Answers: IB Paper 6, Information Engineering, 1997

1. a) $\dfrac{4(s-1)}{(s+1)^2}$

   b) Open-loop unstable, poles at $s = -3, 1$. Closed-loop asymptotically stable, poles at $s = -1, -1$.

   c) $-4 + 4e^{-t} + 8te^{-t}$, $\dot{y}(0) = 4$, $\lim_{t\to\infty} y(t) = -4$.

2. $k_p = 5$, $k_i = 2/3$. max speed deviation= -1.612.

3. $2.76 < k_p < 5.81$; $k_p > 3.48$.

4. a) GM=5, PM=51°.

   b) PM=53°.

5. a) 1.408 Mbit/s.

   b) 6.82 times.

   c) 4.8MHz.

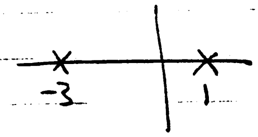
6. b) 2659 channels.

Solutions: 1B Paper 6, 1997

1) a) $$\bar{y}(s) = \frac{\dfrac{4}{s+3}}{1 + \dfrac{4}{(s-1)(s+3)}}\,\bar{w}(s)$$

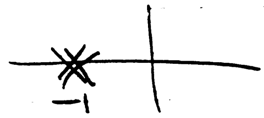
$$= \frac{4(s-1)}{(s+1)^2}\,\bar{w}(s)$$

b) $$\text{OLTF} = \frac{4}{(s-1)(s+3)}$$, poles at



$\Rightarrow$ unstable (1 RHP pole)

CLTF has poles at $s = -1$ (twice)



$\Rightarrow$ asymptotically stable.

c) $$\bar{y}(s) = \frac{4(s-1)}{s(s+1)^2} \qquad \left(\text{as } \bar{w}(s) = \frac{1}{s}\right)$$

$$= \frac{-4}{s} + \frac{4}{s+1} + \frac{8}{(s+1)^2}$$

$$\Rightarrow y(t) = -4 + 4e^{-t} + 8te^{-t}$$

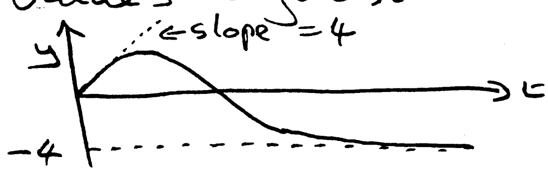
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s \cdot \frac{4(s-1)}{s(s+1)^2} = -4 \quad ✓$$

$$\lim_{t\to 0} \dot{y}(t) = \lim_{s\to\infty} s \cdot \frac{4(s-1)}{(s+1)^2} = 4 \quad ✓$$

(initial value of impulse response!)

- Can check these values against expression for $y(t)$.

