Cambridge University Engineering Department
1997 Part 1B Paper 5 "Electrical Engineering"
ANSWERS

1 (a) (i) -196 (ii) 714 Ω (iii) 1961 Ω

(b) (i) -5.83 (ii) 2330 Ω

2 (b) (i) R = 1 kΩ

(ii) open-loop gain: $\dfrac{4\times10^4}{1+j\dfrac{f}{120}}$; closed-loop gain: $\dfrac{4\times10^4}{3078+j\dfrac{f}{120}}$; -3 dB frequency: 369.36 kHz

(c) (i) -6.8 (ii) -47.6

3 (b) (i) 13,122 A (ii) 9067 V (iii) 20.3 ° last part: 27.6 °

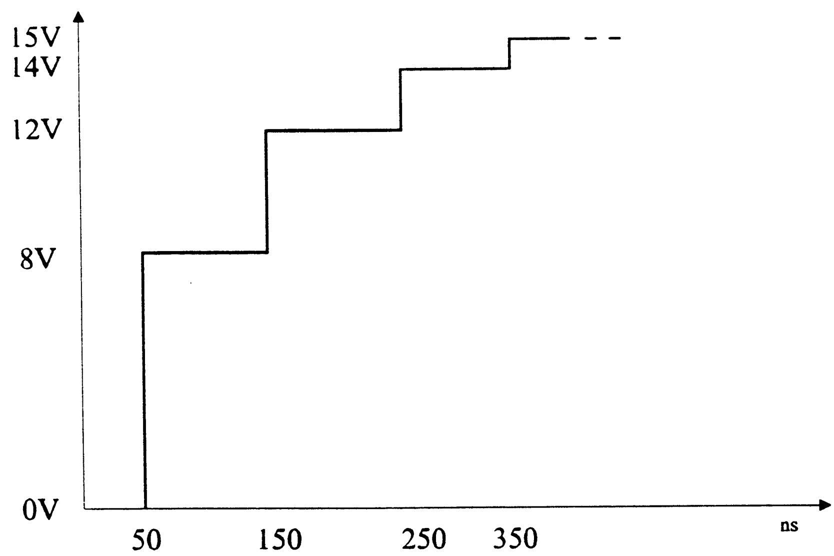
4 (b) (i) 4006 A (ii) 1002 A (iii) 229 MVA last part 0.7 Ω

5 (b) (i) X_m = 184 Ω, R_0 = 861 Ω, R_2' = 3.2 Ω, $X_1 = X_2'$ = 1.95 Ω

(c) (i) 0.01333 (ii) (47.9+j 70.1) Ω (iii) 8.5 A (iv) 20.9 Nm

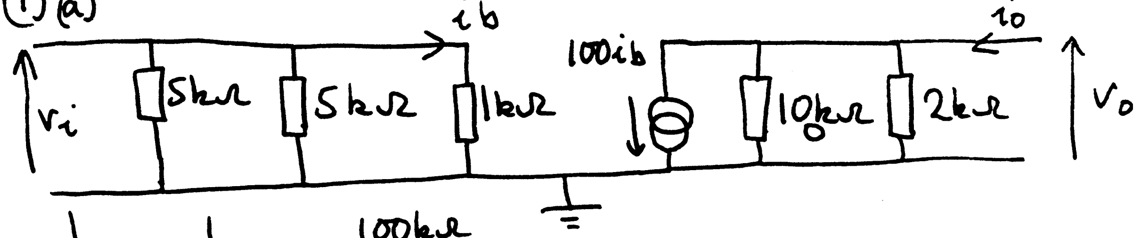
6 (a) characteristic impedance: 50 Ω Transmission frequency: 781.25 kHz

(b) first incident wave: 8V (i) 1.28 W (ii) 1.28 W (iii) 0 W



7 (a) 6.37 pW m^{-2} (i) 12.7 pW (ii) 0.504 μA

(1) (a)



$$\frac{1}{h_{oe}} = \frac{1}{10\times10^{-6}} = 100k\Omega$$

(i) $v_o = -100 i_b \times 100k\Omega /\!/ 2k\Omega = -100 i_b \times \left(\frac{100\times2}{100+2}\right)\times10^3$

