ENGINEERING TRIPOS PART IB - PAPER 8

SOLUTIONS - 1997

1. a) A $\rho = 7.9/4 = 1.975\ kg/l = 1975\ kg/m^3$

$\rho_d = 6.63/4 = 1.658\ kg/l = 1658\ kg/m^3$

Data sheet: $\rho = \rho_w \frac{(G_s + S_r e)}{(1+e)}$; $\rho_d = \rho_w \frac{G_s}{(1+e)}$

$\rho_w = 1000\ kg/m^3$, $G_s = 2.65$, $\therefore e = 0.598$

$\therefore 3157 = 1000\ (2.65 + 0.598\ S_r)$

$\therefore S_r = 0.847$

B $\rho = 1820\ kg/m^3$, $\rho_d = 1325\ kg/m^3$

$\therefore e = 1.00$, $\therefore S_r = 0.99$

b) A $\sigma_v = 2 \times 1.975 \times 9.81 = 39\ kPa$

B $\sigma_v = 4 \times 1.975 \times 9.81 + 2 \times 1.820 \times 9.81$
$= 113\ kPa$

c) A is 2m above the water table. If it had been saturated, its pore pressure would have been

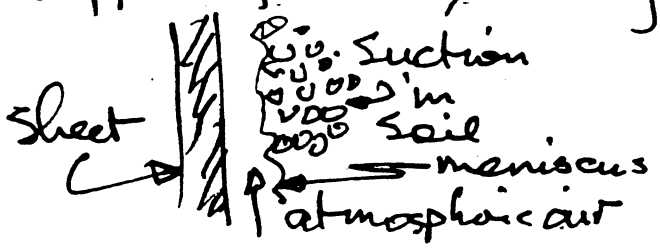
$u = -2 \times 9.81 \simeq -20\ kPa$ (gauge)

If it had been dry ($S_r = 0$), its pore pressure would have been zero (atmospheric). The answer lies somewhere in between, probably closer to $-20\ kPa$.

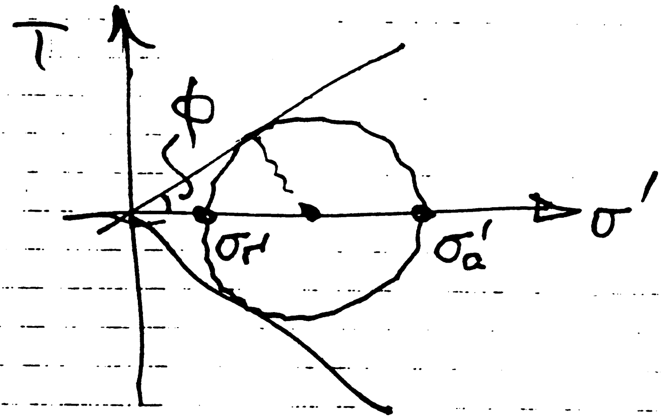
B is 2m below the water table.

$u = 2 \times 9.81 \simeq 20\ kPa.$

Negative pressures can be eliminated by rain or leaking pipes etc. They need not act as suction on supporting sheets, in any event.



d) (i) $\sigma_r' = 50$ kPa ; $\sigma'_{a\,max} = 290$ kPa, $\sigma'_{a\,ult} = 190$ kPa



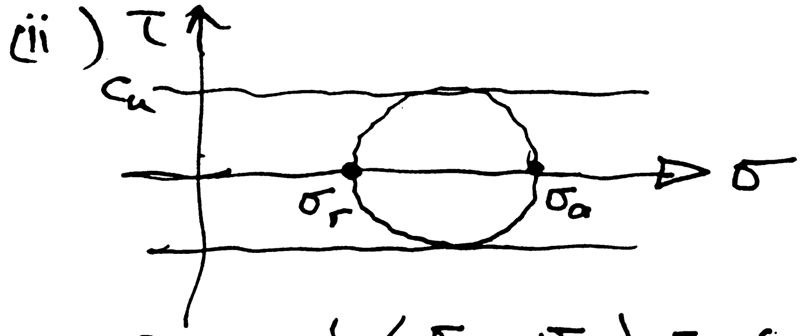
$$\sin\phi = \frac{\text{radius}}{\text{distance to centre}}$$

$$= \frac{(\sigma_a' - \sigma_r')/2}{(\sigma_a' + \sigma_r')/2}$$

$\sin\phi_{max} = 240/340$; $\phi_{max} = 44.9°$

$\sin\phi_{ult} = 140/240$; $\phi_{ult} = 35.7°$

(ii)



$$c_u = \frac{1}{2}(\sigma_a - \sigma_r) = 60 \text{ kPa}$$

e) For A: Take $\sigma_v' = 39$ kPa

Use $\frac{\sigma_h'}{\sigma_v'} = \frac{50}{190}$ corresponding to the ultimate stress ratio in the triaxial test.

Then $\sigma_h = \sigma_h' = 10$ kPa

For B: Take $\sigma_v = 113$ kPa

Use $\sigma_h = \sigma_v - 2c_u = 113 - 120 = -7$ kPa

e) cont.

The 10kPa at A mobilises the ultimate strength, which is less than the peak strength. It is consistent with massive sliding failure. If supports prevent rupture, this is safe. But rainfall or leaks would tend to pond above the clay and the water level might rise. Either punch drain holes, or allow water table to rise, for safety.

The −7kPa at B invokes undrained strength, with negative porepressures mobilised temporarily. Water could certainly drain into a crack against the sheets to give $u = +20$ kPa, $\sigma_h = 20$ kPa. Even higher pressures would occur if the clay drained; a ϕ calculation would be needed.

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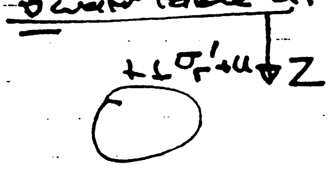
2 a) * Water is mobile
- if restricted it causes pressure
- if permitted it leaks into cavities

* Consider a sealed tunnel etc.
- full water pressure $u = \gamma_w z$
- reduced effective earth pressure $\sigma_r' \leq (\gamma - \gamma_w) z$

e.g. water table at surface



- Terzaghi's effective stress σ' applied like Dalton's Law of Partial Pressures

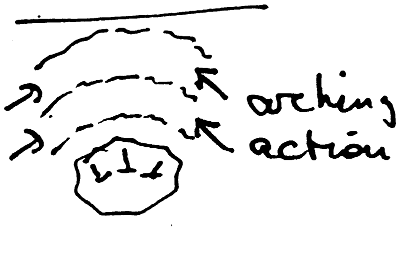
$$\sigma = \sigma' + u$$

total stress, effective stress, pore pressure

For level uniform ground:
σ obtained by vertical equilibrium $\sigma = \gamma z$
u obtained by hydrostatics $u = \gamma_w z$
$\therefore \sigma' = (\gamma - \gamma_w) z$

So if u is high σ' is low.
But stiffness & strength proportional to σ'!
So high water table is bad.

For collapsing tunnel etc. $\sigma < \gamma z$



But if $u = \gamma_w z$ still the water component of lining stress is not reduced, only the effective component. The proportional effect of water at collapse is therefore even greater.

* Consider a leaky tunnel etc.
- seepage into the face as it is constructed can bring soil with it, leading to collapse & void migration
- seepage into lining during service can cause corrosion, and needs pumping away.

* What can be done?
- during construction; pump to lower water table, grout or freeze to restrict inflow, use compressed air (or pressurised bentonite) in headings, shafts etc.
- in service; design lining either to resist pressure or permit leakage.

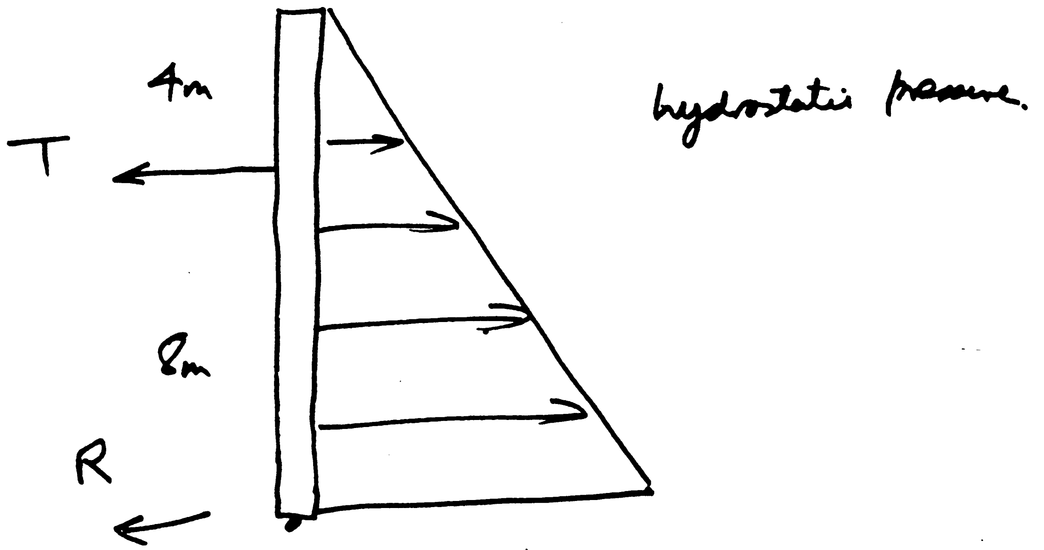
2b) * NATM: excavate freely eg with back hoe in soil or with roadheader in rock; spray primary concrete lining as soon as final surface is exposed; monitor movement and follow up with secondary lining system sufficient to reduce movement towards zero. Advantages – quick, cheap, permits ground to mobilise strength, therefore leads to minimal tunnel lining. Disadvantages – hard to control failure if collapse begins, vulnerable to brittle soils, requires skill & real-time intelligence, may lead to excessive surface settlements in clay.