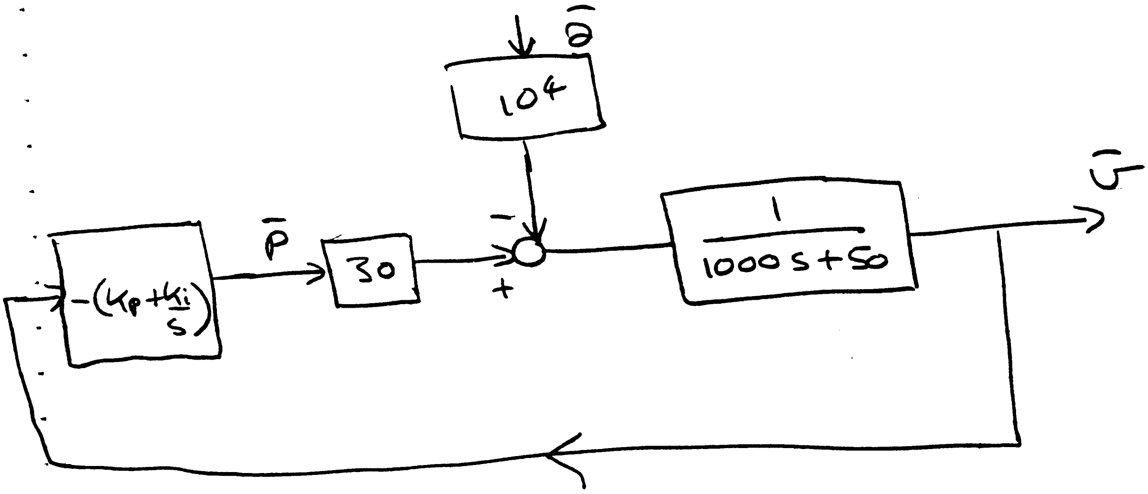
$\Rightarrow$



2) $(1000s + 50)\bar{v}(s) + 10^4\bar{\theta}(s) = 30\bar{p}$

$$\Rightarrow \quad \bar{v} = \frac{1}{1000s+50}\left(-10^4\bar{\theta} + 30\bar{p}\right)$$

$$\bar{p} = -\left(K_p + \frac{K_i}{s}\right)\bar{v}$$



$$\Rightarrow \quad \bar{v} = \frac{-10^4 / 1000s+50}{1 + \frac{30}{1000s+50}\cdot\frac{sK_p+K_i}{s}}\ \bar{\theta}(s)$$

$$= \frac{-10^4 s}{1000s^2 + (50+30K_p)s + 30K_i} \times \bar{\theta}($$

Char eqn is $1000s^2 + (50+30K_p)s + 30K_i = 0$

c.f. $(s+0.1+0.1j)(s+0.1-0.1j) = 0$

$\Leftrightarrow \quad 1000s^2 + 200s + 20 = 0$

$\Rightarrow \quad K_p = 5, \quad K_i = \frac{2}{3}$

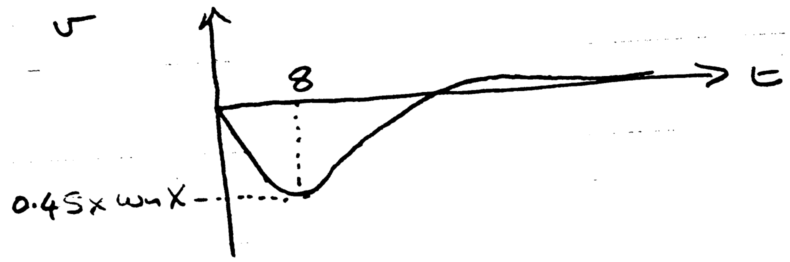
$$\bar{\theta}(s) = \frac{0.05}{s} \Rightarrow \bar{v}(s) = \frac{-0.05 \times 10^4}{1000s^2 + 200s + 20}$$

$$\Rightarrow \bar{v}(t) = \mathcal{L}^{-1} \frac{-25}{50s^2 + 10s + 1}$$

Use Mechanics data book, P7 impulse response of a linear 2nd order system.

$$\omega_n = \frac{1}{\sqrt{50}}, \quad X = -25, \quad c = 0.707\ldots$$

$$\Rightarrow \omega_n X = -5/\sqrt{2}$$



$$\Rightarrow v_{max} \approx -0.45 \times \frac{5}{\sqrt{2}} \approx -1.6$$

Alternatively

$$\bar{v}(s) = \frac{-0.5}{s^2 + 0.2s + 0.02} = \frac{-0.5}{(s+0.1)^2 + 0.1}$$

$$\Rightarrow v(t) = -0.5\, e^{-0.1t} \sin 0.1t$$

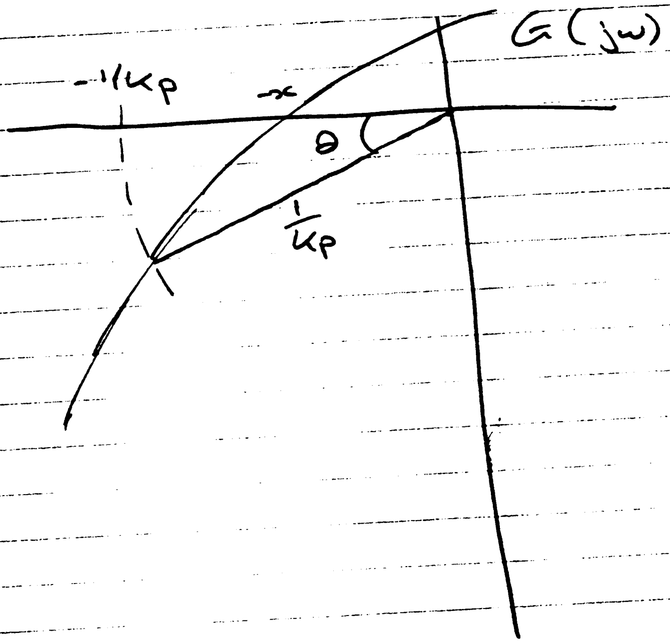
$$\frac{d}{dt} v(t) = -0.5(-0.1e^{-0.1t} \sin 0.1t + 0.1e^{-0.1t} c$$

