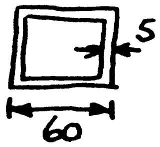
PART IB STRUCTURES, Paper 2, 1997.

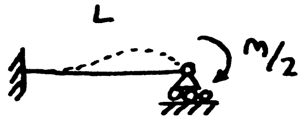
Q1. a) 

$J = \dfrac{4A_e^2}{\oint \frac{ds}{t}}$ $A_e = (55)^2 = 3025 \text{ mm}^2$

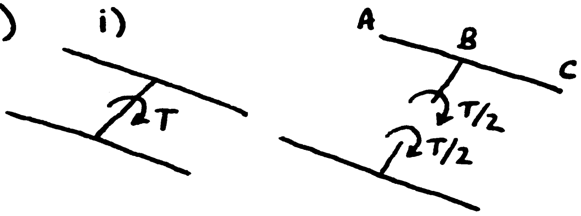
$\oint \dfrac{ds}{t} = \dfrac{4(55)}{5} = 44$

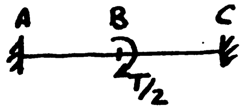
$$J = \frac{4(3025)^2}{44} = \underline{\underline{831\,875 \text{ mm}^4}}$$

(c.f. 86·3 cm^4 Data Book, p17)

b) 

rotation $= \dfrac{M}{2}\,\dfrac{L}{4EI}$ (Databook, p6).

c) i) 



Rotation at B $= \dfrac{T}{2}\left(\dfrac{L}{8EI}\right)$ from pt. b).

Rotation along BD, gives using "$T = GJ\frac{\alpha}{L}$" from data book, p5

$$\alpha = \frac{T}{2}\frac{(L/2)}{GJ}$$

$$\therefore \quad \text{Total} = \frac{TL}{16}\left(\frac{1}{EI} + \frac{4}{GJ}\right)$$

ii) $b = 80\text{mm}$, $t = 6\text{mm}$ $I = \dfrac{b^3 t}{12} = \dfrac{(80)^3 6}{12} = \underline{256\,000 \text{ mm}^4}$

Steel: $E = 210 \text{ GPa} = 210 \times 10^3 \text{ N/mm}^2$
$G = 81 \text{ GPa} = 81 \times 10^3 \text{ N/mm}^2$ } Structures Data Book, p1

$T = 1 \text{ kNm} = 1 \times 10^6 \text{ Nmm}$, $L = 400 \text{ mm}$

$$\therefore \quad \alpha_T = \frac{(1 \times 10^6 \text{ Nmm})(400\text{mm})}{16}\left(\frac{1}{210\times10^3 \cdot 256\times10^3} + \frac{4}{81\times10^3 \cdot 831{\cdot}9\times10^3}\right)$$

$\dfrac{1}{\text{N/mm}^2 \cdot \text{mm}^4}$

$$= \frac{400}{16}\left(\frac{1}{210\times256} + \frac{4}{81\times831{\cdot}9}\right)$$

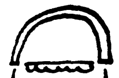
$$= 25\,(1{\cdot}8601\times10^{-5} + 5{\cdot}9361\times10^{-5})$$

$$= 25\,(7{\cdot}7962\times10^{-5}) = 0{\cdot}0019 \text{ radians}$$

$$= \underline{\underline{0{\cdot}11 \text{ degrees}}}$$

Q2.

a) i) STRESSES.

cylinder: hoop 

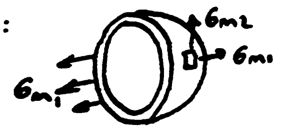
$\sigma_h t = pr$; $\sigma_h = \frac{pr}{t} = 5000\ \mathrm{kN/m^2} \times \frac{200}{5}$

$= \underline{\underline{200\ \mathrm{MPa}}}$

longit: 

$p\pi r^2 = \sigma_L 2\pi r t$; $\sigma_L = \frac{pr}{2t} = \underline{\underline{100\ \mathrm{MPa}}}$

through-the-thickness $\underline{\underline{\sigma_r = 0}}$ (usual thin-walled body assumption)

domes: 

$\sigma_{m_1} = \frac{pr}{2t} = \frac{5000\ \mathrm{kN/m^2} \times 200}{2 \quad\quad 2} = \underline{\underline{250\ \mathrm{MPa}}}$

$\sigma_{m_2} = \sigma_{m_1} = \underline{250\ \mathrm{MPa}}$

$\underline{\sigma_r = 0}$ through-the-thickness.