* TBM – EPB: tunnel boring machine with picks housed on a head in a chamber pressurised by bentonite mud, or air etc. Muck has to pass either through a sealed port into the (unpressurised) tunnel, or through a circulation system under pressure which can segregate the muck from the bentonite mud, before the mud is recirculated. Advantages: elimination of surface subsidence, safe working conditions. Disadvantages: high cost, problems with blocking of muck route.

* Ideal for NATM – tunnel in weak rock, subsidence not critical. Ideal for TBM – tunnel in homogeneous soft clay with high water table, with a city above which demands small subsidence.

* NATM – fault zone leads to unheralded collapse; TBM – erratic boulders jam the picks; one examples of unforeseen problems

Qu 3 (a)



Density of water = 10 KN/m³

Water pressure at depth x = $10 . x$ KN/m²

∴ On a metre wide strip, total horizontal force

$$= \frac{120}{2} . 12 . 1 = 720 \text{ KN}$$

The resultant will act 2/3 of total height from the top i.e. 8m down

∴ R & T will both be 360 KN

∴ The bending moment will be

$$M = \frac{10x}{2} . x . \frac{x}{3} - 360\{x-4\}$$ — Macaulay brackets.

$$\text{Shear Force } F = \frac{10x}{2} . x - 360\{x-4\}^0$$

which is zero when $x = 8.48$ m

when $M = -596$ KNm/m width

Possible bar sizes 32 mm bars = 804 mm²/bar

= 7.9 bars/m

≡ 127 mm spacing.

40 mm bars = 1256 mm²/bar

= 5.03 bars/m

≡ 198 mm spacing.

Thus, 32 ϕ @ 125 mm
or 40 ϕ @ 175 mm } both acceptable.

– Practical considerations. Bigger bar requires more thickness to give both required depth and cover. Bigger spacing gives more possibility of cracking.

∴ <u>32ϕ @ 125</u> more likely

In bending the other way, at the tie position Neutral axis will be at a much smaller depth, since moment much lower. $(M = 106.7 \cdot 10^6$ Nmm$)$

Initially, guess $x = 0.2$ (value not critical)

$$0.87 \cdot 460 \cdot A_s \cdot 316 \left(1 - \frac{0.2}{2}\right) = 106.7 \cdot 10^6$$

$\Rightarrow A_s = 937$ mm²/m.

Recalculate.

$$x = 2.175 \cdot \frac{460}{40} \cdot \frac{937}{1000 \cdot 316}$$

$$= 0.074$$

Recalculate As

$$0.87 \cdot 460 \cdot A_s \cdot 316 \left(1 - \frac{0.074}{2}\right) = 106.7 \cdot 10^6$$

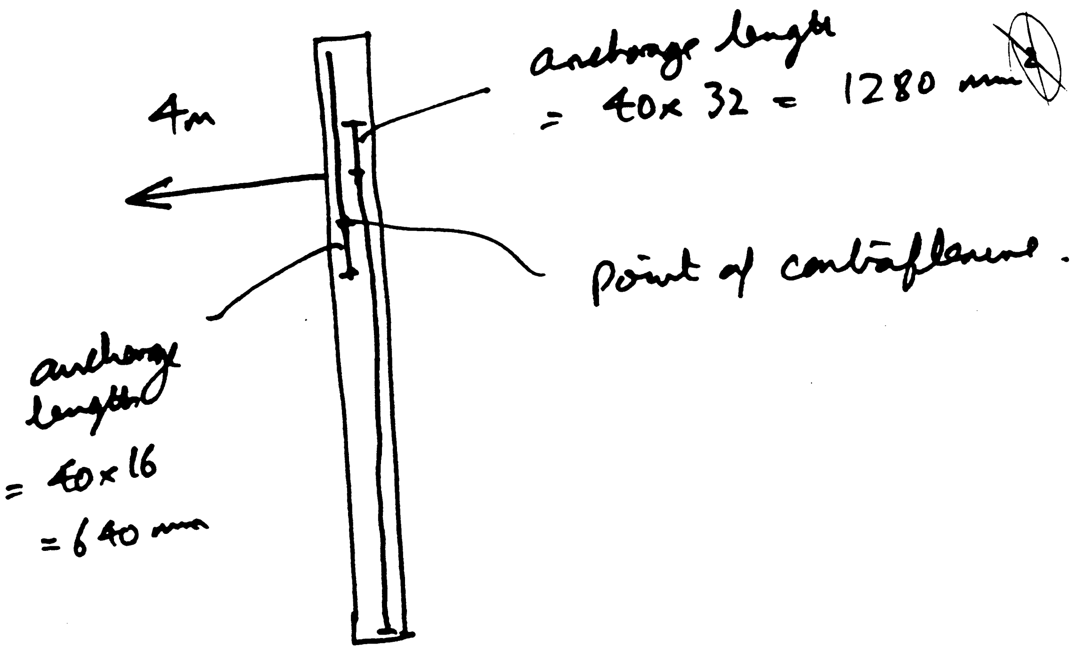
$\Rightarrow$ $A_s = 876\ mm^2/m$

(could iterate further, but not really necessary as integral No of bars is needed).

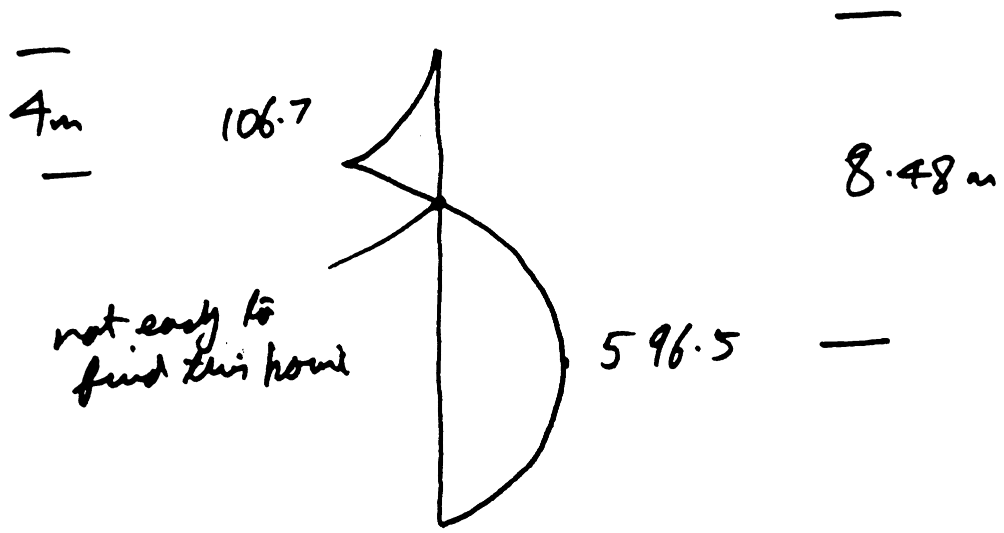
16 mm bars $= 201\ mm^2$ $\equiv 4.35$ bars/m

$= 229$ mm spacing.

Seems reasonable **16 ϕ @ 225 mm**



Moment distribution is (in kNm/m)



This is the loading with no safety factors applied to the load. However, it is impossible to overload a water tank, since it would overtop. ∴ No increase needed.

[note to marker — this argument should be explicitly stated. They could argue that the factor includes an allowance for ignorance and general uncertainty in modelling, in which case a value up to about 1.2 should be accepted].

(b) $M = 0.15 \; f_{cu} \, b d^2$

Say $M = 600$ kN m/m (rounded)

$f_{cu} = 40$ N/mm^2

$b = 1000$ mm

$\therefore \; 600.10^6 = 0.15 .\; 40 .\; 1000 .\; d^2$

$\Rightarrow d = \underline{316 \text{ mm}}$

This is effective depth.

Allow : 50 mm cover (since water retaining)

+ 15 mm for bar radius

$\Rightarrow$ Total thickness $\approx$ <u>380 mm</u>

can be confirmed after steel has been designed.

(c) If we take the effective depth as 316 mm, then the neutral axis will be at half depth (i.e. $xd = 158$ mm)

$\therefore \; 0.87 . f_y . A_s d \left(1 - \frac{x}{2}\right) = 600 . 10^6$

$0.87 . 460 . A_s . 316 . \frac{3}{4} = 600 . 10^6$

$\Rightarrow$ 6328 mm^2 of steel /m width

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Q 3 (d)

(i) A SLS criterion limiting the stresses in the concrete would be a better design philosophy than ULS in a water retaining structure such as this since this would result in smaller crack widths + thus reduce the likelihood of corrosion of the reinforcement & the possibility of leakage through the wall. ULS design only considers strength + does not examine cracking at all.

(ii) This would require a thicker section resulting in reduced curvatures. The amount of reinforcement required would depend on calculations however you would expect more steel in the section. This would result in lower steel stresses + narrower crack widths. In reinforced concrete cracking is inevitable hence the aim of the designer is to limit the size of these cracks.