$$= 0 \text{ at } \sin 0.1t = \cos 0.1t$$

$$\text{ie } 0.1t = \frac{\pi}{4} \Rightarrow t = \frac{5\pi}{2}$$

Substituting in $v(t)$ gives $v_{max} = -1.612$

3) a)



PM is θ, GM is $\frac{1/K_p}{x} = \frac{1}{K_p x}$

b) for PM > 45°, need $\frac{-1}{K_1} < \frac{-1}{K_p} < \frac{-1}{K_2}$

ie $K_1 < K_p < K_2$

$K_1 \approx \frac{114\,mm}{42\,mm} = 2.7$ (2.756)

$K_2 \approx \frac{114\,mm}{19\,mm} = 6$ (5.806)

for final constraint, need $a > b$

i.e. $\frac{-1}{K_p} > \frac{-1}{K_3}$

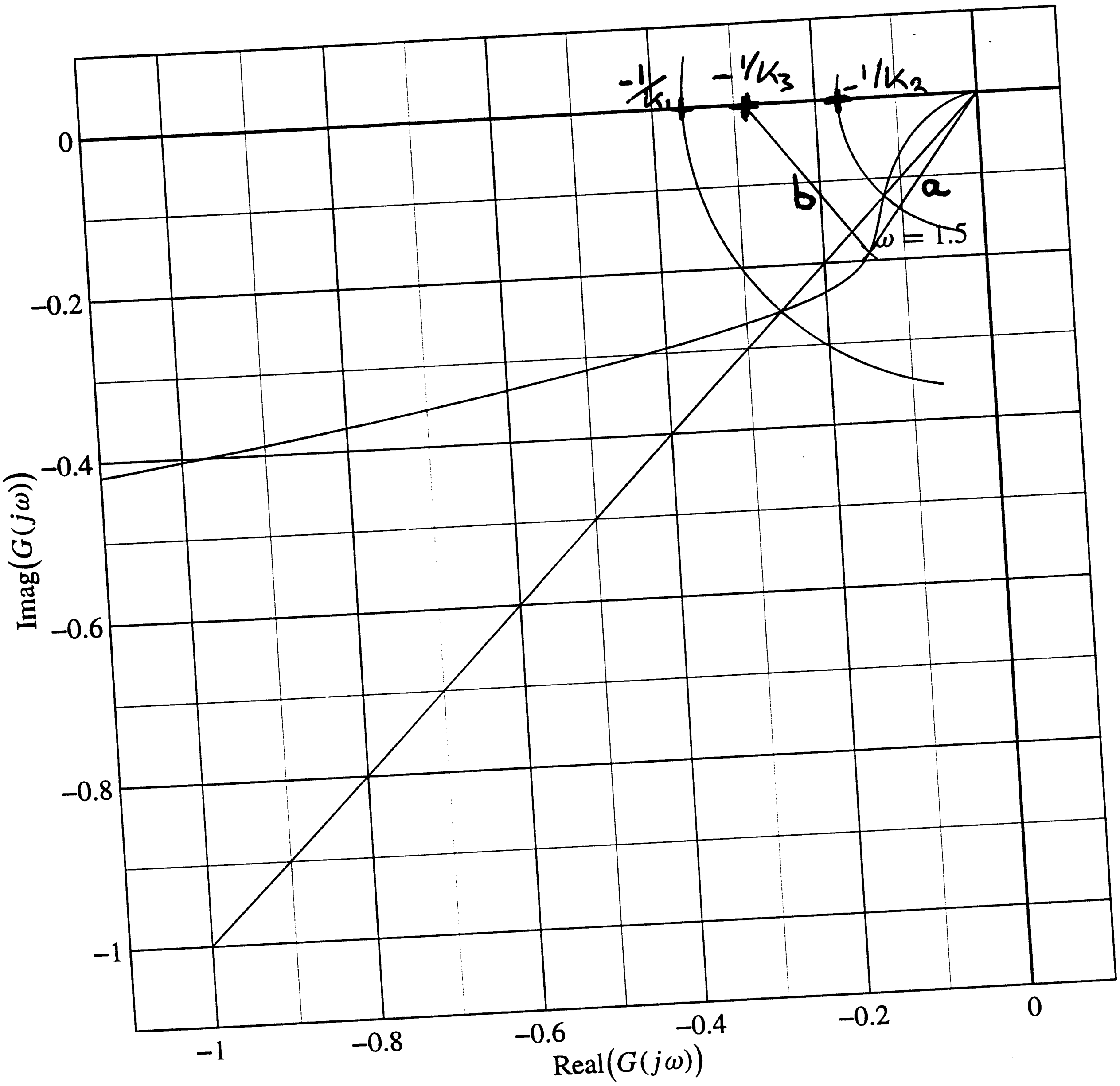
$\Rightarrow K_p > \frac{114\,mm}{33\,mm} = \underline{\underline{3.45}}$ (3.478)