These are all principal stresses.

(Slightly more accurate answers can be obtained if one distinguishes between internal, external and mid-wall diameters but for thin-walled assumptions, this is unnecessary .)

ii) STRAINS

Using $\varepsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{yy} - \nu\sigma_{zz})$ Data Book, p1.

Al. alloy; $E = 70\ \mathrm{GPa}$, $\nu = 0{\cdot}33$ Data Book, p1.

CYLINDER:

$\varepsilon_{longit} = \frac{1}{70 \times 10^3\ \mathrm{MPa}}\left(100 - 0{\cdot}33(200)\right)\mathrm{MPa} = \underline{486 \times 10^{-6}}$ (DIMENSIONLESS) (= "486 microstrain")

$\varepsilon_{hoop} = \frac{1}{70 \times 10^3}\left(200 - 0{\cdot}33(100)\right) = \underline{2386 \times 10^{-6}}$

$\varepsilon_r = \frac{1}{70 \times 10^3}(-0{\cdot}33)(100 + 200) = \underline{-1414 \times 10^{-6}}$

DOMES:

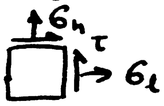
$\varepsilon_{m_1} = \frac{1}{E}\left(\sigma_{m_1} - \nu\sigma_{m_2} - \nu.0\right) = \frac{(1-\nu)\sigma_m}{E} = \frac{0{\cdot}67\ (250\ \mathrm{MPa})}{70 \times 10^3\ \mathrm{MPa}}$

$= \underline{2393 \times 10^{-6}}$

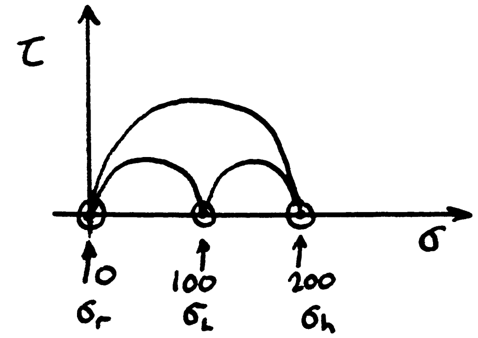
$\varepsilon_{m_2} = \varepsilon_{m_1} = 2393 \times 10^{-6}$

$\varepsilon_r = \frac{1}{70 \times 10^{+3}}(-0{\cdot}33)(250 + 250) = \underline{-2357 \times 10^{-6}}$

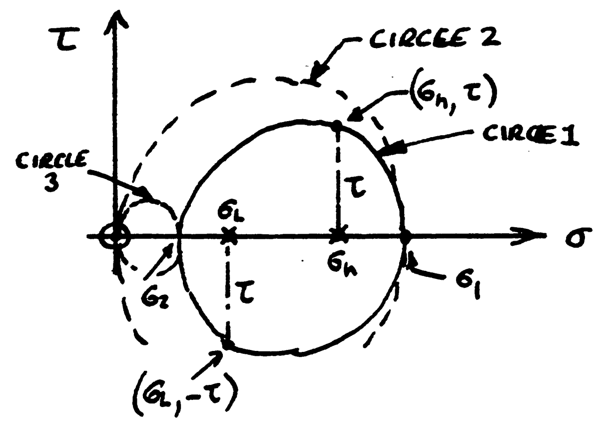
Q2 (cont'd.).

b) Torques only affect cylindrical body, causing shear stresses on  hoop/longit direction.