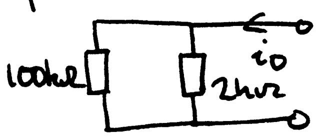
$i_b = \frac{v_i}{1000} \Rightarrow v_o = \frac{-100 v_i}{1000} \times \frac{200}{102} \times 10^3 \doteq -196 v_i$

$\therefore$ gain $= \frac{v_o}{v_i} = -196$

(ii) r_{in} is clearly (by inspection) $= 5k\Omega /\!/ 5k\Omega /\!/ 1k\Omega = 714\Omega$

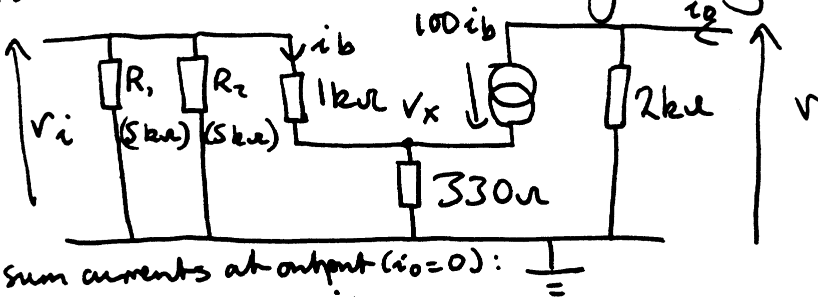
(iii) short-circuit input $\Rightarrow v_i = 0 \Rightarrow i_b = 0 \Rightarrow h_{fe} i_b = 0$

circuit becomes



$\Rightarrow \frac{v_o}{i_o} = R_{OUT} = 100k\Omega /\!/ 2k\Omega \doteq 1961\Omega$

(b) with C_E omitted and neglecting hoe:-



sum currents at output ($i_o = 0$):

$\frac{v_o}{2000} = -100 i_b$ (1) $\qquad \frac{v_i - v_x}{1000} = i_b$ (2)

$i_b(1+100) = \frac{v_x}{330}$ (3); $\qquad$ (2) $\Rightarrow v_i = v_x + 1000 i_b$

so $v_i = 330 i_b \times 101 + 1000 i_b$ using (3)

$\Rightarrow v_i = 34330 i_b$ subs. in (1) $\Rightarrow v_o = -2000\times100 i_b$

$= -\frac{200000 v_i}{34330}$

(i) so $\frac{v_o}{v_i} = -5.83$

R_E - provides a stable operating point (dc conditions)

C_E - "bypass" capacitor ensures high ac gain by providing a short-circuit path for ac signals to ground (b)(ii) We already have $v_i = 330 i_b \times 101 + 1000 i_b$

$\Rightarrow i_b = \dfrac{v_i}{330 \times 101 + 1000}$

$\left[\text{so } \dfrac{v_i}{i_b} = 34.33\,k\Omega \right]$

$\dfrac{v_{in}}{i_{in}}$ is $\therefore R_1 // R_2 // 34.33\,k\Omega$

i.e. $r_{in} = 2.5\,k\Omega // 34.33\,k\Omega$

$\doteqdot \underline{\underline{2.33\,k\Omega}}$

(2)(a) Common mode gain: 2 inputs of amp. at same voltage $V_1 = V_2 = V_c$
output voltage $= A_c V_c$ where A_c is c.m. gain.
Differential gain: the factor by which an amplifier multiplies the difference between the 2 inputs. $V_1 = V_2 = V_d/2$
$V_0 = A_d V_d$ where $V_d = V_1 - V_2$; A_d is diff. gain.
High CMRR: small differential signal from transducer has superimposed 50Hz mains noise on top of each input to amp (∴ a common mode signal). High CMRR will attenuate this 50Hz noise, but amplify transducer signal. (e.g.)

