ENGINEERING TRIPOS PART IB

Monday 2 June 1997 2 to 4

Paper 2

STRUCTURES

Answer not more than **four** *questions.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

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1 (a) Calculate the elastic torsion constant J of the square hollow section shown in Fig. 1(a). [5]

(b) Figure 1(b) shows a beam of length $2L$ with fixed ends. A couple M is applied at its centre. If the beam remains everywhere elastic and deflections are small, show that the central slope θ is given by

$$\theta = \frac{ML}{8EI}$$

where EI is the relevant flexural rigidity of the beam. [7]

(c) A bar BQ of length L is welded to two identical parallel strips AC and PR each of length $2L$ as shown in Fig. 1(c). These strips are fully fixed at their supports A, C, P and R. A torque T is applied at the midsection D of the bar BQ.

(i) If the structure remains everywhere elastic, show that the midsection D twists through an angle α given by

$$\alpha = \frac{TL}{16}\left(\frac{1}{EI} + \frac{4}{GJ}\right)$$

where GJ is the torsional rigidity of bar BQ and EI is the relevant flexural rigidity of strips AC and PR. [3]

(ii) The bar BQ of Fig. 1(c) has the square hollow section shown in Fig. 1(a). The strips PR and AC have cross-sectional dimensions b = 80 mm and t = 6 mm as shown in Fig. 1(c). The length $L = 400$ mm. All members are made of steel. Assuming the structure remains everywhere elastic, calculate the angle of twist (in degrees) at the midsection D when T = 1 kNm. [5]

(Cont.

Fig. 1(a)

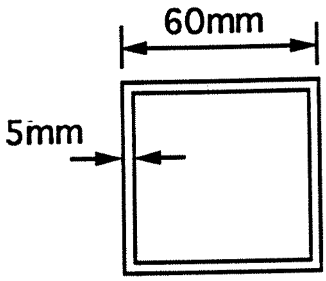


Fig. 1(b)

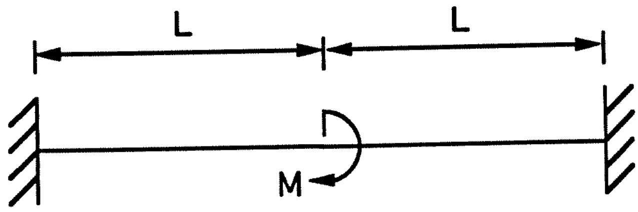
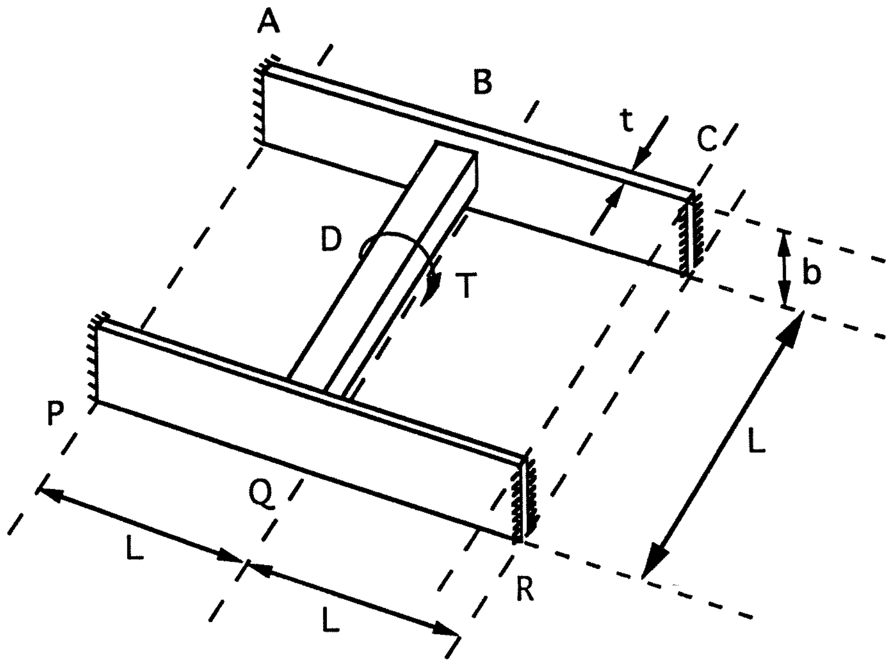


Fig. 1(c)



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2 The experimental apparatus shown in Fig. 2 consists of a 1 m long aluminium alloy circular cylindrical tube with internal diameter 400 mm and wall thickness 5 mm. It is capped at each end by hemispherical aluminium alloy domes welded to the main body. The wall thickness of the domes is 2 mm.