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1997

Pt. IA Paper 8 Section B Answers

4. b) ~ 0.065 kgm

   d) ~ 12 Hz for a 1% speed fluctuation

5. ~~a) vibration from pig motion, ocean currents or noise from electronic circuits~~

   b) low-pass ~ 72 Hz

   high-pass ~ 3.4 Hz

   c) range ~ 0.5 – 2.5 kNm⁻¹ depending on choice of c and 3dB frequency

   ~~d) mechanical – low-pass only~~
   ~~electronic – easier to tune~~

6. –

Answers for (4) + (5)

+ Crits 4, 5, 6.

Please add + give to Ann Dowling's Sec –

See you next Friday Bill

Q4 (a) The eccentric moment for the eccentric mass may be derived by considering only the semi-cylindrical part of the mass, since the cylindrical part has a centre of mass on the axis of rotatation and therefore zero eccentric mass.

$$m.r = (m.r)_{semi-cylinder} + \cancelto{0}{(m.r)_{cylinder}}$$

For a semi-cylinder $m = \rho \cdot \frac{\pi}{2}\left(\frac{D}{4}\right)^2 \cdot \frac{D}{2} = \frac{\rho \pi D^3}{64}$

$$r = \frac{2}{3} \cdot \frac{D}{4} \cdot \frac{\sin \pi/2}{\pi/2} = \frac{D}{3\pi}$$ (5.4.2 p.16 Mechanics Data book)

Hence $m.r = \frac{\rho \pi D^3}{64} \cdot \frac{D}{3\pi} = \underline{\underline{\frac{\rho D^4}{192}}}$

(b) For a steel mass where $D = 0.2\,m$; $\rho = 7800\ \text{kg/m}^3$

Hence $m.r = \frac{7800 \times 0.2^4}{192} = \underline{\underline{0.065}}\ \text{kg m}$

(c) The speed fluctuation may be derived from consideration of the energy balance of the rotating system.
i.e. change in potential energy when centre of mass moves from highest to lowest position equals the change in kinetic energy.

Q4 (c) cont.

Change in potential energy = m g h, where h is the distance moved vertically by the centre of mass of the eccentric semi-cylinder. The cylindrical part of the mass shows no change in energy.

$\therefore$ Δ potential energy = m g h

$$= \overbrace{\rho \cdot \frac{\pi}{2} \cdot \left(\frac{D}{4}\right)^2 \cdot \frac{D}{2}}^{m} \cdot g \cdot \overbrace{2 \cdot \frac{D}{3\pi}}^{h}$$

note m is the mass of the semi-cylinder.

Change in kinetic energy $= \frac{1}{2} J \left(\omega_{max}^2 - \omega_{min}^2\right)$

If $\omega_{max} = \omega + \Delta\omega$ and $\omega_{min} = \omega - \Delta\omega$

then Δ kinetic energy $= \frac{1}{2} J \left((\omega+\Delta\omega)^2 - (\omega-\Delta\omega)^2\right)$

$= 2 J \omega \Delta\omega$

where J is the inertia of the whole eccentric mass

Now $J = 3 m k^2$ (total mass ≡ 3 semi-cylinders) where m is the mass of a semi-cylinder and k its radius of gyration

Hence $J = 3m \cdot \overbrace{\left(\frac{1}{2} \cdot \left(\frac{D}{4}\right)^2\right)}$ (5.5.1 p 17 Mechanics Data book)

and Δ kinetic energy $= 3 m \left(\frac{D}{4}\right)^2 \omega \Delta\omega$

Q4 (c) cont.

$$\Delta \text{ kinetic energy} = \Delta \text{ potential energy}$$

$$\therefore \quad 3\,m\left(\frac{D}{4}\right)^2 \omega\,\Delta\omega = m.g.\,2\,\frac{D}{3\pi}$$

rearrangement gives $\dfrac{\Delta\omega}{\omega} = \dfrac{32g}{9\pi\omega^2 D}$

(d) For the steel mass, assuming a 1% fluctuation is acceptable:

$$\frac{\Delta\omega}{\omega} = 0.01 = \frac{32g}{9\pi\omega^2 \times .2}$$

Hence $\omega = 74.5$ rad sec^{-1}

or $f = 11.9$ Hz

This is a lower limit, since a higher frequency would decrease $\Delta\omega/\omega$.

Q5. (a) Accelerometer noise typically arises from vibration of the pig moving along the pipe (high frequencies). Movement of spans due to ocean currents may introduce low frequency noise. There may also be electrical or mechanical noise generated within the vibrating pig.

(b) The voltage gain for the first amplifier is given by:

$$\text{gain} = -\frac{1}{1+j\omega RC_1}$$

which represents a low-pass filter with a turning frequency of $1/RC_1$ rad sec^{-1}.

The voltage gain for the second amplifier is given by:

$$\text{gain} = -\frac{1}{1+1/j\omega RC_2}$$

which represents a high-pass filter with a turning frequency of $1/RC_2$ rad sec^{-1}

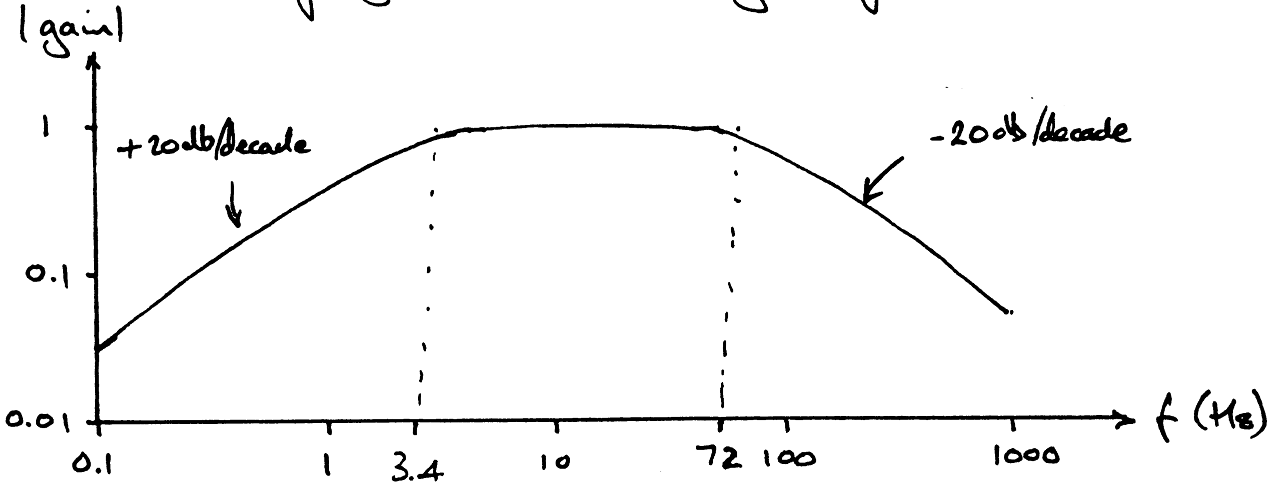
For $R = 10\,k\Omega$, $C_1 = 0.22\,\mu F$ and $C_2 = 4.7\,\mu F$:

low-pass turning frequency $= \frac{1}{2\pi}\cdot\frac{1}{RC_1} = \underline{\underline{72\,Hz}}$

high-pass turning frequency $= \frac{1}{2\pi}\cdot\frac{1}{RC_2} = \underline{\underline{3.4\,Hz}}$

Q5. (b) cont.

Sketch of gain vs frequency for filter:



(c) The equation of motion for the mass is given by:

$$m\ddot{y} = k(x - y) + \lambda(\dot{x} - \dot{y})$$

$$\therefore \quad \frac{m}{k}\ddot{y} + \frac{\lambda}{k}\dot{y} + y = \frac{\lambda}{k}\dot{x} + x$$

$$\text{c.f.} \quad \frac{1}{\omega_n^2}\ddot{y} + \frac{2c}{\omega_n}\dot{y} + y = \frac{2c}{\omega_n}\dot{x} + x$$

(4.6.3 p12 Mechanics Data book case (c))

Tuning frequency $\approx \omega_n = \sqrt{\frac{k}{m}} = 72 \times 2\pi$ rad sec^{-1}

$$\therefore \quad k = (2\pi \times 72)^2 \times 0.01 = \underline{\underline{2.1 \text{ kN/m}}}$$

Also $\frac{2c}{\omega_n} = \frac{\lambda}{k}$

$$\therefore \quad c = \frac{2\pi \times 72}{2} \cdot \frac{3}{2100} = 0.32$$

Q5 (c) cont.

The previous answer is an oversimplification of the problem, but would result in an adequate design even if the peak gain is a little high.