(b) Ideal op-amp: $V_+ = V_-$ so $V_i = \frac{R}{12+R} V_0 \Rightarrow \frac{V_0}{V_i} = \frac{12+R}{R} = 1 + \frac{12}{R}$

for gain of 13, $\Rightarrow 13 = 1 + \frac{12}{R} \Rightarrow \underline{\underline{R = 1\,k\Omega}}$.

$A = \frac{A_0}{1 + jf/120}$ (open-loop) $\Rightarrow$ closed loop gain $= \frac{A}{1+AB}$

where $B = \frac{R}{12+R}$; closed-loop gain $= \frac{A_0/(1+jf/120)}{1 + \frac{A_0 B}{(1+jf/120)}}$

$\therefore B = \frac{1}{12+1} = 1/13$

$$= \frac{A_0}{(1+A_0B) + jf/120} = \frac{4\times10^4}{1 + 4\times10^4 \cdot \frac{1}{13} + jf/120} = \frac{4\times10^4}{3078 + jf/120}$$

$$= \frac{13}{1 + jf/_{120\times3078}} = \underline{\underline{\frac{13}{1+jf/369360}}} \Rightarrow \underline{\underline{f_{3dB} = 369.36\,kHz}}$$

(c)(i) $V_+ = \frac{R_2}{R_1+R_2} \cdot V_4 = V_-$. Summing currents at "–" termin

$$\frac{V_x - V_-}{R_1} = -\frac{V_0 - V_-}{R_2} \quad \therefore \frac{\left(V_x - \frac{R_2}{R_1+R_2} \cdot V_4\right)}{R_1} = \frac{\left(-V_0 + \frac{R_2}{R_1+R_2} V_4\right)}{R_2}$$

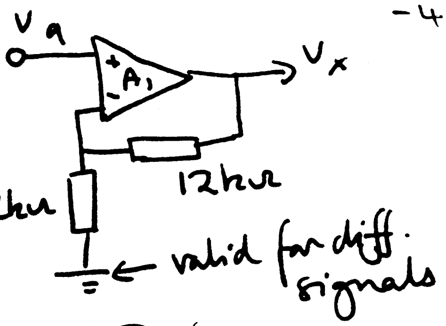
$$\therefore \frac{(R_1+R_2)V_x - R_2V_4}{R_1} = \frac{-V_0(R_1+R_2) + R_2V_4}{R_2}$$

$$\therefore R_2(R_1+R_2)V_x - R_2^2 V_4 = -V_0 R_1(R_1+R_2) + R_1R_2V_4$$

so $R_2(R_1+R_2)[V_x - V_4] = -V_0 R_1(R_1+R_2)$

$$\Rightarrow \frac{V_0}{V_x - V_4} = -\frac{R_2}{R_1} = -\frac{68}{10} = \underline{\underline{-6.8}}$$

(2) (c) (iii) Consider ½ circuit:-

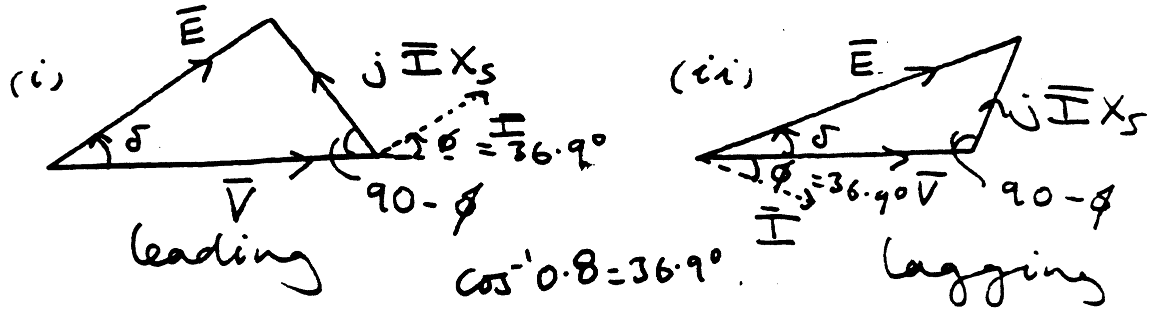


as before, $\frac{V_x}{V_a} = 1 + \frac{12}{2} = +7$ $\quad \frac{4k\Omega}{2} = 2k\Omega$

so overall gain $\frac{V_o}{V_a} = +7 \times -6.8 = \underline{\underline{-47.6}}$