(a) The apparatus is internally pressurised to a gauge pressure of 5000 kPa. Using the usual assumptions for thin-walled structures:

(i) calculate the three principal stresses at a typical point in the cylindrical body and in the end-caps; [6]

(ii) calculate the corresponding strains in the principal directions. [6]

(b) Equal and opposite axial torques of magnitude T are applied to each end of the cylindrical tube at sections A-A and B-B shown. Assuming that the aluminium alloy obeys Tresca's criterion and has a yield stress of 260 MPa, calculate the maximum torque that can be sustained before inelastic behaviour occurs in the following cases:

(i) the gauge pressure is 5000 kPa; [4]

(ii) the gauge pressure is 2500 kPa. [4]

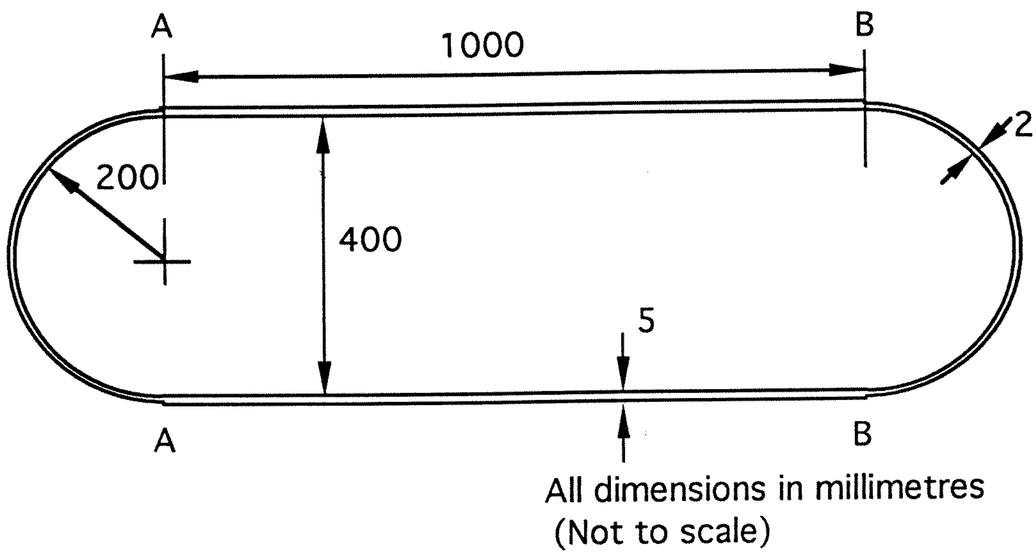

Fig. 2

3 (a) The continuous beam ABCDE has point loads P and Q applied as shown in Fig. 3(a). Span AC is of length L and has plastic moment of resistance M_p. Span CE is of length $0.7L$ and has plastic moment of resistance $0.8\ M_p$.

Consider three appropriate collapse mechanisms and use the work equation to determine the corresponding collapse conditions. [8]

On an interaction diagram of QL/M_p versus PL/M_p, plot the failure lines corresponding to collapse by these mechanisms. (Only the positive quadrant is required). [4]

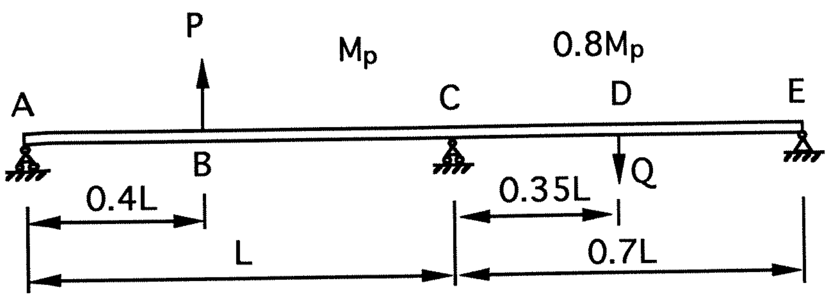

Fig. 3(a)

(b) A bridge under construction contains a continuous beam ACE as shown in Fig. 3(b). The beam is made of universal beam sections:

Span AC: 686×254 UB 152

Span CE: 686×254 UB 125

all of steel with yield stress 340 MPa.

