{1}{\rho} \right]$

  (b) Maximise $M_2 = \frac{\sigma_f}{\rho}$

  (e) Consider a carbon fibre blade.

**1997 Part IB Paper 4**

**Answers**

1 a) $\frac{4}{\pi d^2} \cdot \frac{M-\rho V}{\rho}$

b) $\frac{d}{4}\sqrt{\frac{\rho g}{\pi M}}$ (Hz)

2 b) 3.72m/s, 2.82m

c) 90kN in downstream direction

3 b) $\frac{\pi g R^4 (\rho_s - \rho_w)}{8\mu}$

c) $\frac{g^{1/2} R^{3/2} \rho_w}{\mu}$

4 b) $V(r) = \frac{V_0 r_1}{r}$

c) $h = r_1 \ln\left(\frac{r_2}{r_1}\right)$

d) $\frac{1}{2}\rho V_0^2\left[1 - {r_1^2}/{r_2^2}\right]$, $\frac{1}{2}\rho V_0^2\left[r_2^2 - r_1^2\right]$

5 c) 2230K

6 b) 372W, 496K

c) 669K

**Cambridge University Engineering Department**
**1997 Part 1B Paper 5 "Electrical Engineering"**
**ANSWERS**

1 (a) (i) -196 (ii) 714 Ω (iii) 1961 Ω

(b) (i) -5.83 (ii) 2330 Ω

2 (b) (i) R = 1 kΩ

(ii) open-loop gain: $\dfrac{4\times 10^4}{1+j\dfrac{f}{120}}$ ; closed-loop gain: $\dfrac{4\times 10^4}{3078+j\dfrac{f}{120}}$ ; -3 dB frequency: 369.36 kHz

(c) (i) -6.8 (ii) -47.6

3 (b) (i) 13,122 A (ii) 9067 V (iii) 20.3 ° last part: 27.6 °

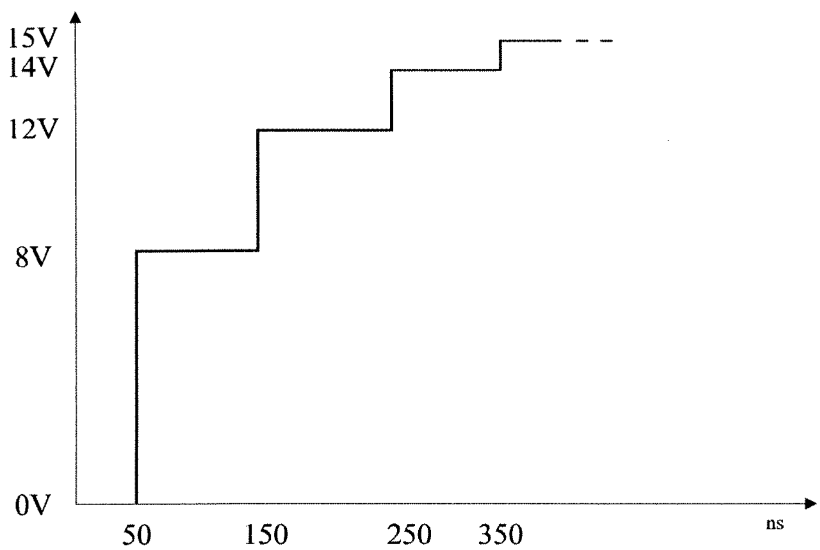
4 (b) (i) 4006 A (ii) 1002 A (iii) 229 MVA last part 0.7 Ω

5 (b) (i) $X_m$ = 184 Ω, $R_0$ = 861 Ω, $R_2'$ = 3.2 Ω, $X_1 = X_2'$ = 1.95 Ω

(c) (i) 0.01333 (ii) (47.9+$j$ 70.1) Ω (iii) 8.5 A (iv) 20.9 Nm

6 (a) characteristic impedance: 50 Ω Transmission frequency: 781.25 kHz

(b) first incident wave: 8V (i) 1.28 W (ii) 1.28 W (iii) 0 W




7 (a) 6.37 pW $m^{-2}$ (i) 12.7 pW (ii) 0.504 μA

Answers: IB Paper 6, Information Engineering, 1997

1. a) $\dfrac{4(s-1)}{(s+1)^2}$

   b) Open-loop unstable, poles at $s = -3, 1$. Closed-loop asymptotically stable, poles at $s = -1, -1$.

   c) $-4 + 4e^{-t} + 8te^{-t}$, $\dot{y}(0) = 4$, $\lim_{t\to\infty} y(t) = -4$.