To satisfy both criteria, need

$3.478 < K_p < 5.806$

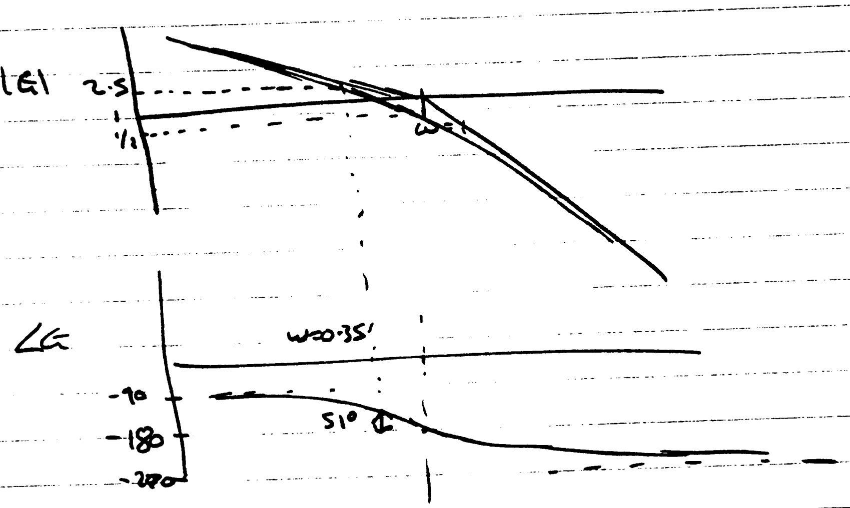
ENGINEERING TRIPOS PART IB

Thursday 5 June 1997, Paper 6, Question ??.



Extra Copy of Fig. ??

4) a)



$\angle G = -180°$ at $\omega = 1$

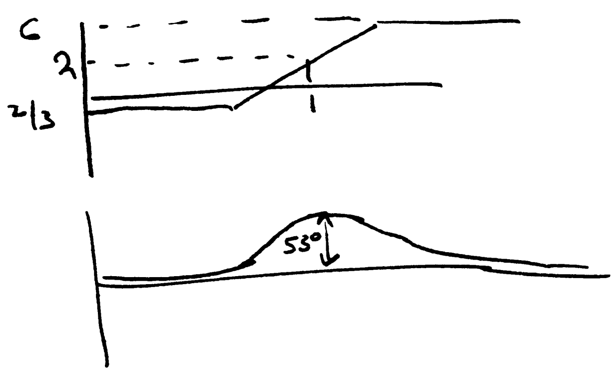
$|G(j\omega)| = \frac{1}{2}$ at $\omega = 1$