A better result would be obtained by choosing a sensible value for c for a moderate peak gain, eg. $c = 0.8$. This suggests that the 3dB point would then be a little above $2\omega_n$. To approximate:

$$2\omega_n = 72 \times 2\pi \text{ rad sec}^{-1}$$

$$k = (\pi \times 72)^2 \times 0.01 = \underline{\underline{510 \text{ N/m}}}$$

A more accurate answer could be found by letting

$$\frac{Y}{X} = \frac{1}{\sqrt{2}} = \frac{\{1 + (2c\omega/\omega_n)^2\}^{1/2}}{\{[1-(\omega/\omega_n)^2]^2 + (2c\omega/\omega_n)^2\}^{1/2}}$$

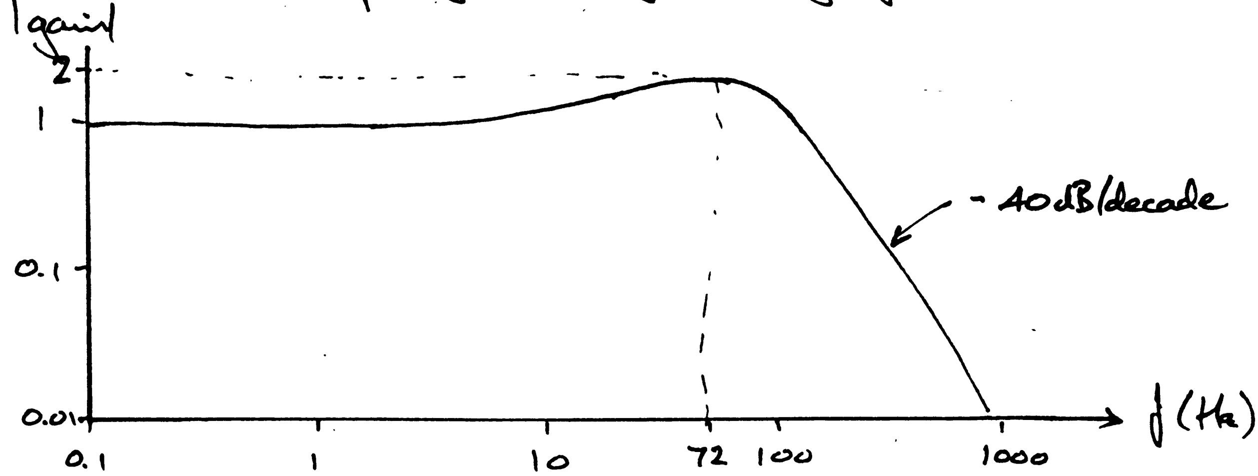
This is messy to solve, but possible. The graphical method is much easier.

In reality the 3dB frequency for the mechanical system would be increased to avoid the gain peak occuring at frequencies of interest.

A range of numeric answers would be acceptable.

Q5. (c) cont.

Sketch of gain vs frequency for filter:

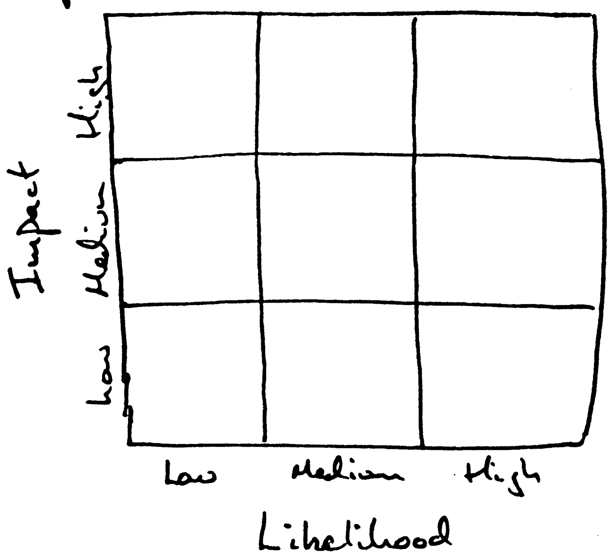


(d) The resonant frequency of the pipe is 15 Hz. Both filters will remove high-frequency noise above 72 Hz. Only the electronic filter will remove low frequency noise.

The electronic filter will most likely be most practical since it is easier to construct and tune to the correct frequencies and may be integrated with the charge amplifier which is already a part of the pig.

Q5. (a) (i) A likelihood / impact grid is a plot of known project risks against axes of likelihood of occurrence and impact.

e.g.



Project risk may be factors which effect development overrun, budget overspend, product or service performance, quality and safety.

Risks which appear in the upper-right sector of the plot must be actively reduced by either minimising their impact or by reducing their likelihood of occurrance.

The impact and likelihood scales are qualitative and should be chosen to illustrate the full range of risks.

Q6 (a) (ii) Phased development helps to reduce risk, especially if areas of high risk are investigated and tested early. Use of prototypes and models can be used to eliminate technical risks before committing to more detailed design.

In general:

Develop the concept → Develop the function → Develop the form → Develop the means of production

The stages of BS 7000 are consistent with this, with different emphases for different types of product.

(iii) HAZOP takes a product or process or service which is assumed to be sound and systematically assesses the consequences and likely causes of abnormal behaviour.

For each design parameter in each part of the product the questions of NONE, MORE OF, LESS OF, PART OF, MORE THAN and OTHER THAN are considered. If a deviation results in an undesirable effect (safety or operational) then the cause is investigated and the design modified to eliminate the effect. HAZOP is a team exercise.

Q5 (a)(iii) cont.

Results are usually tabulated:

Question	Effect	Cause	Action
MORE OF			

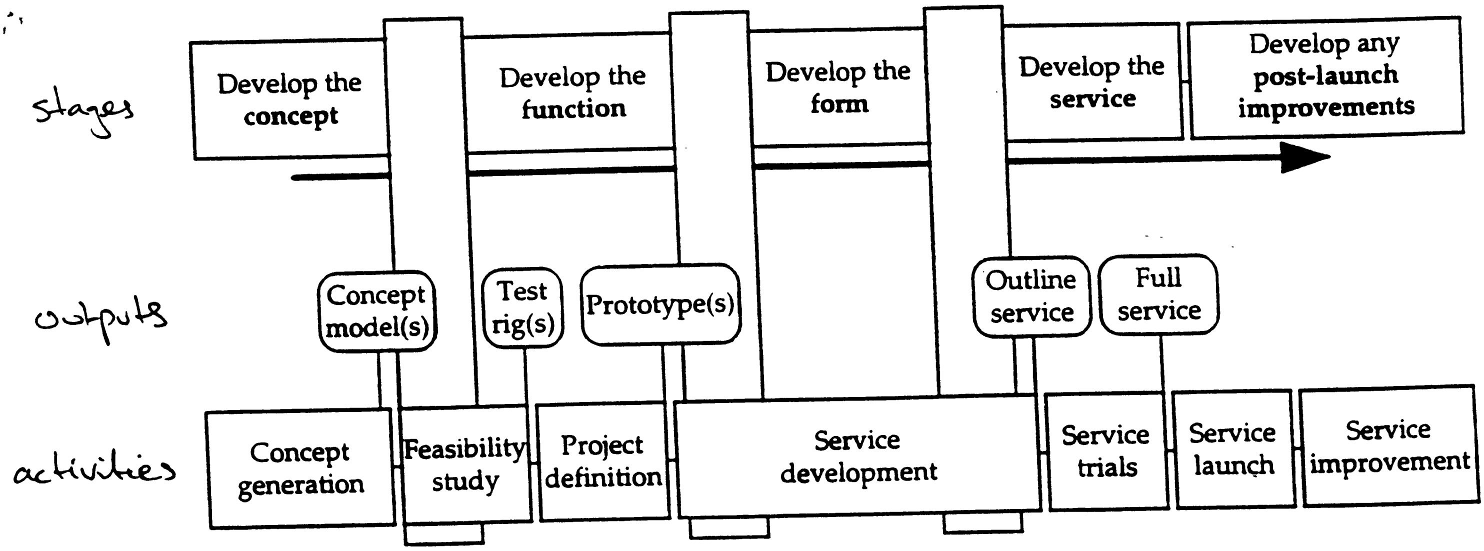
(iv) FMEA assesses the consequences of failure of a single component in a system. Failure may be total or partial and may effect more than one physical property. A component may be a stage in a service provision. FMEA may be a team or individual exercise.

Results are usually tabulated:

Failure mode	Effect	Action

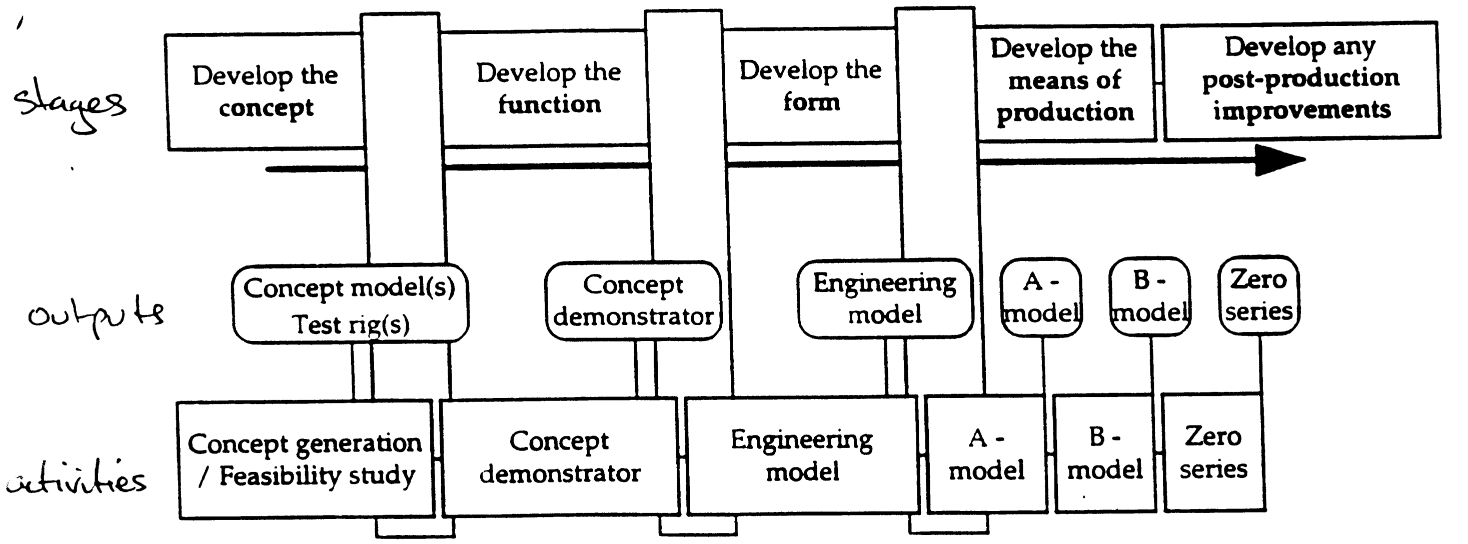
Failure modes are usually assessed for likelihood of occurrance before design actions are specified.

