ENGINEERING TRIPOS PART IB

Tuesday 3 June 1997 2 to 4

Paper 4

FLUID MECHANICS AND HEAT TRANSFER

Answer not more than **four** *questions.*

Answer at least **one** *question from each section.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

1 The float pictured in Fig. 1 consists of a bulb, of volume V and mass M, and a cylindrical section of diameter d and negligible mass. When placed in water, it settles in the equilibrium position shown, with length Y of the cylindrical section submerged. The density of the water is ρ, and the acceleration due to gravity is g.

(a) Derive an expression for the length Y. [6]

(b) The float is displaced vertically downwards and then released. After release, it oscillates vertically, with the bulb always submerged and the top of the cylindrical section always out of the water. Find an expression for the natural frequency of the oscillation. [8]

(c) In practice, it is observed that the oscillation gradually dies away. Suggest two physical mechanisms which might cause this behaviour, and briefly explain the reasoning behind your answers. [6]

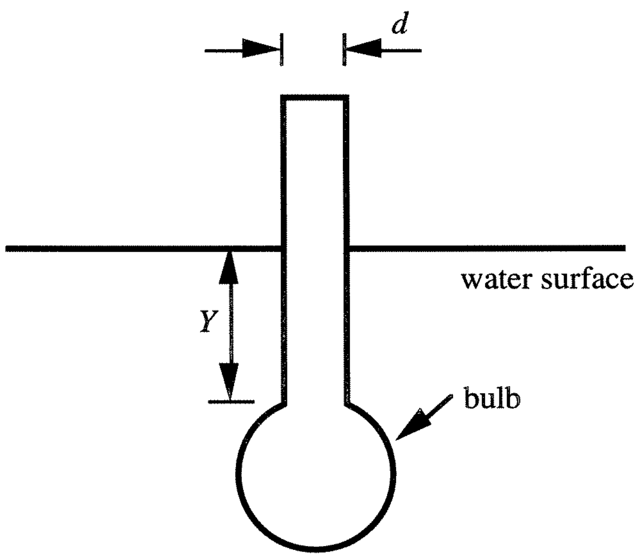

Fig. 1

2 (a) State Bernoulli's equation, explaining the meaning of each term and giving the conditions under which the equation is valid. [4]

(b) A river of width 50 m flows past a bridge which is supported on five equispaced piers, as shown in Fig. 2. At station A, upstream of the bridge, the flow is uniform with depth 2 m and speed 2 m/s . The flow between the piers is uniform at the exit plane (station B), with depth 1.50 m . Assuming that viscous effects are negligible between stations A and B, calculate

(i) the flow speed at station B, and

(ii) the width w of each pier. [6]

(c) Downstream of the bridge, at station C, the flow is again uniform across the entire width of the river, with depth 1.88 m and speed 2.12 m/s . Calculate the magnitude and direction of the force exerted by the river on the bridge. You may assume that friction forces at the river bed and banks are negligible. [6]

(d) Between stations B and C the flow loses mechanical energy. Why? [4]

