ENGINEERING TRIPOS PART IB

Monday 2 June 1997 9 to 11

Paper 1

MECHANICS

Answer not more than **four** *questions.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

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1 (a) The thin uniform circular annulus shown in Fig. 1(a) has an external radius a and an internal radius b . The mass of the annulus is m . Find from first principles the moment of inertia of the annulus about its diameter A-A. If you make use of the perpendicular-axis theorem then derive it from first principles. [10]

(b) A centrifuge comprises a thin uniform circular disc whose mass is 96 kg and whose radius is 1 m. The disc spins at a constant angular speed of $\omega = 3600$ rad/s on a vertical shaft through its centre. In what way does the earth's rotation (shown as Ω in Fig. 1(b)) affect the operation of this centrifuge when it is used at the equator? Compute the magnitude of the couple acting on the disc and indicate on a sketch its direction. [10]

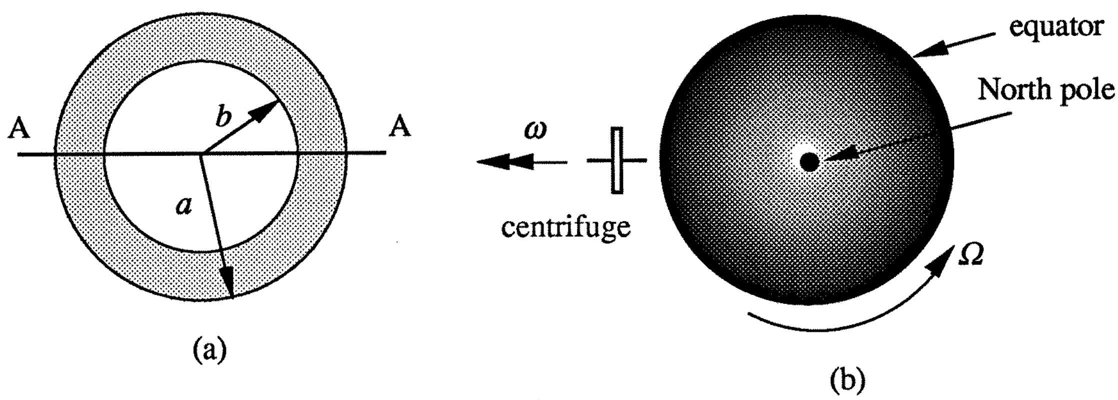

Fig. 1

2 (a) An insect of mass 1g is walking across an LP record (diameter 0.3 m) as it is spinning steadily on its turntable clockwise at $33\frac{1}{3}$ revolutions per minute. Describe carefully how the centripetal and Coriolis forces acting upon the insect change as it moves at constant speed diametrically from one edge of the record to the opposite edge supposing that it takes the insect 1 minute to traverse the diameter. What force does it experience as it passes by the centre of the record? [10]

(b) Three rotors A, B and C are fixed to a shaft each separated by a distance of 100 mm as shown in Fig. 2. The out of balance of rotors A, B and C are respectively 0.005 kg m , 0.012 kg m and 0.013 kg m. Determine the relative angular position of the rotors on the shaft required in order to achieve static balance. In this statically-balanced configuration what is the magnitude of the dynamic out-of-balance couple when the shaft is spinning at 1000 rad/s? [10]

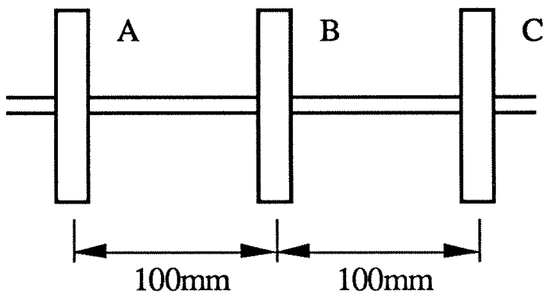

Fig. 2

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3 A cyclist is wondering why his chain falls off when he goes around corners. He suspects that the Coriolis acceleration may be having a significant effect. Figure 3(a) shows the chain wheel and chain of his bicycle. A detail of these components is shown in Fig. 3(b).