2. $k_p = 5$, $k_i = 2/3$. max speed deviation= $-1.612$.

3. $2.76 < k_p < 5.81$; $k_p > 3.48$.

4. a) GM=5, PM=51°.

   b) PM=53°.

5. a) 1.408 Mbit/s.

   b) 6.82 times.

   c) 4.8MHz.

6. b) 2659 channels.

**ENGINEERING TRIPOS PART IB 1997**

**PAPER 7 MATHEMATICAL METHODS**

**ANSWERS AND HINTS**

1. $a = 3$, $b = 3/2$, or *vice versa*, $\phi = \sin(x + 3y)$.

2. Identities required are $\nabla \times (\boldsymbol{u}_1 \times \boldsymbol{u}_2)$, $\nabla \times (\phi \boldsymbol{u})$.
Also note $\nabla r^n = n r^{n-1} \nabla r = n r^{n-2} \boldsymbol{r}$
$\nabla(fg) = f\nabla g + g\nabla f$ is useful for last part.

3. (ii) Note that, if $\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a} \cdot \boldsymbol{c}$, $\boldsymbol{b} \neq \boldsymbol{c}$ in general.
(iii) Define $\bar{\boldsymbol{r}}$ rigorously in terms of $\boldsymbol{r}$ first.

4. $\phi = 8\ell n\, x + 2$
Using central differencing for interior points and backward differencing for $d\phi/dx$ at $x = 4$, the numerical solution is $\phi_1 = 2{\cdot}00$, $\phi_2 = 7{\cdot}55$, $\phi_3 = 10{\cdot}79$, $\phi_4 = 13{\cdot}09$

5. a) $\alpha \Delta t / \Delta x^2 = 1/6$ (Note that $\partial^2 u / \partial t^2 = \alpha \partial^4 u / \partial x^4$ ).
b) $A \Delta t / \Delta x < 1$ (Note that disturbance propagates downwind only).

6. For the discrete representation:
$|F(o)| = 1{\cdot}0$, $|F(\pi)| = 0{\cdot}6533$, $|F(2\pi)| = 0$, $|F(3\pi)| = 0{\cdot}2706$, $|F(4\pi)| = 0$,
(Other values are symmetrical).
(Do not forget to multiply the $F_k$ by the sampling time).

7. For the second part, direct integration is most straightforward, noting that $F(2\pi k/T)$ is not a function of $t$. For the last part, note that the equation is valid for <u>all</u> $\omega$. Hence, if $F(\omega)$ is sampled at intervals of $\Delta\omega = 2\pi/T$, $F(2\pi k/T)$ can be calculated for all $k$ and $F(\omega)$ reconstructed in its entirety

8. Construct a tree diagram to find the probability that the game ends with $n$ bags and hence that

$$m(t) = \sum_{n=2}^{\infty} \frac{(n-1)}{n!} t^n$$

Then show that this is the same as the given expression.
Finally, mean $(N) = e$ , var $(N) = 3e - e^2$.

**1997 Part IB Paper 8**
**Solutions**

**1.** (a) 1820 kg/m$^3$, 0.99. (b) 39kPa, 133kPa (d) (i) 44.9°, 35.7°
(ii) 60kPa

(e) 10kPa, -7kPa

**3.** (a) T = 360kN
Mm... = 596.5kNm
(b) 316mm

**4.** (b) ~ 0.065 kym (d) ~ 12 Mz for a 1% speed fluctuation

**5.** **(b)** Lowpass ~ 72Hz
Highpass ~ 3.4 Hz
(c) range ~ 0.5 - 2.5 kNm$^{-1}$ depending on choice of c and 3db frequency

**6.**

**7.** (c) About 40:1

**8.** (b) 61.2° (c) 69.9° (d) 1084 K, 3.16 (e) 2.4

**9.** (b) .88, 259.8K (c) (i) 514.3 & 92% (ii) 1.49 kg/s (d) 3.3 kg/s

**11.** (c) $Kg = \frac{Dg}{\delta g}$, $Ke = \frac{D\ell}{\delta \ell}$

(d) 9 x 10$^{-6}$m, 1.2 x 10$^{-2}$ms$^{-1}$

**12.** (c) h = 3.2m

**13.** Resolution ~ 0.4$\mu$m, Depth of focus ~ 2.77$\mu$m

**14.** $\approx 10^{-8}$m

**15.** L = 1$\mu$, C = 2 x 10$^{-15}$F, R = 10$^3\,\Omega$