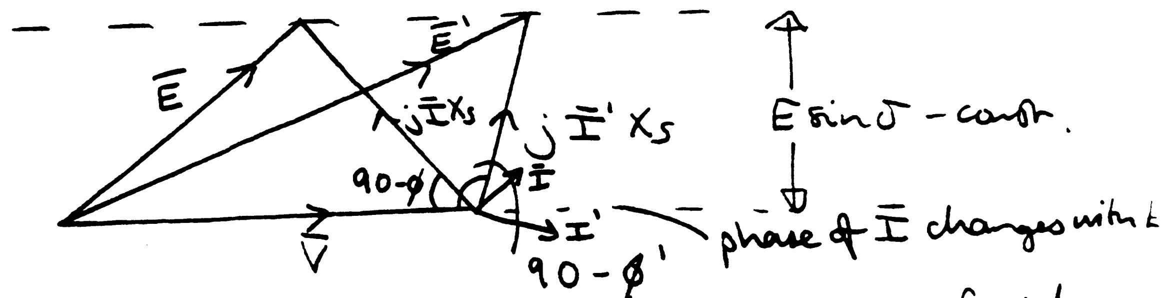
First stage gives v. high input impedance (and reduces problems due to resistor mismatches in 2nd stage)

(3) (a)



$\cos^{-1} 0.8 = 36.9°$

for constant power, $E \sin\delta$ is constant



by changing excitation voltage using field regulator, the shape of the diagram changes and so ϕ varies. Hence for constant power, Q varies because of change in p.f

(b) (i) $\sqrt{3}\, V_L I_L \cos\phi = 300\times10^6 \Rightarrow I_L = \dfrac{300\times10^6}{0.6\times\sqrt{3}\times22\times10^3} = \underline{\underline{13,12}}$

(ii) using phasor diagram above and cosine rule:-

$$E^2 = V^2 + (IX_s)^2 - 2V(IX_s)\cos(90-\phi) \quad (\sin\cos^{-1}$$

$$\Rightarrow E^2 = \frac{(22\times10^3)^2}{3} + (13122\times0.4)^2 - \frac{2\times22\times10^3}{\sqrt{3}}\times(13122\times0.4)\sin$$

$$\Rightarrow \underline{\underline{E = 9067\text{V}}} \text{ per phase}$$

(iii) using sine rule: $\dfrac{\sin\delta}{IX_s} = \dfrac{\sin(90-\phi)}{E}$

$\Rightarrow \sin\delta = \dfrac{IX_s\cos\phi}{E} = \dfrac{13122\times0.4\times0.6}{9067} \Rightarrow \underline{\underline{\delta = 20.3}}$

for increase in power, $\dfrac{\sin\delta_2}{\sin\delta_1} = \dfrac{400}{300} \Rightarrow \delta_2 = \sin^{-1}\left[\dfrac{4}{3}\times\sin 20\right.$

$\Rightarrow \underline{\underline{\delta_2 = 27.6°}}$

(4)(a) "fast acting": to prevent damage to equipment and danger
"resetable": automatic clearing of faults – breakers reclose after fault has gone
"redundancy": tripping one part of a system doesn't adversely affect another part (multiple trips etc.)
(+others)

(b) $MVA_b = 100\,MVA$.

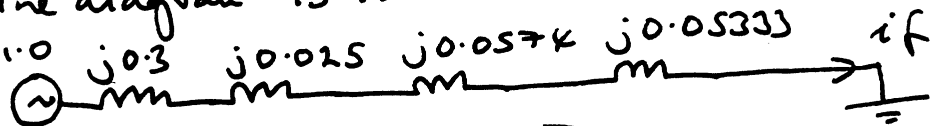
generator: 1.0 pu volts, $j0.3$

200 MVA transf: $j\dfrac{0.05\times 100}{200} = j0.025\,pu$

line: $j50\times 0.2\,\Omega = j10\,\Omega \Rightarrow j\dfrac{10}{132^2/100} = j0.0574\,pu$