ORIGINAL MOHR'S CIRCLES (at 5000 kPa)



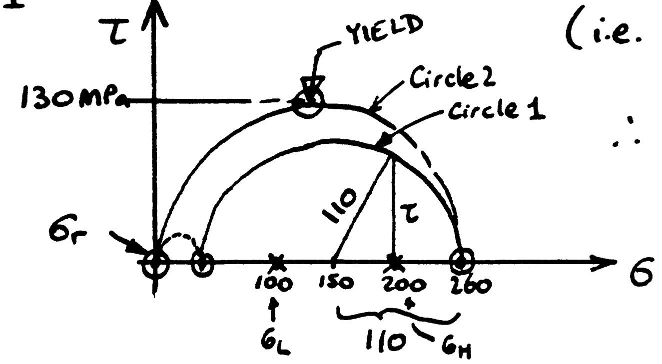
MOHR'S CIRCLES WITH TORQUE.



Now $\sigma_y = 260$ MPa, $\therefore$ $\tau_{max} = 130$ MPa (Tresca).

i) <u>At 5000 kPa pressure,</u> centre of circle 1 is at 150 MPa $\left(=\frac{\sigma_L+\sigma_H}{2}\right)$

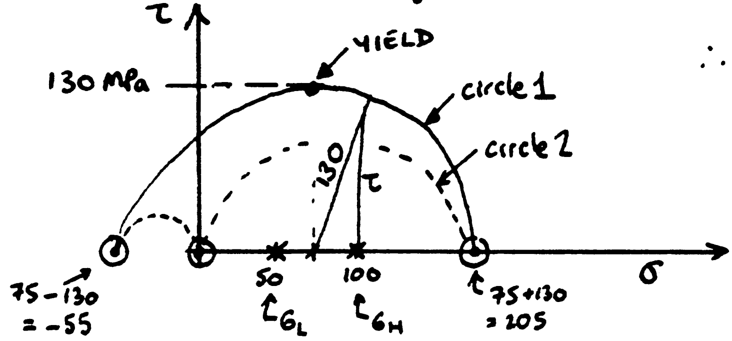
$\therefore$ Radius of Circle 1 can only grow to 110 MPa before σ_1 reaches 260 MPa (i.e. circle 2 reaches τ_{max})



$\therefore$ $\tau = \sqrt{110^2 - 50^2} = \underline{\underline{98 \text{ MPa}}}$

ii) <u>At 2500 kPa</u>, centre of circle 1 is at 75 MPa

$\therefore$ Radius of circle 1 can grow to full 130 MPa.



$\therefore$ $\tau = \sqrt{130^2 - 25^2} = \underline{\underline{127.6 \text{ MPa}}}$

$T = 2\pi r^2 t \tau$

$2\pi r^2 t = 2\pi (202.5)^2 (5)$

$= 1.288 \times 10^6 \text{ mm}^3$

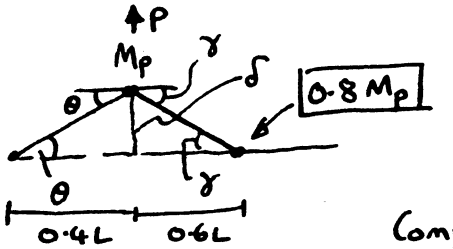
$\therefore$ i) $T = 98 \times 1.288 \times 10^6$ Nmm $= \underline{126 \text{ kNm}}$

ii) $T = 127.6 \times 1.288 \times 10^6$ Nmm $= \underline{164 \text{ kNm}}$

Q3

a)

Mech 1:



Ext. W.D. $= P.\delta$

Int. W.D $= M_p(\theta+\gamma) + 0.8M_p\gamma$

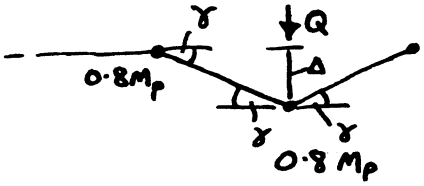
Compatibility: $\theta = \dfrac{\delta}{0.4L}$ $\quad \therefore\ \delta = 0.4L\theta = 0.6L\gamma$

$\therefore\ \theta = \dfrac{0.6}{0.4}\gamma = 1.5\gamma$

Equating Int. with Ext. $\quad P(0.6L\gamma) = M_p(1.5\gamma + \gamma) + 0.8M_p\gamma = 3.3M_p\gamma$

$$\therefore\ \frac{PL}{M_p} = \frac{3.3}{0.6} = 5.5 \qquad (1)$$

Mech 2:



Ext. W.D $= Q.\Delta$

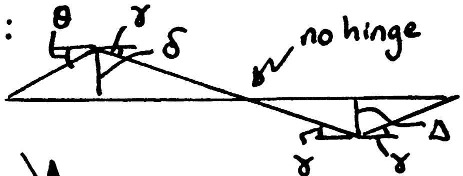
Int. W.D $= 0.8M_p(3\gamma)$

Compat: $\gamma = \dfrac{\Delta}{0.35L}$ $\quad \therefore\ \Delta = 0.35L\gamma$

$\therefore\ Q(0.35L\gamma) = 2.4M_p\gamma$

$$\therefore\ \frac{QL}{M_p} = \frac{2.4}{0.35} = 6.8571 \qquad (2)$$

Mech 3:

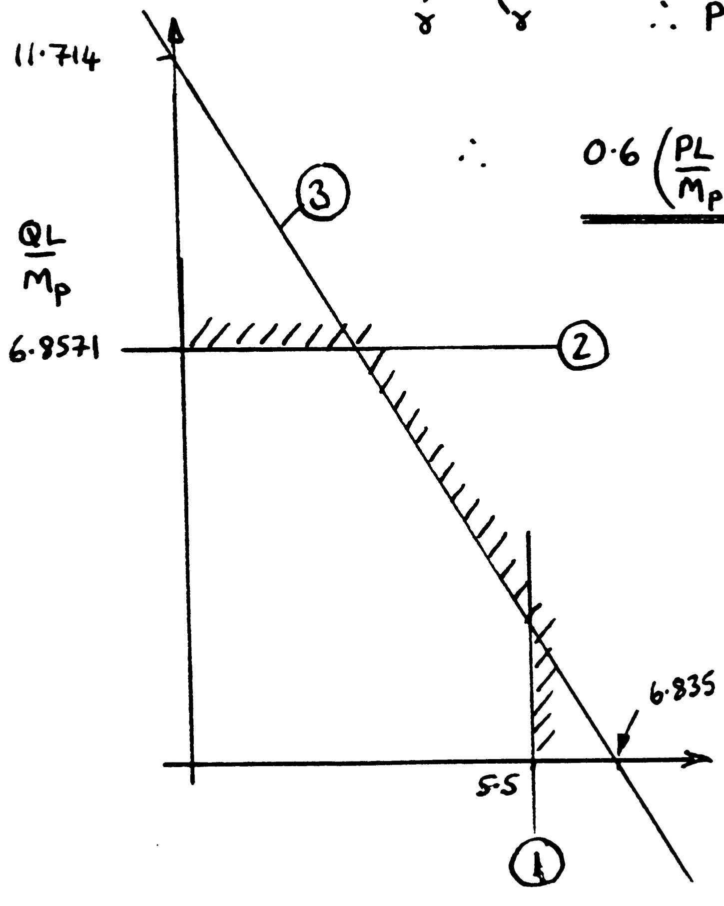


Ext. W.D. $= P\delta + Q.\Delta$

Int. W.D. $= M_p(\theta+\gamma) + 0.8M_p(2\gamma)$

$\therefore\ P(0.6L\gamma) + Q(0.35L\gamma) = M_p(2.5\gamma) + 1.6M_p\gamma$

$$\therefore\ 0.6\left(\frac{PL}{M_p}\right) + 0.35\left(\frac{QL}{M_p}\right) = 4.1 \qquad (3)$$



Q3 (cont'd)

b) From Data Book

AC, $M_{p1} = 4997 \times 10^{-6} \times 340 \times 10^{6} = 1.699 \times 10^{6}\,Nm = 1699\,kNm$

CE, $M_{p2} = 3996 \times \ldots \ldots \ldots = 0.8\,M_{p1}$ ✓

Dimensions in same proportion as part a).