A light gantry is built over the support A as shown, to carry a load W. If the load Q = 450 kN find the load W at collapse. (Neglect the self-weight of the beams). [8]

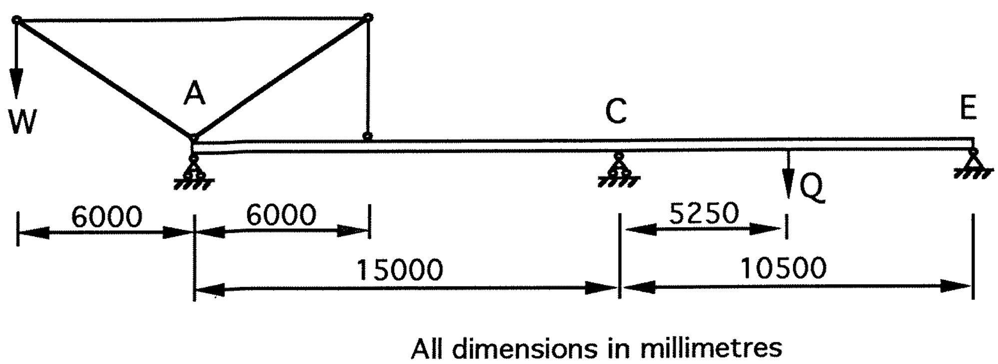

Fig. 3(b)

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4 The flat plate ABCD of length $2L$, width L and uniform thickness t is simply supported along AB and DC, fixed along BC and free along AD. It carries a uniformly distributed load w per unit area.

(a) By considering the yield-line mechanism based upon the parameter α shown in Fig. 4, show that the plate collapses in such a mechanism if

$$w > \frac{6m}{L^2}\frac{(\alpha+2)}{\alpha(3-\alpha)}$$

where m is the plastic moment of resistance per unit length. [10]

(b) Assume $\alpha = 1$ and thus obtain an upper bound on the collapse load w per unit area for a steel plate of this geometry and support conditions, having dimensions $L = 300$ mm and $t = 6$ mm and a yield stress of 250 MPa. [6]

(c) Sketch two other possible collapse mechanisms (but do not calculate the corresponding collapse loads). [4]

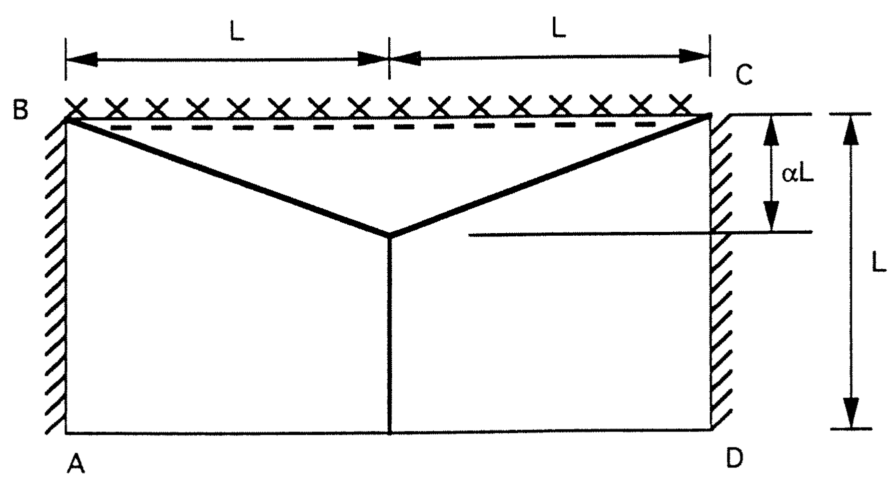

Fig. 4

5 The Perry-Robertson formula for column buckling under a compressive load P may be written

$$(\sigma_Y - \sigma)(\sigma_E - \sigma) = \eta \sigma_E \sigma$$

where

σ_Y	=	yield stress
σ_E	=	Euler stress = $\pi^2 E/(L/r)^2$
σ	=	maximum allowable mean stress P/A
η	=	Perry imperfection factor
A	=	cross-sectional area
E	=	Young's modulus
L	=	effective length of column
r	=	relevant radius of gyration of column cross-section