Q6 (b) Stages for service development are shown below:



Reference to BS 7000 is also appropriate.

(c) Stages for high-volume product development are:



Key differences are the needs to develop production systems which may take place in parallel with product development. There may be a range of models and demonstrators to aid development of different parts of a product, and to assist in developme of the production systems. Patents are important to revie

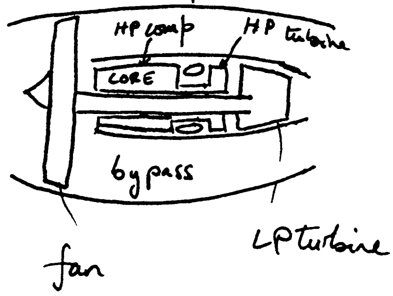
7 (a) $\eta_P = \frac{\text{useful work}}{\Delta KE_{gas}}$ $\eta_{cycle} = \frac{\Delta KE_{gas}}{\dot{m}_f \, LCV}$ $\eta_{ov} = \frac{\text{useful work}}{\dot{m}_f \, LCV}$

and useful work is the work done against the drag on the aircraft $= (\dot{m}_{air} + \dot{m}_{fuel}) V_J - \dot{m}_{air} V_1 \simeq \dot{m}_{air}(V_J - V_1)$
← flight speed

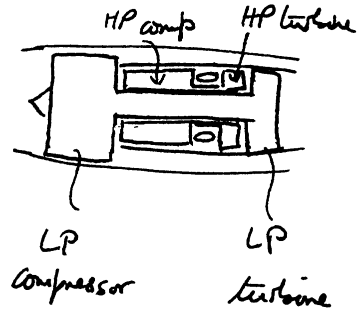
Flight speed does not scale directly with engine operating point.
∴ η_{cycle} same (same op. point), η_P & η_{ov} different

(b) $BPR = \frac{\dot{m}_{bypass}}{\dot{m}_{core}}$

Current bpr ≃ 8



Low bpr



(c) Cycles which deliver high power are also efficient only if the pressure ratio is high. A typical value is 40:1. Increasing the design value of turbine inlet temperature increases cycle efficiency to some extent (not by that much if today's typical values are taken as datum), but most importantly significantly increases the net power output from the ideal cycle. This power can be used to increase the power available to the fan and thus increase the propulsive efficiency through an increase in bypass ratio.

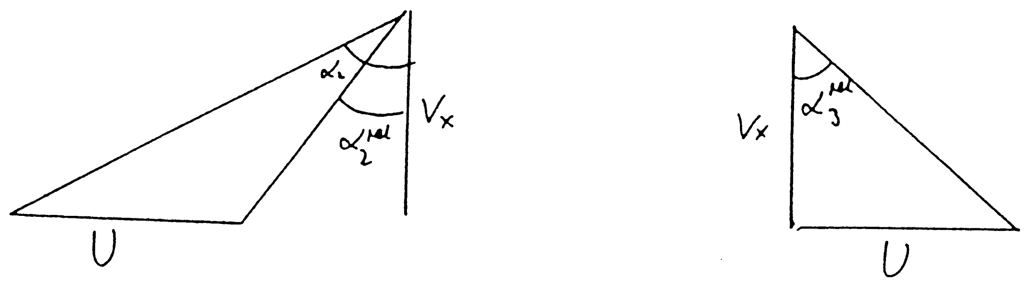
(d) In addition to low levels of propulsive efficiency and increased noise, a low bypass ratio engine has (for given core pressure ratio and turbine inlet temperature) a low ratio of $\frac{\text{top of climb thrust}}{\text{cruise thrust}}$ and of $\frac{\text{take off thrust}}{\text{cruise thrust}}$.

(e) $\eta_P \simeq \frac{2V_1}{V_J + V_1}$ increases as V_J is reduced. High bypass ratios reduce V_J and thus have better η_P (& η_0) than low bypass ones.

Bypass ratios are limited by: ratio of LP turbine speed / fan speed needs to be kept reasonable to avoid too many turbine stages / a gearbox; weight of engine increases with bypass ratio; greater size of engine (which implies higher manufacturing costs, more nacelle drag, a high and expensive undercarriage and potential problems with existing airport facilities).

8. (a)

Stator Exit ②　　　　Rotor Exit ③



(b) $\tan \alpha_3^{rel} = \dfrac{U}{V_x} = \dfrac{1}{.55} \Rightarrow \underline{\underline{61.2^\circ = \alpha_3^{rel}}}$

(c) $W_x = U(V_{\theta_2} - V_{\theta_3})$ & $V_{\theta_3} = 0$

$\therefore \dfrac{V_{\theta_2}}{U} = \dfrac{W_x}{U^2} = 1.5 \Rightarrow \dfrac{V_{\theta_2}}{V_{x_2}} = \dfrac{V_{\theta_2}/U}{V_{x_2}/U} = \dfrac{1.5}{.55} = \tan \alpha_2$

$\Rightarrow \underline{\underline{\alpha_2 = 69.9^\circ}}$

(d) Work output (per unit mass) = Change of enthalpy (per unit mass) (i.e. stagnation enthalpy)

$W_x = C_p \, \Delta T_0$

$\therefore \Delta T_0$ per stage $= \dfrac{W_x}{C_p} = \dfrac{1.5\,U^2}{C_p} = \dfrac{1.5\;350^2}{1005} = 183\,K$

ΔT_0 is same for each stage

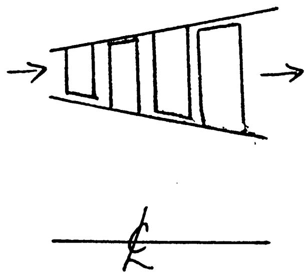
$\therefore \Delta T_0$ turbine $= 366$

$T_{0in} = 1450\,K \Rightarrow \underline{\underline{T_{0out} = 1084\,K}}$

$\eta_{is} = \dfrac{T_{0in} - T_{0out}}{T_{0in} - T_{0ut}^{is}} \Rightarrow \eta_{is} = \dfrac{1 - \frac{T_{0out}}{T_{0in}}}{1 - \left(\frac{1}{r}\right)^{\gamma-1/\gamma}}$　　r = pressure ratio

$\therefore \dfrac{1}{r} = .316 \Rightarrow \underline{\underline{r = 3.16}}$

(e)



mass flow in = mass flow out

i.e. $\rho_{in} V_{x in} A_{in} = \rho_{out} V_{x out} A_{out}$

Axial velocity constant $\Rightarrow \dfrac{A_{out}}{A_{in}} = \dfrac{\rho_{in}}{\rho_{out}}$

$$\frac{\rho_{in}}{\rho_{out}} = \frac{p_{in}}{T_{in}} \frac{T_{out}}{p_{out}} \simeq \frac{p_{0in}}{p_{0out}} \frac{T_{0out}}{T_{0in}}$$ using stagnation values

$$\therefore \frac{\rho_{in}}{\rho_{out}} = \frac{A_{out}}{A_{in}} = 3.16 \frac{1084}{1450} \simeq 2.4$$

9 (a) For high power cycles typical of high bypass ratio engines T_{04} should be as high as possible. A typical cruise value might be 1450 K. In addition, high power settings – takeoff, top-of-climb, involve temperatures higher than this. Creep is a real problem and T_{04} is chosen as a compromise between high efficiency and adequate turbine life.

To achieve high T_{04}, turbine blades are cooled, using air bled from the compressor, both internally and by ejecting this air through holes in the turbine blade to form a layer of cool air surrounding the blade. Better life is obtained using cast blades which are directionally-solidified or single crystal.

(b) $p_{02} = p_a\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$; $p_{02} = 47.5$, $p_a = 28.7 \Rightarrow \underline{M = .88}$

$T_{02} = \left(\frac{p_{02}}{p_a}\right)^{\frac{\gamma-1}{\gamma}} T_a$, $T_a = 225 \Rightarrow \underline{\underline{T_{02} = 259.8\ K}}$

(c) $T_{023} = 300$, $P_{023} = 75$, $\dot{m} = 100$ kg/s

$\frac{P_{03}}{P_{023}} = 25$, $\frac{P_{04}}{P_{045}} = 5.5$, $\eta_c = .88$, $T_{04} = 1450$.