$M_p = 1699\,kNm.$

$Q = 450\,kN$

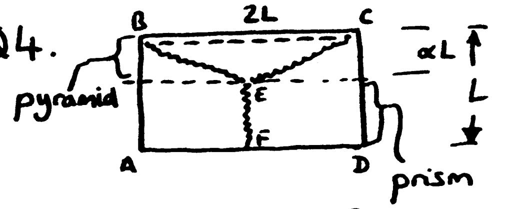
$\frac{QL}{M_p} = \frac{450\,kN \times 15m}{1699\,kNm} = 3.973 \quad (= q, \text{ say})$ (and $p \equiv \frac{PL}{M_p}$)

From graph, (part a). Mechanism 3 governs.

Algebraically: $p = \frac{4.1 - 0.35q}{0.6} = \frac{2.7095}{0.6} = 4.516$

$\therefore \quad W = P = \frac{4.516\,(1699\,kNm)}{15m} = \underline{\underline{511\,kN}}$

Q4.



Let EF fall by δ.

W.D. by loads

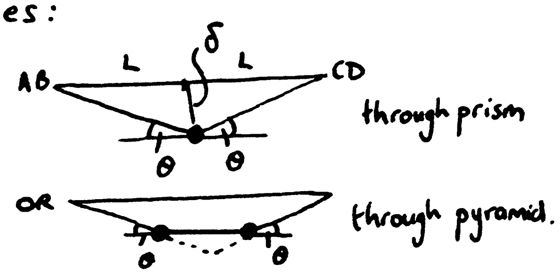
$= W.(1-\alpha)L\,.\,2L.\ \delta/2$ (prism)

$+\ W.\,\alpha L\,.\,2L.\ \delta/3$ (pyramid)

$= wL^2\delta\left(1 - \alpha + \frac{2\alpha}{3}\right) = \underline{\underline{wL^2\delta\left(\frac{3-\alpha}{3}\right)}}$

W.D. by hinges:

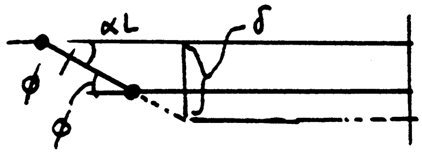
"Horiz." section:



Compat: $\theta = \delta/L$

W.D. $= m(2\theta)L = \underline{\underline{2m\delta}}$

"Vert." section



Compat: $\phi = \frac{\delta}{\alpha L}$

W.D. $= m(2\phi)2L$

$= \underline{\underline{4m\frac{\delta}{\alpha}}}$

$\therefore$ Total. W.D. by hinges $= 2m\delta + 4m\frac{\delta}{\alpha} = \underline{\underline{2m\delta\left(\frac{\alpha+2}{\alpha}\right)}}$

Equating external with internal:

$wL^2\frac{\delta}{3}(3-\alpha) = 2m\delta\left(\frac{\alpha+2}{\alpha}\right)$

$\therefore \quad \underline{\underline{w = \frac{6m}{L^2}\,\frac{(\alpha+2)}{\alpha(3-\alpha)}}}$

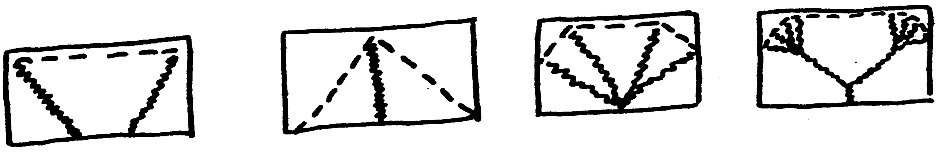
Q4 (cont'd)

b) $\alpha = 1 \quad \rightarrow \quad w = \frac{6m}{L^2}\left(\frac{3}{(1)(2)}\right) = \frac{9m}{L^2}$

$m =$  $\Rightarrow \quad m = \frac{\sigma_y t^2}{4}$

$\therefore \quad m = 250\ N/mm^2 \ \frac{(6)^2}{4}\ mm^2 = 2250\ N = 2.25\ kNm/m.$

$L = 0.3\,m \qquad \therefore \quad w = \frac{(2.25\ kN)\,9}{(0.3\,m)^2} = \underline{\underline{225\ kPa}}$

c)  etc.