150 MVA transf: $j\dfrac{0.08\times 100}{150} = j0.05333\,pu$

one-line diagram is ∴



$$i_f = \frac{1}{j(\underbrace{0.3+0.025+0.0574+0.05333}_{[0.43573]})} \doteq j2.29\,pu$$

base current at 33kV bus $= \dfrac{100\times 10^6}{\sqrt{3}\times 33\times 10^3} = 1749.5\,A$

(i) so I_f (in bus) $= 2.29\times 1749.5 = \underline{\underline{4006\,A}}$

(ii) I_f' (in line) $= 2.29\times \dfrac{100\times 10^6}{\sqrt{3}\times 132\times 10^3} = \underline{\underline{1002\,A}}$

(iii) fault MVA (breaker rating) $= 1\,pu \times 2.29 = 2.29\,pu \Rightarrow 2.29\times 100 = \underline{\underline{229\,MVA}}$

For 3500A in bus, $\Rightarrow \dfrac{3500}{1749.5} = 2\,pu$.

so $2\,pu = \dfrac{1}{X} \Rightarrow X = 0.5$ so extra reactance (pu)

is $0.5 - 0.43573 = 0.06427\,pu$

$\Rightarrow 0.06427\times\left(\dfrac{33^2}{100}\right) = \underline{\underline{0.7\,\Omega}}$

Z_{base} at bus

(5) (a) R_1: stator winding resistance

R_2': referred rotor resistance

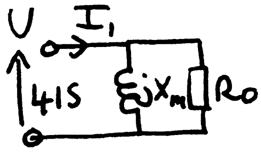
X_m: three phase magnetising reactance

X_1: stator leakage reactance :-

X_2': referred rotor leakage reactance :-

R_0: iron-loss resistance

(b) 1500 rpm:



$$\frac{3\times V_1^2}{R_0} = 600 \Rightarrow \frac{3\times 415^2}{R_0} = 600$$

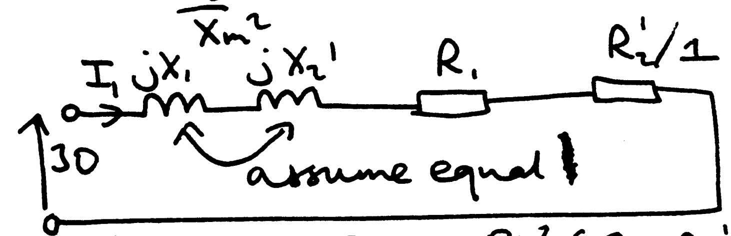
$$\Rightarrow \underline{\underline{R_0 = 861\,\Omega}}$$

currents in quadrature

so $I_1^2 = I_{X_m}^2 + I_{R_0}^2 \therefore \left(\frac{4}{\sqrt{3}}\right)^2 = \left(\frac{415}{861}\right)^2 + \left(\frac{415}{X_m}\right)^2$

$\Rightarrow \frac{16}{3} = 0.2323 + \frac{415^2}{X_m^2}$ $\qquad \Rightarrow \underline{\underline{X_m = 184\,\Omega}}$

0 rpm:



$3I_1^2(R_1 + R_2') = 333 \Rightarrow \left(\frac{8}{\sqrt{3}}\right)^2 (2 + R_2') = 111$

given as $2\,\Omega$ $\qquad \Rightarrow \underline{\underline{R_2' = 3.2\,\Omega}}$

$$\frac{30^2}{(X_1+X_1)^2 + (2+3.2)^2} = \left(\frac{8}{\sqrt{3}}\right)^2 \Rightarrow \frac{900}{5.2^2 + 4X_1^2} = \frac{64}{3} \Rightarrow \underline{\underline{X_1 = X_2' = 1.95\,\Omega}}$$

(c) (i) $s = \frac{1500-1480}{1500} = \frac{20}{1500} = \underline{\underline{0.01333}}$

(ii) $\bar{Z}_1 = R_1 + jX_1 + \frac{jX_m(jX_2' + R_2'/s)}{j(X_m + X_2') + R_2'/s}$