$\Rightarrow$ stable for $k_p < 2$

$k_p = 0.4$ $\quad GM = \frac{2}{0.4} = \underline{\underline{5}}$

$PM = \underline{\underline{51°}}$ (as on sketch)

b) For k only



Now $|K(j\omega)G(j\omega)| = 1$ at $\omega = 1$

$\Rightarrow PM = \underline{\underline{53°}}$

PM the same in both cases, DC gain v. similar. b) is better because of higher bandwidth. (faster response)

Solutions:

5 (a) High quality audio requires a bandwidth of approx 20 kHz. Nyquist's rule says that signals must be sampled at a rate that is at least twice the signal bandwidth to avoid aliassing distortion. Hence the sample rate for audio should be at least 40 kHz. In practice a rate of 44 kHz is used because practical anti-aliassing filters need transition bands of non-zero width.

64K sampling levels are needed in order for the noise due to quantisation to be below the level that is audible. The peak-to-peak quantising error will be approximately 1/64K of the peak-to-peak signal voltage, which leads to the noise being approximately 97 dB below the maximum signal. This is an adequately low noise level.

64K levels requires $\log_2(64K) = 16$ bits per sample.

Stereo requires 2 channels.

Bit rate for CD player $= 16 \times 2 \times 44 \,.\, 10^3 = 1.408 \,.\, 10^6$ bit/s = 1.408 Mbit/s.

(b) 256 colours requires $\log_2(256) = 8$ bits per pixel.

Pixel rate for video $= 400 \times 300 \times 10 = 1.2 \,.\, 10^6$ pixel/sec.

Bit rate for video $= 8 \times 1.2 \,.\, 10^6 = 9.6 \,.\, 10^6$ bit/sec.

Hence, speed-up factor of CD player $= 9.6/1.408 = 6.82$ times.
(In practice this means a $\times 8$ CD player would be needed.)

(c) In order to pass at least the fundamental component of an alternating bitstream ...101010..., the reader must have a bandwidth of at least half the bit rate.

Hence minimum bandwidth $= 9.6 \,.\, 10^6 \;/\; 2 = 4.8$ MHz.

In practice a greater bandwidth would be desirable in order to allow optimal recovery of the clock rate and rejection of noise.

6 (a) Carson's rule defines the approximate bandwidth required by a frequency modulated signal when the peak frequency deviation is f_D and the frequency of the modulating signal is f_M. It really only applies for modulation by a single sine-wave and arises from the properties of Bessel functions. In the case of a more complicated modulating signal, we can get an upper estimate of bandwidth if f_M is the maximum frequency present in the input.

If the carrier frequency is f_C then the occupied frequency band extends from $(f_C - f_D - f_M)$ to $(f_C + f_D + f_M)$.

(b) $f_D = 5$ kHz.

Max bandwidth of a typical telephone speech signal is 3.4 kHz, so set $f_M = 3.4$ kHz.

Therefore from Carson's rule, bandwidth $= 2(5 + 3.4) = 16.8$ kHz.

Hence, with the 2 kHz gap, min channel spacing $= 16.8 + 2 = 18.8$ kHz.

So the maximum number of channels in $900 - 850 = 50$ MHz is:

$$\frac{50\ .\ 10^6}{18.8\ .\ 10^3} = 2659 \text{ channels}$$

The 2 kHz gap is needed to prevent signals in adjacent channels from interfering with the wanted signal. Interference would occur without the gap, because some power does exist beyond the bandwidth given by Carson's rule and also because practical receiver filters cannot cut off abruptly.

(c) Amplitude modulation requires a bandwidth of $2f_M$, because its spectrum comprises 2 sidebands, above and below the carrier, each of the same shape and bandwidth as the modulating signal spectrum. Single sideband modulation requires a bandwidth of only f_M, since it comprises only one of these sidebands. Hence both of these modulation methods need less bandwidth than FM and could be used to allow more users – e.g.:

Channel spacing with AM $= 2 \times 3.4 + 2 = 8.8$ kHz, giving 5681 channels.

Channel spacing with SSB $= 3.4 + 2 = 5.4$ kHz, giving 9259 channels.

The penalties that would be paid for this are worse rejection of noise and interference and, in the case of SSB, poorer speech quality and higher complexity for the transmitter and receiver.