Q5.

a) 

$\sigma_y = 345\ MPa$

$E = 210 \times 10^3\ MPa$

356 x 368 UC 177

$I_{xx} = 57153\ cm^4 = 57153 \times 10^4\ mm^4$

$I_{yy} = 20470\ cm^4 = 20470 \times 10^4\ mm^4$

$r_{xx} = 159\ mm$

$r_{yy} = 95.2\ mm$

$A = 225.7\ cm^2 = 22570\ mm^2$

Buckling about MINOR axis.

$\frac{L}{r} = \frac{3500\ mm}{95.2\ mm} = 36.76$

$\sigma_E = \frac{\pi^2 E}{(L/r)^2}$

$= \frac{\pi^2 (210 \times 10^3\ MPa)}{(36.76)^2}$

$= \underline{\underline{1533\ MPa}}$

$\sigma_y = 345\ MPa$ (given)

$\eta = 0.0055\left(\frac{L}{r}\right) = 0.0055(36.76) = 0.2022$

Work in GPa.

$(0.345 - \sigma)(1.533 - \sigma) = 0.2022(1.533)\,\sigma$

$0.5289 - 1.878\,\sigma + \sigma^2 = 0.310\,\sigma$

$\sigma^2 - 2.188\,\sigma + 0.5289 = 0$

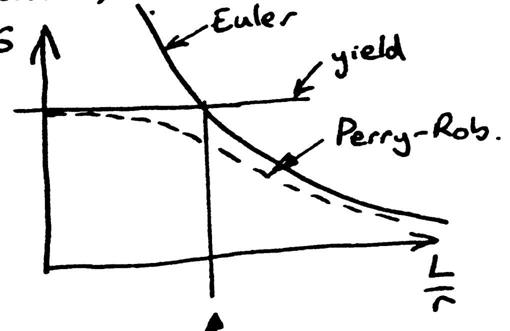
$\therefore \quad \sigma = \frac{2.188 \overset{\text{lowest}}{-} \sqrt{(2.188)^2 - 4(0.5289)}}{2} = \frac{2.188 - 1.635}{2} = 0.277\ GPa$

$= 277\ MPa.$

$\therefore$ Max. allowable axial load $= 277\ N/mm^2 \times 22570\ mm^2 = \underline{\underline{6246\ kN}}$

Q5 (cont'd)

b)



↑ change-over pt, $\sigma_E = \sigma_y$

$$\frac{\pi^2 E}{(L/r)^2} = \sigma_y \qquad \therefore \left(\frac{L}{r}\right) = \pi\sqrt{\frac{E}{\sigma_y}} = \pi\sqrt{\frac{210\times10^3}{345}} = 77.5$$

(c.f. $L/r = 36.76$, this column)

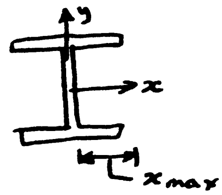
∴ over here, ∴ **stocky**.

Alternatively, $\sigma_{Perry-Rob} = 277$ MPa, which is a little less than $\sigma_y = 345$ MPa, but much less than $\sigma_E = 1533$ MPa

∴ **Stocky** (or stocky / intermediate)

c) $$\sigma_{max} \left(= \sigma_y\right) = \frac{P}{A} + \frac{M_{max}\, x_{max}}{I_{yy}}$$

↑ Perry criterion



Write $\sigma \equiv P/A$.

$Pv = M$

$$v_{max} = \frac{\delta_0}{1-P/P_E} = \frac{\delta_0}{1-\sigma/\sigma_E}$$

Amplification.

$$\therefore \sigma_y = \sigma + \frac{P\,\delta_0}{1-\sigma/\sigma_E}\,\frac{x_{max}}{I}$$

$$\therefore (1-\sigma/\sigma_E)(\sigma_y - \sigma) = \frac{P\,\delta_0\, x_{max}}{I_{yy}} = \left(\frac{P}{A}\right)\frac{\delta_0\, x_{max}}{(I/A)} = \sigma\,\underbrace{\frac{\delta_0\, x_{max}}{r_{yy}^2}}_{\equiv\,\eta \text{ Perry factor}}$$

$$\therefore (\sigma_E - \sigma)(\sigma_y - \sigma) = \eta\,\sigma_E\,\sigma$$

as given with $$\eta \equiv \frac{\delta_0\, x_{max}}{r_{yy}^2}$$

Q5 cont'd.