(i) $\eta_c = \dfrac{\left(\frac{P_{03}}{P_{023}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_{03}}{T_{023}} - 1} \Rightarrow \frac{T_{03}}{T_{023}} = 1 + \frac{1}{\eta_c}\left(25^{\frac{\gamma-1}{\gamma}} - 1\right) = 2.714$

(& so $T_{03} = 814.3$)

Compressor $\Delta T = 514.3$, Turbine work = Compressor work

$\Rightarrow$ Turbine $\Delta T = \underline{514.3}$ ($\Rightarrow T_{045} = 935.7$)

$$\eta_T = \frac{T_{04} - T_{045}}{T_{04} - T_{045}^{IS}} = \frac{1 - \frac{935.7}{1450}}{1 - \left(\frac{1}{5.5}\right)^{\frac{\gamma-1}{\gamma}}} = \underline{\underline{92\%}}$$

(ii) $\dot{m}_f\, LCV = \dot{m}_{air}\, C_p\,(T_{04} - T_{03}) \Rightarrow \dot{m}_f = \dfrac{100 \times 1005(1450 - 814.3)}{43 \times 10^6}$

i.e. $\underline{\underline{\dot{m}_f = 1.49\ \text{kg/s}}}$

(d) $\dot{m}_f$ is seen by cycle as an <u>energy</u> flux $\dot{m}_f$ LCV (and <u>not</u> as a mass flow).

Assuming choked nozzles, etc, suitable variables with which to non-dimensionalise $\dot{m}_f$ LCV will be those appropriate to the flow at inlet to the engine.

Since energy flux = power = force × velocity

& force — $p_{02} D^2$, velocity — $\sqrt{\gamma R T_{02}}$

∴ In non-dimensional terms

$$\frac{\dot{m}_f \, LCV}{p_{02} D^2 \sqrt{\gamma R T_{02}}} = fn\,(op.\ point)$$

For the <u>same operating point</u> $\quad \dot{m}_f \propto p_{02} \sqrt{T_{02}}$

For ground test $\quad T_{02} = T_a \quad p_{02} = p_a \quad$ (288K & 100 kPa)

$$\therefore \quad \dot{m}_f = \dot{m}_f^{cruise} \frac{p_a}{p_{02}^{cruise}} \sqrt{\frac{T_a}{T_{02}^{cruise}}} = 1.49 \, \frac{100}{47.5} \sqrt{\frac{288}{259.8}}$$

$$= \underline{\underline{3.3 \ kg/s}}$$

10 Some points are as follows :-

H_2O

no impact ∵ $\ll$ natural emissions, condensing vapour

no control (generally)

CO_x

CO_2 is a greenhouse gas, emissions *may* contribute to global warming, ~7% of natural emissions

- control by reducing combustion (turn down central heating, alternative energy sources, higher efficiency) or dash for gas

CO poison

- control by better design of combustion systems (mixing, lean burn), or catalytic oxidation

NO_x

Photochemical smog, acid rain, only ~ 5% of natural emissions

- control : non-catalytic reduction, selective catalytic reduction, design burner to reduce residence time at high temperature

SO_x

$\gg$ natural emissions. Acid rain

- deforestation (maybe), acidification of lakes (maybe), damage to buildings
- control : switch fuel, flue gas desulfurisation, limestone in fluidised combustor, wash fuel

11 (a)

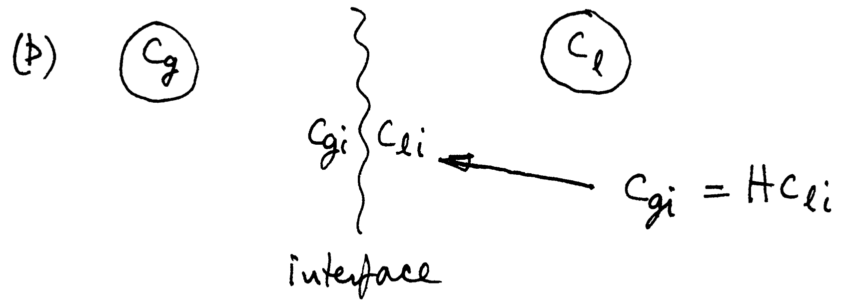
$$N_A = (N_A + N_B)\, x_A - D_{AB} \frac{dc_A}{dz}$$

$$N_B = (N_A + N_B)\, x_B - D_{BA} \frac{dc_B}{dz}$$

$$N_A + N_B = 0 \rightarrow D_{AB} \frac{dc_A}{dz} + D_{BA} \frac{dc_B}{dz} = 0$$

$$c_A + c_B = \text{uniform} \rightarrow \frac{dc_A}{dz} + \frac{dc_B}{dz} = 0 \rightarrow D_{AB} = D_{BA}.$$

(b)



$$N = k_g (c_g - c_{gi}) = k_l (c_{li} - c_l) = K_g (c_g - Hc_l)$$

$$\left.\begin{aligned} \frac{N}{k_g} &= c_g - c_{gi} \\ \frac{HN}{k_l} &= Hc_{li} - Hc_l \end{aligned}\right\} \quad N\left(\frac{1}{k_g} + \frac{H}{k_l}\right) = c_g - Hc_l$$

$$\rightarrow \frac{1}{K_g} = \frac{1}{k_g} + \frac{H}{k_l}$$

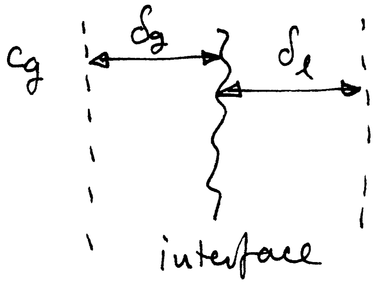
suspect: large H — liquid side control

small H — gas side control

strictly: $\frac{H}{k_l} \gg \frac{1}{k_g}$ — liquid side control

$\frac{H}{k_l} \ll \frac{1}{k_g}$ — gas side control

(c) Whitman



$$N = k_g (c_g - c_{gi}) = D_g \frac{c_g - c_{gi}}{\delta_l}$$

so $k_g = \dfrac{D_g}{\delta_g}$, and similarly $k_l = \dfrac{D_l}{\delta_l}$

NH_3 $\quad k_g = K_g = 4.0 \times 10^{-2}$ m/s

$$\delta_g = \frac{D_g}{k_g} = \frac{2.36 \times 10^{-5}}{4 \times 10^{-2}} = 5.9 \times 10^{-4} \text{ m}$$

CO_2 $\quad k_l = K_l = H K_g = 1.05 \times 1.9 \times 10^{-4}$

$= 1.995 \times 10^{-4}$ m/s

$$\delta_l = \frac{D_l}{k_l} = \frac{1.8 \times 10^{-9}}{1.995 \times 10^{-4}} = 9.0 \times 10^{-6} \text{ m}$$

SO_2 $\quad \dfrac{1}{K_g} = \dfrac{\delta_g}{D_g} + \dfrac{\delta_l}{D_l} = \dfrac{5.9 \times 10^{-4}}{1.46 \times 10^{-5}} + \dfrac{9.03 \times 10^{-3} \times 9.0 \times 10^{-6}}{1.9 \times 10^{-9}}$

$= 40.4 + 42.8 = 83.2$

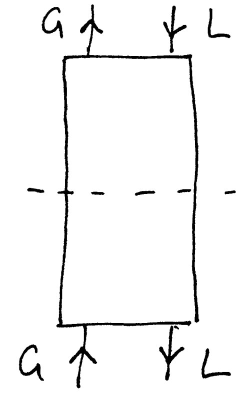
(↑ ↑ resistance from both sides)

$K_g = 1.2 \times 10^{-2}$ m/s

? Check that the ignored terms for NH_3 and CO_2 are negligible (they are) ?

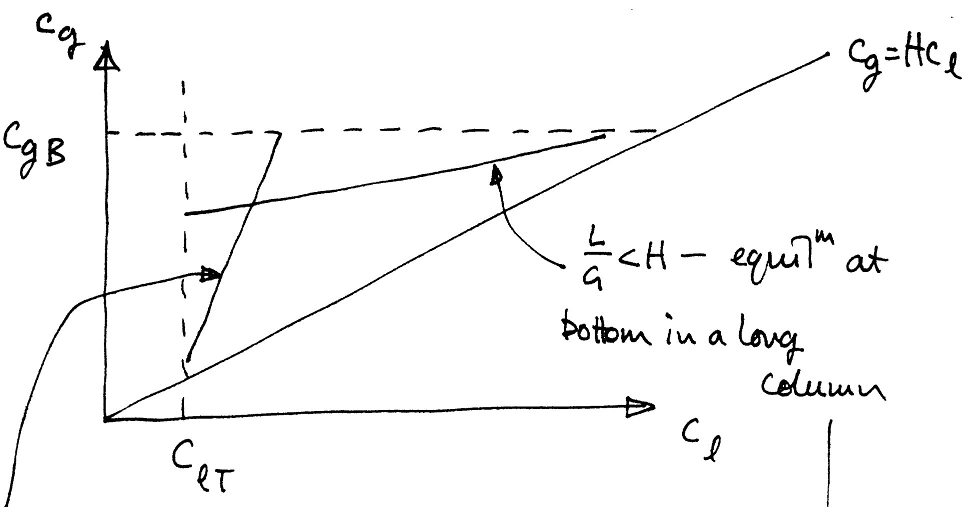
12 (a) Vertical cylindrical column is packed with solids eg rachig rings to increase interfacial area. Gas passes up, liquid down, mass transfer from one stream to the other. Gas impedes the flow of liquid (loading), eventually, at high enough gas flowrate, giving flooding.