$\therefore \bar{Z}_1 = 2 + j1 + \frac{j100\left(\frac{2}{0.01333} + j1\right)}{j101 + 2/0.01333} \doteq \underline{\underline{(47.9 + j\,70.1)\,\Omega}}$

(iii) $I_L = \sqrt{3} \times \frac{415}{|47.9 + j70.1|} = \sqrt{3}\times 4.89 \doteq 8.5\,\text{A}$

(5)(c)(iv) using current divider rule:-

$$I_2' = 4.89 \frac{|j100|}{|j100 + j1 + 2/0.01333|} \doteq 2.7A.$$

$$T_e = \frac{3 I_2'^2 R_2'}{s\,\omega_s} = \frac{3 \times 2.7^2 \times 2}{2\pi \times \frac{1500}{60} \times 0.01333} \simeq \underline{\underline{20.9\,Nm}}$$

(3)(a) Z_0: The ratio between the voltage and current of a unidirectional wave on a transmission line at any point.

$C = 80\times10^{-12}\ \mathrm{Fm^{-1}}$, $L = 2\times10^{-7}\ \mathrm{Hm^{-1}}$

so $Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{2\times10^{-7}}{80\times10^{-12}}} = \underline{\underline{50\Omega}}$

$\lambda = \frac{2\pi}{\beta}$ where $\beta = \omega\sqrt{LC}$ so for $\lambda = 16\times20\,\mathrm{m}$

$\Rightarrow 20\times16 = \frac{2\pi}{2\pi f\sqrt{LC}} = \frac{1}{f\sqrt{LC}}$ so $f = \frac{1}{20\times16\times\sqrt{2\times10^{-7}\times80\times10^{-12}}}$

$= \underline{\underline{781.25\,\mathrm{kHz}}}$

(b) $\rho_L = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow$ (i) for $Z_L \to \infty$, $\rho_L \to +1 \Rightarrow |\rho_L|^2 \to 1$.

incident wave is $\frac{Z_0}{Z_0+Z_G}\times32 = \frac{50}{50+150}\times32 = \underline{\underline{8V}}$

so incident power $= \frac{8^2}{Z_0} = \frac{8^2}{50} = \underline{\underline{1.28W}}$ = reflected power since $|\rho_L|^2 = 1$.

(ii) $\rho_L = \frac{0-Z_0}{0+Z_0} = -1 \Rightarrow |\rho_L|^2 = 1 \Rightarrow$ reflected power $= \underline{\underline{1.28W}}$

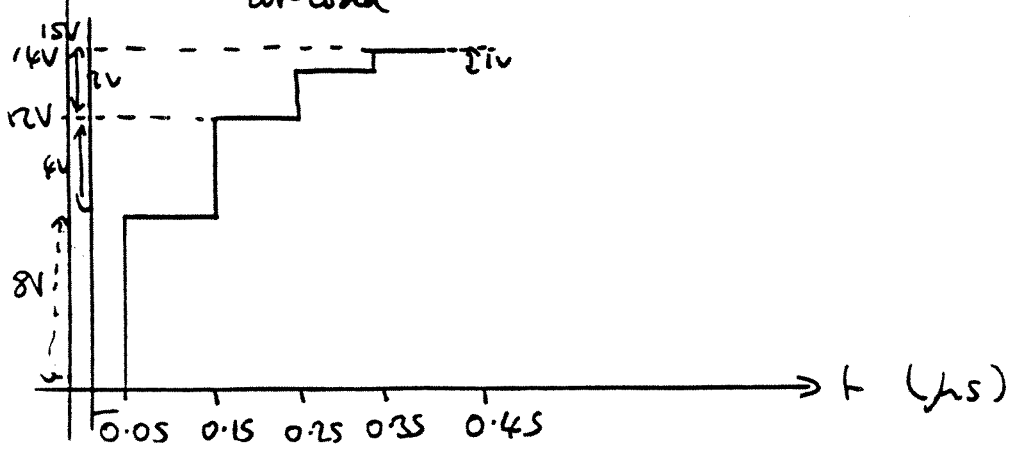
(iii) $\rho_L = \frac{Z_0-Z_0}{Z_0+Z_0} = 0 \Rightarrow |\rho_L|^2 = 0 \Rightarrow$ reflected power $= \underline{\underline{0W}}$