$\eta = 0.0055\left(\frac{L}{r}\right)$ given and $\eta = \frac{\delta_0 x_{max}}{r^2}$

$\therefore\ 0.0055 \frac{L}{r} = \frac{\delta_0 x_{max}}{r^2}$ $\therefore\ \delta_0 = 0.0055 \frac{Lr}{x_{max}}$

$$= 0.0055 \frac{(3500)(95.2)}{(372.1)/2} \text{ mm}$$

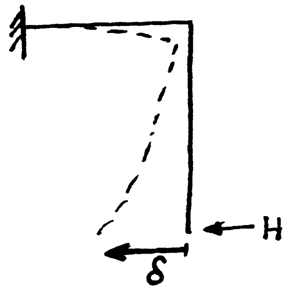
$$= 9.85 \text{ mm} \approx \underline{\underline{10 \text{mm}}}$$

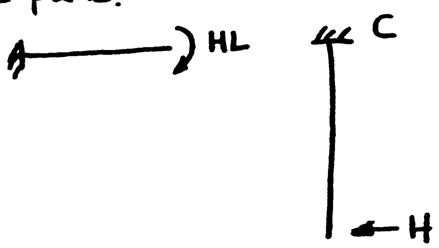
$\delta = \frac{\delta_0}{1 - P/P_E}$; $P_E = 1533 \text{ N/mm}^2 \times 22{,}570 \text{ mm}^2$

$$= 34{,}600 \text{ kN}$$

$$\therefore\ \delta = \frac{9.85 \text{ mm}}{1 - \frac{6000}{34600}} = 11.9 \text{mm} \approx \underline{\underline{12 \text{mm}}}$$

Q6. a) One indeterminacy – take horizontal reaction at foot.
Symmetry – consider half the frame.





Rotation at C, $\theta_c = HL . \frac{L}{EI_b}$

Deflection of leg CD as cantilever $= \frac{HL^3}{3EI_c}$

$$\therefore\ \delta = \underbrace{-\alpha \Delta T L}_{\text{due to ramp}} + \frac{HL^2}{EI_b} . L + \frac{HL^3}{3EI_c} = 0 \quad (\text{Compat.})$$

$$\therefore\ \frac{HL^3}{E}\left(\frac{1}{I_b} + \frac{1}{3I_c}\right) = \alpha \Delta T L$$

$$\therefore\ H = \frac{E\alpha\Delta T}{L^2}\left(\frac{3I_c I_b}{3I_c + I_b}\right)$$

Q6 b) $I_b \gg I_c$

$$\therefore \quad \frac{E\alpha\Delta T}{L^2}\left(\frac{3I_cI_b}{\underbrace{3I_c}_{\text{neglect}}+I_b}\right) \rightarrow \frac{E\alpha\Delta T}{L^2}\,\frac{3I_c\cancel{I_b}}{\cancel{I_b}} = H$$

(i.e. )

Max moment in leg, $M = HL$

$$\text{Max stress, } \sigma_{max} = \frac{My}{I_c} = \frac{3I_cE\alpha\Delta T}{L^2}\,\frac{L}{I_c}\,.\overset{t_c}{\left(\frac{t}{2}\right)}$$

$$= \frac{3}{2}\,\frac{E\alpha\Delta T}{L}\,.\,t_c \qquad (\text{proportional to } t_c)$$

$$\sigma_{max} \propto t_c$$

$$\therefore \quad \frac{\sigma_{max1}}{\sigma_{max2}} = \frac{t_{c1}}{t_{c2}}$$

$$\therefore \quad t_{c2} = \frac{\sigma_{max2}}{\sigma_{max1}}\,t_{c1} = \frac{80}{130}\,.\,1.2\,\text{mm} = \underline{\underline{0.74\,\text{mm}}}$$

(i.e. <u>reduce</u> leg thickness to reduce the stresses !).