(b)



$$G c_g + L c_{lB} = L c_l + G c_{gB}$$

$$c_g = \frac{L}{G} c_l + c_{gB} - \frac{L}{G} c_{lB}$$



$\frac{L}{G} > H$ – equil^m at top in a long column,
— then $c_{lT} = 0 \rightarrow c_{gT} \simeq 0$, and all pollutant removed

so $c_{lB} \simeq \frac{1}{H} c_{gB}$

$$c_{gT} = \frac{L}{G} c_{lT} + c_{gB} - \frac{L}{G} c_{lB}$$

$$\simeq 0 + c_{gB} - \frac{L}{GH} c_{gB} = \left(1 - \frac{L}{GH}\right) c_{gB}$$

(0 ← maybe)

(c) $\frac{HG}{L} = \frac{9.03\times10^{-3}\times 1.1}{0.012} = 0.828 < 1$ can remove 'all' pollutant if column long enough

$$\left. \begin{aligned} G\left(c_{gB}-c_{gT}\right) &= K_g A \left\{ \frac{(c_{gB}-Hc_{lB}) - c_{gT}}{\ln\left[(c_{gB}-Hc_{lB})/c_{gT}\right]} \right\} \\ G\left(c_{gB}-c_{gT}\right) &= L\, c_{lB} \end{aligned} \right] c_{lT}=0$$

$$\rightarrow \quad c_{lB} = \frac{G}{L}\left(c_{gB}-c_{gT}\right)$$

$$G\left(c_{gB}-c_{gT}\right) = K_g A \frac{\left(1-\frac{HG}{L}\right)\left(c_{gB}-c_{gT}\right)}{\ln\left[\frac{\left(1-\frac{HG}{L}\right)c_{gB} - \frac{HG}{L}c_{gT}}{c_{gT}}\right]}$$

$$\ln\left[\left(1-\frac{HG}{L}\right)\frac{c_{gB}}{c_{gT}} + \frac{HG}{L}\right] = \frac{K_g A}{G}\left(1-\frac{HG}{L}\right)$$

$$A = \frac{G}{K_g\left(1-\frac{HG}{L}\right)} \ln\left[\left(1-\frac{HG}{L}\right)\underbrace{\frac{c_{gB}}{c_{gT}}}_{10} + \frac{HG}{L}\right]$$

$$= 712.1 \text{ m}^2 = \frac{\pi d^2}{4} a h$$

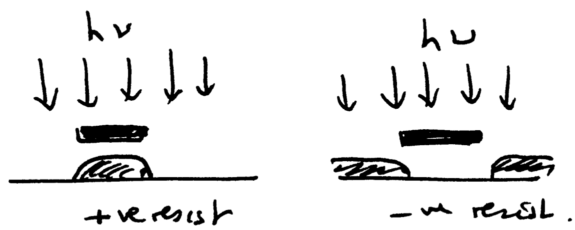
$$h = \frac{4\times 712.1}{\pi\times 1.2^2\times 194} = \underline{3.2 \text{ m}}.$$

To get the same reduction in SO_2 at higher G, you need to keep G/L constant ... but then you run the risk of flooding. At fixed L, you will not get so good SO_2 removal.

Cribs Paper 8 - Section E

13 (a) and (b)

Positive and Negative Resist



Photoresist is a radiation sensitive compound. They can be classified as positive or negative, depending on how they respond to radiation.

For positive resists the exposed regions become more soluble and thus more easily removed in the development process. The net result is that patterns formed in the positive resist are the same as those on the mask. For negative resists the exposed regions become less soluble and the patterns formed in the negative resist are the reserve of the mask patterns (see above figures).

Depth of field - the region over which the image remains in focus. If variation in height of the surface is > depth of focus then get degradation of image.

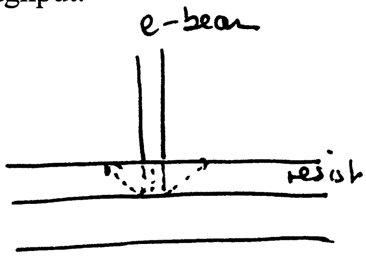
Resolution - minimum feature size that can be reproduced repeatedly.

(i) UV optical exposure is the standard industrial process. High throughput and whole wafer exposure is possible.

The spectrum from a typical mercury UV lamp shows the strongest emission at $\lambda = \approx$ 365 nm. The practical limit in UV lithography however is diffraction at the edges of the mask used for contact printing.

eg 0.5 microns for 365 nm radiation

(ii) E-beam exposure is a well developed but expensive process. Serial exposure using an e-beam focused to a small spot of typical diameter 0.1 micron. The desired pattern is traced using raster or vector scan. Low throughput.



eg For 30 keV $\lambda = \dfrac{h}{(2mE)^{\frac{1}{2}}}$

thus $\lambda = 7\times10^{-12}\ m$

The practical limit in e-beam lithography is NOT the diffraction limit to the spot because the wavelength is so small but the scattering of secondary electrons which expose the resist when they are reflected as shown in the diagram.

(iii) X-ray lithography is new technology which can expose whole wafers and therfore is capable of high throughput BUT it is in its infancy and still numerous problems to be overcome.

The practical limit in X-ray lithography may be ≈ 0.1 micron determined by the range of secondary electrons produced in the resist but at present the main problem is how to make an effective, stable mask.

$$\text{Resolution} = \left(\frac{0.5\lambda}{NA}\right) \approx 0.4\ \mu\text{m}$$

$$\text{Depth of focus} = \frac{\lambda}{(NA)^2} \approx 2.77\ \mu\text{m}$$

Resolution not good enough therefore use shorter wavelength light if photolithography to be used or X-ray or .....

Depth of focus also not good enough therefore would use step and repeat or other lithography process.

14. a) Electron diffraction by crystal. Line spectrum of optical emission by atoms - Bohr atom.

b) $E = 0.1.1.6.10^{-19}$ J

$E = (h/\lambda)^2/2m$

$\lambda = h/\sqrt{2m^*E} = 6.6.\ 10^{-34}/\sqrt{(2.0.9.10^{-30}.0.1.0.1.1.6.10^{-19})} = 1.3.10^{-8}$m

c) classical - reflection by the barrier
quantum - finite probability of tunnelling, because the KE is positive again on the RHS of barrier.

d) LHS $\psi = A\exp(jkx) + B\exp(-jkx)$
(incoming + reflected wave)

inside barrier
$\psi = C\exp(-k'x) + D\exp(k'x)$
(right going evanescent wave + reflected evanescent wave)

RHS outside barrier
$\psi = F\exp(jkx)$

A, B, C, D, F all real constants to be found by setting continuity conditions ψ and $d\psi/dx$ being continuous across each junction barrier.

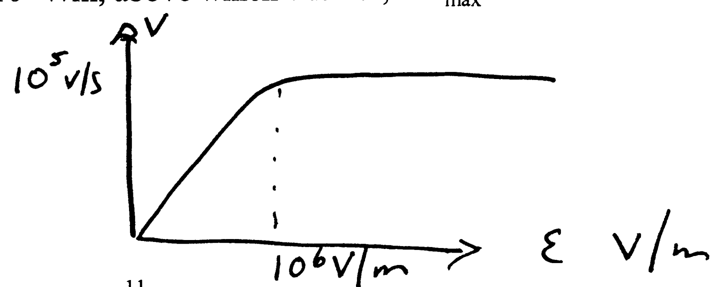
$k = \sqrt{(2m^*/E)}/\hbar$, $k' = \sqrt{(2m^*[V-E])}/\hbar$

Tunnelling probability = F^2/A^2

Probability drops by factor e^2 when Lk' = 1. k' ≈ 10^8 m-1, from above. So L = 10^{-8}m.

15. $\mu = 0.1 m^2/V.s$
$v_{max} = 10^5 m/s$

a) Sketch shows linear increase of velocity with field E, until v = 10^5 m/s and E = 10^6 V/m, above which v is flat, at v_{max}.



b) $\tau = 10^{-11}$ s

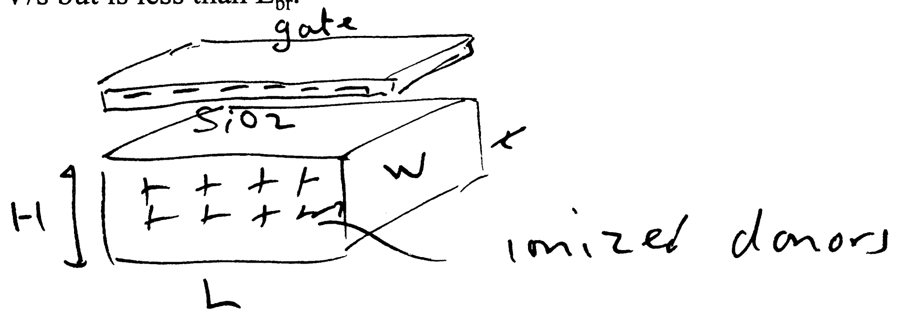
V = 2V

Assume velocity is in the scattering limited regime for the moment, then check this.

$L = v_{max}.\ \tau = 10^5.\ 10^{-11} = 10^{-6}$ m

$E = V/L = 2/10^{-6}$ = V/s. This exceeds the threshold for scattering limited regime which is 10^6 V/s but is less than E_{br}.

c)



Enclosed charge to a depth x is = Q = q. A. x

electric displacement at bottom surface, D = 0

By Gauss' theorem, electric displacement at top surface is

A..D. = Q = q. A.x

so $E = D/\varepsilon = \frac{qAx}{A\varepsilon} = \frac{qx}{\varepsilon}$

$dV = -E.dx$

$V = \int E.dx = -\int (q.x/\varepsilon).dx$

$= q.H^2/2\varepsilon$

capacitance per unit area $C' = Q/(V.A); Q = q.A.H;$

so $C' = q.A.H/(q.A.H.H/2\varepsilon) = 2\varepsilon/H$

$C = L.W.C'$

$L = 10^{-6}$m, W/L = 10,
so $W = 10^{-5}$m and $C = 10^{-6}.10^{-5}.2.10^{-10}/10^{-6} = 2.10^{-15}$F.

d) $R = L\rho/(W.H)$, $\rho = 0.01$ ohms.m

so $R = 10^{-6}.\ 10^{-2}/(10^{-5}.10^{-6}) = 10^{3}$ ohms

and $\tau' = RC = 2.10^{-15}.\ 10^{3} = 2$ps.