(because line is "matched" or "correctly terminated"; ∴ no reflections)

(c) propogation time $= \frac{20}{2\times10^8} = 0.1\mu s$

if $Z_L = 150\Omega \Rightarrow \rho_L = \frac{150-50}{150+50} = 0.5$; $Z_G = 150\Omega$ also so $\rho_G = 0.5$

all reflections ∴ produce an attenuation by a factor of ½ (at load and source). First incident wave is ~~8V~~ 8V 1st reflected at load is 4V, then 2V at source etc.



54- (7) (a) The gain of an antenna is the factor by which the maximum radiated intensity exceeds that of an isotropic antenna if they emit equal power from an equal distance.

peak intensity $= \dfrac{G \times Power}{4\pi r^2} = \dfrac{2000 \times 25}{4\pi.(25 \times 10^3_{\times 10^3})^2} = \underline{\underline{6.37}}\ {}^{p}\mathrm{W\,m^{-2}}$

(i) peak power received $= A_{eff} \times$ intensity

$= 2 \times 6.37 \doteq \underline{\underline{12.7\ {}^{p}\mathrm{W}}}$

(ii) perfect matching $\Rightarrow$ all power received goes to receiving electronics

so $12.7 \times 10^{-12} = \frac{1}{2} VI = \frac{1}{2} I^2 Z_0$

$\Rightarrow I^2 = \dfrac{2 \times 12.7 \times 10^{-12}}{Z_0} \doteq 713\ {}^{n}\mathrm{A} \Rightarrow$ RMS $I = \underline{\underline{504\ {}^{n}\mathrm{A}}}$

(b) $\mathrm{curl}\, \underset{\sim}{H} = \dfrac{\partial \underset{\sim}{D}}{\partial t} = \varepsilon_0 \dfrac{\partial \underset{\sim}{E}}{\partial t} = j\omega \varepsilon_0 \underset{\sim}{\bar{E}}$ for sinusoids.

so $\mathrm{curl}\, \underset{\sim}{\bar{H}} = j\omega\varepsilon_0 \underset{\sim}{\bar{E}}$

$$\mathrm{curl}\, \underset{\sim}{\bar{H}} = \begin{vmatrix} \hat{u}_x & \hat{u}_y & \hat{u}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & H_0 e^{-j\beta z} & 0 \end{vmatrix} = -\hat{u}_x \frac{\partial}{\partial z}(H_0 e^{-j\beta z}) + \hat{u}_z \frac{\partial}{\partial x}(H_0 e^{-j\beta z})$$

$\to 0$.

so $\mathrm{curl}\, \underset{\sim}{\bar{H}} = -\hat{u}_x H_0 (-j\beta) e^{-j\beta z}$

$= \hat{u}_x j\beta H_0 e^{-j\beta z}$

$\Rightarrow \hat{u}_x j\beta H_0 e^{-j\beta z} = j\omega \varepsilon_0 \underset{\sim}{\bar{E}}$

so $\underset{\sim}{\bar{E}} = \dfrac{\hat{u}_x \beta}{\omega \varepsilon_0} H_0 e^{-j\beta z}$ both $\left[\dfrac{\beta}{\omega}\right] \equiv \sqrt{\mu_0 \varepsilon_0}$

reciprocal of prop. vel.

so $\underset{\sim}{\bar{E}} = \dfrac{\hat{u}_x \sqrt{\mu_0 \varepsilon_0}}{\varepsilon_0} H_0 e^{-j\beta z}$

$= \hat{u}_x \underbrace{\sqrt{\dfrac{\mu_0}{\varepsilon_0}}}_{=\eta_0} H_0 e^{-j\beta z}$

reintroduce implied sinusoidal time variation

$\Rightarrow \underline{\underline{E(z,t) = \hat{u}_x \eta_0 H_0 e^{+j(\omega t - \beta z)}}}$