(a) Calculate the maximum axial compressive load that may be carried by a pin-ended column 3.5 m long with Universal Column section 356 × 368 UC 177. Assume both ends are fully restrained against displacement in all lateral directions. The column is made of steel with yield stress 345 N/mm^2. Take the Perry imperfection factor $\eta = 0.0055(L/r)$. [12]

(b) Would you describe this column as stocky, slender or intermediate between these two extremes? Explain your answer. [2]

(c) The x and y directions are defined in Fig. 5. Let z be the distance along the column measured from one end. Assume that the initial lateral imperfection in the x direction is of the form $\delta_0 \sin \pi z/L$. Show that the Perry factor above implies that the initial lateral imperfection magnitude δ_0 is approximately 10 mm, and estimate the magnitude of the midspan lateral deflection in the x direction when the column carries a compressive load of 6000 kN. [6]

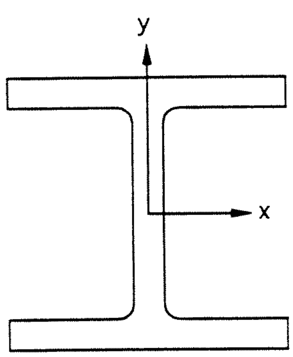

Fig. 5

Cross-section of column

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6 (a) The pin-footed portal frame in Fig. 6(a) is initially stress-free. It is made of a material of Young's modulus E and coefficient of thermal expansion α. The relevant flexural rigidity of the beam BC is EI_b whereas that of each leg AB and CD is EI_c. The temperature of the whole structure is increased by ΔT.

Assuming that the material remains everywhere elastic, that there is no instability of any kind, and that axial compressibility and shear deformation may be neglected, show that the horizontal reaction at the foot A is

$$\frac{E\alpha\Delta T}{L^2}\left(\frac{3I_cI_b}{3I_c+I_b}\right)$$ [14]

(b) All members of a computer component sketched in Fig. 6(b) have solid rectangular cross-section and the thickness t_b of the cross-member is much greater than the thickness t_c of the legs. In the initial design, t_c = 1.2 mm, but the maximum bending stress in the legs at the maximum operating temperature is calculated to be 130 MPa. Select a new leg thickness such that the maximum stress will reach only 80 MPa under these operating conditions. [6]

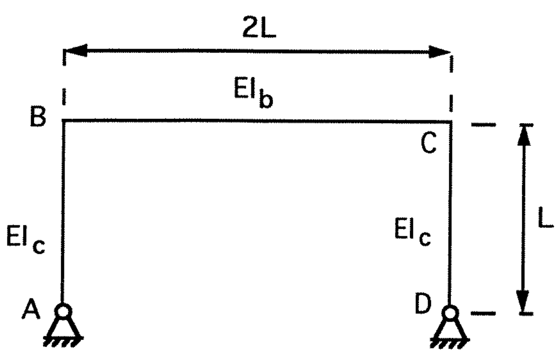

Fig. 6(a)

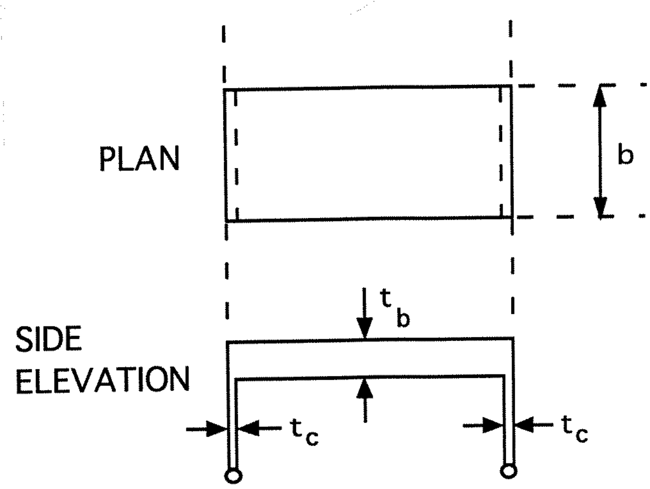

Fig. 6(b)

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