Thus the simplest RC time constant delay due to an internal device capacitance is less than the transit time, and the transit time dominates.

1B Paper 8 1997 - Exam Cribs

Question 16

1st Part: Description of hashing is bookwork. It is assumed that information is to be stored and retrieved based on a 'key' (eg a number, a name, etc). The basic principle is then to use a one-many mapping (called a hash function) to partition the space of these search keys into a number of subsets. Within each subset, the data is typically stored in a conventional list or array structure. Given a search key, an item is found by first locating the subset using the hash function and then making an exhaustive search for the specific item required (eg by a simple linear search). If each subset is small, then a linear search is still quite quick. A good answer might also mention the distinction between closed and open hashing schemes.

The hash function needs to be chosen such that it evenly distributes the data items amongst the subsets and that it is cheap to compute.

[5 marks /20]

The posed storage problem appears to be different to the examples given in lectures in that there are two lookup keys: SA and DA. Whilst this would severely complicate some schemes, using hashing, it only affects the hash function. It does not necessitate any modifications to the basic data structure.

2nd Part: Data Structure [6 marks /20]

```
CONST hashmax = ????;  { a large prime number }

TYPE  Link = ^Entry;

    Entry = RECORD { linked list of equivalent hash entries }
                sa,da: Address;
                sc: SecurityCode;
                next: Link
            END;

    HashIndex = 1..hashmax;
    HashTable = ARRAY [HashIndex] OF Link;   { all NIL initially }

VAR   htab: HashTable;
```

3rd Part: Function [6 marks /20]

```
FUNCTION LookUp(sa: Address; da: Address) : SecurityCode;
  VAR h:HashIndex;
    l:Link;
    Boolean found;
```

```
BEGIN
  h := (sa * da) MOD hashmax + 1;   { or similar }
  l := htab[h]; found := FALSE;
  WHILE NOT found AND (l <> NIL) DO BEGIN
    found := (sa = l^.sa) AND (da = l^.da);
    IF NOT found l := l^.next
  END;
  IF found THEN
    LookUp := l^.sc
  ELSE
    LookUp := 0
END;
```

4th Part: Comments [3 marks /20]

The effects of correlation in SA and DA would depend on the design of the hash function but in general, it will make the allocation of SA/DA pairs to hash lists less evenly distributed thereby increasing retrieval time. Note however that the "mod primenumber" operation will minimise the effect of such correlations.

Question 17

a) In a connection-oriented protocol, a connection is first established. This allows the two communicating systems to retain per-connection state information about one another thereby enabling error correction, ie requests for retransmission of lost or corrupted packets identified by a unique sequence number; and data flow control by indicating how many additional packets or bytes may be sent.

In a connectionless protocol, each packet is an independent event and no state information is retained about the other end.

The advantage of connection-oriented protocols is that they can provide flow control and error handling. The advantage of connectionless protocols is simplicity: the lack of the need to set up a connection and for either end to retain any state information. In the case of lower layer protocols such state information would need to be maintained by all the gateways along the route between two end systems. Also Broadcast schemes are inherently connectionless and are difficult and wasteful to implement in a connection-oriented framework. [6 marks /20]

b) A transport layer packet consists of the header for this layer, preceded by those for the lower layers and followed by the data being transferred. This data may in turn contain header information for higher layers and in the same way, the transport layer header plus the data is viewed by the layer below (the network layer) as simply data. The overall packet thus looks like:

```
 -----------------------------
|  Physical Layer Header     |
```

```
-----------------------------
| Data Link Layer Header   |
-----------------------------
| Network Layer Header     |
-----------------------------
| Transport Layer Header   |
-----------------------------
|         Data         |
-----------------------------
```

The physical layer header will typically at most amount to a unique pattern of ones and zeros used to synchronise the local oscillators at sender and receiver and to identify the start and end of packets. There may also be a checksum for low level error detection.

The main essential fields in the next three layers are those associated with addressing and in ISO/OSI reference model terms identify the Service Access Point (SAP) of the interface to the layer above. There may also typically be fields giving the length of the packet and/or header and a checksum for the header and/or complete packet. Looking at the address fields in more detail:

Data Link Layer - the source and destination MAC (or "hardware") address plus information to identify the Network Layer protocol.

Network Layer - the source and destination network address (used to route packets from end system to end system via gateways connecting different physical networks) plus information to identify the Transport Layer protocol.

Transport Layer - strictly the address information here identifies the Session Layer SAP but in practice the normally bears a one-to-one correspondance with the application and thus identifies a particular instance of a particular application, eg a particular login session. [7 marks /20]

c) The main function needed of the network analyser is similar to that of the traffic command described in lectures except that grouping other than by MAC source and MAC destination address will be required. The specific information needed in each case is:

i) very like that in the traffic command except that only the source address is of interest in grouping the packets. The hardware must be set to receive all packets.

ii) this is a similar problem except that we no longer require the hardware to receive every packet; indeed it may be used in its normal operational mode as it will receive all broadcasts.

iii) we now need first to use the technique in (i) to identify which computer is causing the problem and then, as a second separate investigation, group each packet originated by it by unique transpor layer address (ie unique combination of network layer and transport layer header address information). It will not be possible with normal network interface hardware to do the filtering required in this second stage (this provides filtering on destination rather than source MAC address) so all packets will have to be received and then filtered using software. [7 marks /20]

Question 18

In polled mode, the cpu repeatedly tests a status bit to determine when it can write or read data. In interrupt-mode, the cpu can proceed with other tasks. When the device is ready for input or output, an interrupt is generated. This causes the current cpu state to be saved and an interrupt service routine executed. The latter normally terminates by restoring the cpu state to resume the interrupted program. Polling wastes cpu but is also fast since it avoids the need to save and restore state information.
[3 marks /20]

The device registers are simply described by Pascal packed records

```
TYPE Resolution  = 0..7;

   StatusReg = PACKED RECORD
                 dataReady: Boolean;
                 errCode:  0..7;
                 unused: 0..15
               END;

   ControlReg= PACKED RECORD
            reset: Boolean;
                 intEnable: Boolean;
                 xRes: 0..7;
                 yRes: 0..7
               END;

VAR data   : Byte @ 0FE000H;
   stat  : StatusReg @ 0FE001H;
   cont   : ControlReg @ 0FE001H;
```

[6 marks /20]

The required interface module would be very simple since all it needs is an initialisation routine, a function to indicate how many screen presses are available and a procedure to get the next screen press in the queue, ie

```
MODULE TouchScreen;
EXPORT
  TYPE Position = RECORD
              x,y:Byte
                 END;
```

```
  PROCEDURE InitTS(xres, yres: Resolution);
  FUNCTION  NumTouches: integer;
  PROCEDURE GetPosition(VAR pos: Position);
END;
```

[4 marks /20]

The interrupt service routine simply reads the data register twice and stores a position in the buffer. Only real problem is what to do about overflow.

```
CONST buffersize = 100;
VAR   buffer : ARRAY[1..100]OF Position;
      used,next : Integer;

INTERRUPT PROCEDURE TSInt@0200H;
  VAR p:Position;
BEGIN
  IF stat.errCode <> 0 THEN error ...;
  p.x = data; p.y = data;
  IF (used = buffersize) THEN error ...;
  buffer[next] = p;
  next = next MOD buffersize + 1;
  used = used + 1
END;
```

[5 marks /20]

In a multiprocess environment, an event would be defined for the buffer not empty condition. When a process calls GetPosition and NumTouches is zero, the process is sent to sleep. TSInt then sends the buffer not empty event every time it is executed in order to wakeup any sleeping processes.

[2 marks /20]

1997 Part IB Paper 8 (Section A)
Solutions

1. (a) 1820 kg/m^3, 0.99. (b) 39kPa, 133kPa (d) (i) 44.9°, 35.7° (ii) 60kPa

(e) 10kPa, -7kPa

3. (a) T = 360 kN (b) 316mm
Mm..= 596.5 kNm

4. (b) ~ 0.065 kym (d) ~ 12 Hz for a 1% speed fluctuation

5. (b) low-pass ~ 72 Hz
high-pass ~ 3.4 Hz

(c) range ~ 0.5 - 2.5 kNm^{-1} depending on choice of c and 3db frequency

6. -

7. (c) About 40:1

8. (b) 61.2° (c) 69.9° (d) 1084 K, 3.16 (e) 2.4

9. (b) .88, 259.8K (c) (i) 514.3 & 92% (ii) 1.49 kg/s (d) 3.3 kg/s

11. (c) $Kg = \frac{Dg}{\delta g}$, $Ke = \frac{D\ell}{\delta \ell}$

(d) 9 x 10^{-6}m, 1.2 x 10^{-2}ms^{-1}

12. (c) h = 3.2m

13. Resolution ~ 0.4 μm, Depth of focus ~ 2.77 μm

14. $\approx 10^{-8}$m

15. L = 1 μ, C = 2 x 10^{-15}F, R = 10^3 Ω