(i) What is the acceleration of the centre Q of the chain wheel when the bicycle is moving at a constant forward speed V around a corner of radius R ? Assume that the bicycle remains vertical when cornering. [2]

(ii) Find a general expression for the acceleration of a general point P on the chain wheel at an angle θ from the highest point A as shown. The radius of the chain wheel is a . You might find it useful to use the unit vectors $\boldsymbol{k}$ and $\boldsymbol{e}_1$ fixed in the frame of the bicycle and the unit vector $\boldsymbol{e}_2$ fixed in the chain wheel as shown. [12]

The chain wheel has a radius of 100mm and it is rotating at $\dot{\theta} = 10$ rad/s when the bicycle is moving with forward speed $V = 5$ m/s on a corner of radius $R = 25$ m .

(iii) Compute the various components of acceleration of the chain at the two points A and B shown and identify the Coriolis acceleration in both cases. [4]

(iv) How do the magnitudes of the various inertia forces acting on the chain compare with the weight of the chain? [2]

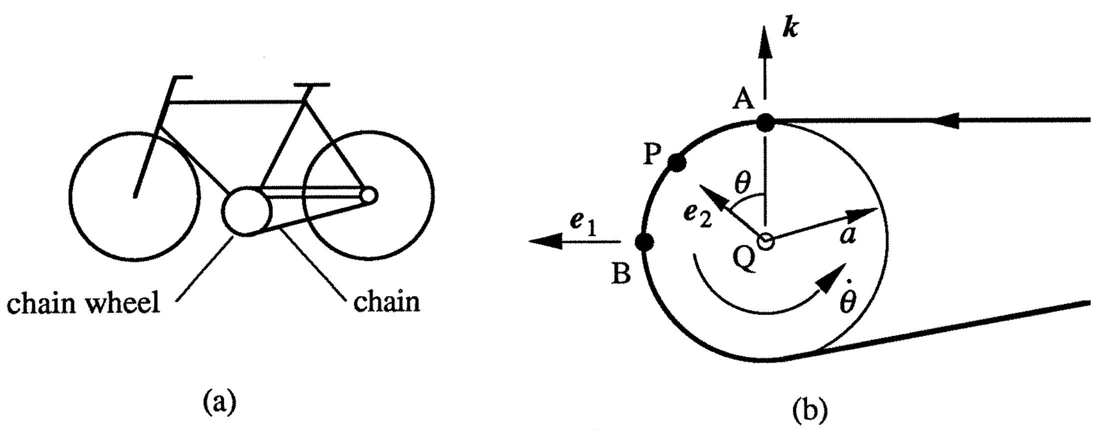


Fig. 3

4 A small bead of mass m is free to slide on a smooth horizontal ring of radius a as shown in Fig. 4. The ring is fixed at O to a vertical shaft which rotates with angular velocity ω and acceleration $\dot{\omega}$. At any instant the bead is at P and the angle OAP is θ . Unit vectors $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are defined along OA and AP respectively.

(i) Find vector expressions for the position, velocity and acceleration of the bead. [8]

(ii) By considering the forces which act on the bead find an expression for $\ddot{\theta}$ in terms of θ, ω and $\dot{\omega}$. [6]

(iii) At a specific instant in time the bead is at $\theta = \pi/2$ and it is not sliding. Show that the bead will never begin to slide on the ring provided that $\dot{\omega} = \omega^2$. [2]

(iv) At time $t = 0$ the bead is at $\theta = \pi/2$ and it is not sliding . The shaft angular velocity ω is negative $(\omega < 0)$. Show that it is possible to bring the shaft to rest $(\omega = 0)$ without ever causing the bead to slide. Sketch the graph of ω with time. [4]

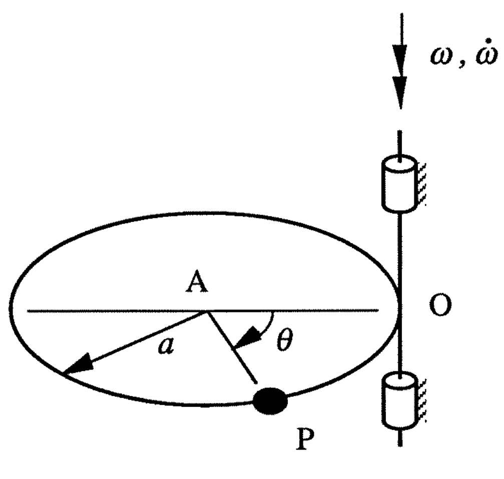


Fig. 4

5 The floor-scraper mechanism shown in Fig. 5 comprises a heavy scraper bar AB of length $4a$ attached at A to a drive rotor of radius a . The centre of the rotor is distance $2a$ above the smooth horizontal floor upon which end B of the scraper slides freely. The rotor spins at a constant angular velocity ω. For correct operation it is important that the scraper maintains contact with the floor at all times.

For the mechanism in the configuration shown (with OA horizontal) find the angular velocity of the scraper bar and show that the magnitude of the angular acceleration of the bar is [4]

$$\frac{\omega^2}{12\sqrt{3}}$$ [6]

Assuming AB to be a uniform bar four metres in length find the contact force at B and thus deduce the angular speed ω above which contact is lost in this configuration. [8] [2]

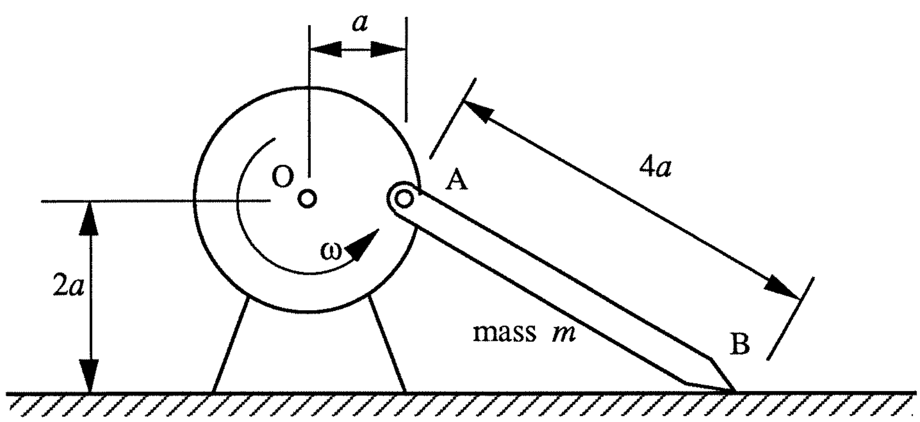


Fig. 5

6 The solid, uniform cylindrical roller shown in Fig. 6(a) has a mass of 20 kg and a radius of 0.1 m. It rests at C on a rough horizontal surface. A horizontal copper wire AB is held fixed at A and fastened to the cylinder at its highest point B. The cylinder is then struck at time $t = 0$ with an *ideal* horizontal impulse I at D as shown. The wire begins to stretch and it breaks at time $t = 12$ ms. The force in the wire is measured as shown in Fig. 6(b) and during the period of time $0 < t < 12$ ms the effect of friction at C should be neglected.

After the wire has broken the roller skids for some time before rolling away with no slip at a speed of 1 m/s.

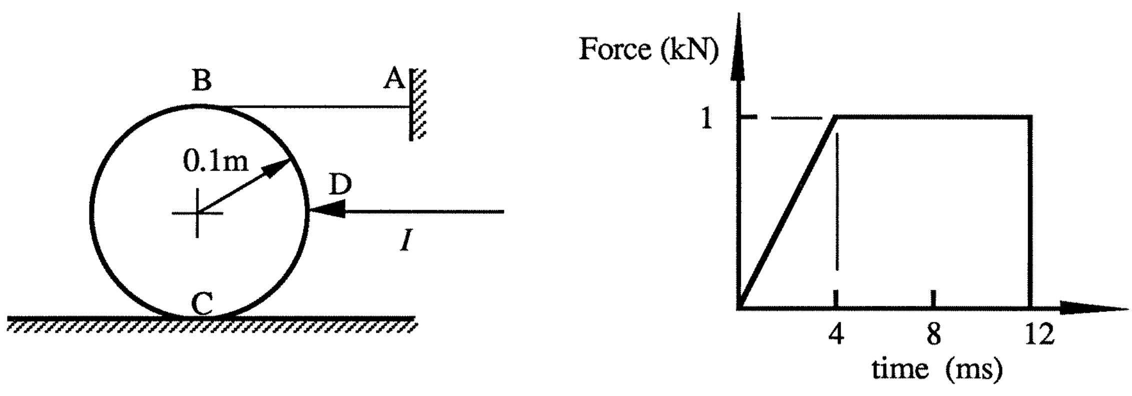
(i) What is the magnitude of the impulse given to the roller by the wire? [4]

(ii) Calculate the angular velocity of the roller at time $t = 12$ ms. [4]

(iii) Find the velocity of the centre of the roller immediately after the application of impulse I. [6]

(iv) What is the magnitude of I ? [4]

(v) When might it *not* be appropriate to neglect friction at C for the period of time $0 < t < 12$ ms? [2]



(a)

Fig. 6

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