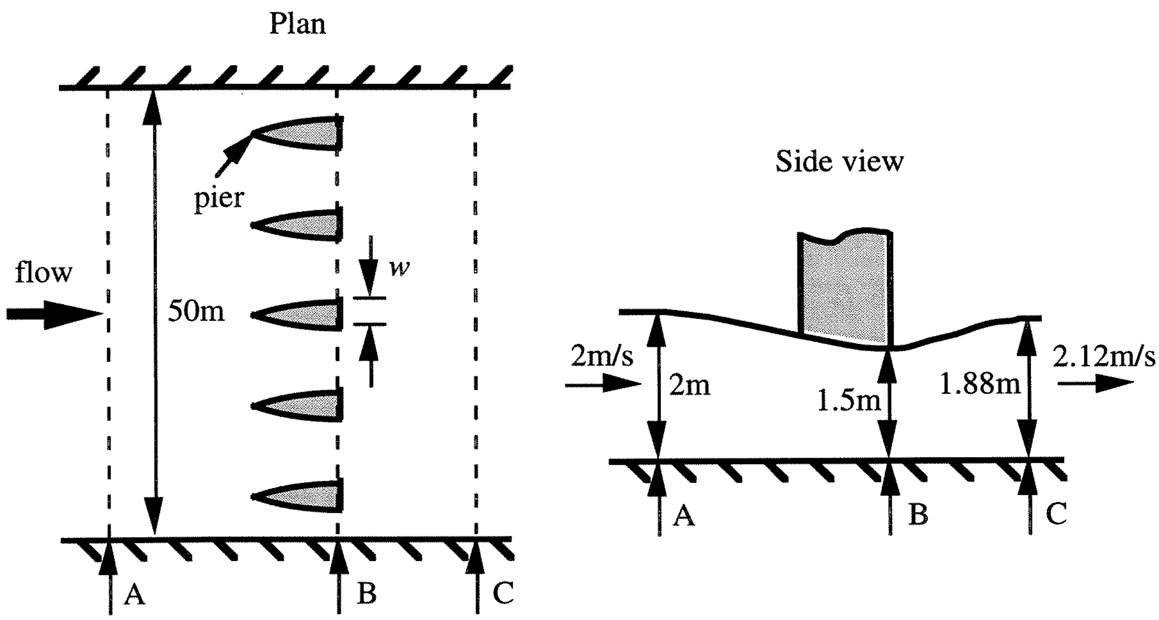

Fig. 2

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3 Waste sludge is discharged into water through a long vertical pipe of length L and radius R (Fig. 3). The densities of the sludge and water are ρ_s and ρ_w respectively, and ρ_s is greater than ρ_w. The pressure in the sludge is uniform at any given cross-section of the pipe, and is equal to the external pressure at entry and exit. The flow in the pipe is steady and laminar, with velocity independent of vertical position . The dynamic viscosity of the sludge is μ and the acceleration due to gravity is g.

(a) By considering the equilibrium of a cylindrical volume of fluid between the pipe entry and exit, or otherwise, show that the flow velocity V at radius r obeys the equation

$$\frac{\mathrm{d}V}{\mathrm{d}r} = -\frac{gr}{2\mu}\left(\rho_s - \rho_w\right).$$ [6]

(b) Find an expression for the volumetric flow rate of the sludge. [8]

(c) Your expression for the volumetric flow rate is to be tested in a laboratory scale experiment. The fluids representing the sludge and the water have densities which differ from ρ_s and ρ_w , but whose ratio is equal to ρ_s/ρ_w. By casting the expression for the flow rate in a suitable dimensionless form, find the other dimensionless group that must be kept constant if dynamical similarity is to be achieved. [6]

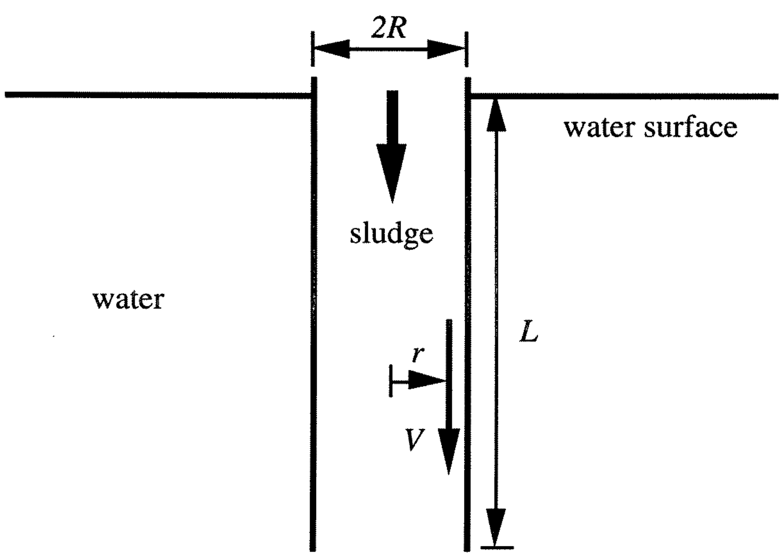

Fig. 3

4 Figure 4 shows a horizontal cross-section through a thrust-reverser for a jet-propelled underwater vehicle. The thrust-reverser consists of a semi-cylindrical surface, of radius r_2, supported at its mid-point, B. A uniform water jet of density ρ, velocity V_0 and width h enters the thrust reverser at A, and exits at C.

(a) The flow may be assumed to be steady and inviscid, and to have semi-circular streamlines between A and C. Show, from first principles, that the pressure p in the jet at radius r satisfies the relation

$$\frac{dp}{dr} = \frac{\rho V^2}{r},$$

where $V(r)$ is the jet velocity. [4]

(b) The pressure in the uniform part of the jet is equal to the ambient pressure, p_a. Write down an equation linking the pressure and velocity in the semi-circular region to p_a and V_0. Hence find $V(r)$ in terms of V_0, r and r_1. You may assume that the pressure at the inner edge of the semi-circular region ($r = r_1$) is also equal to p_a. [6]

(c) Use your expression for $V(r)$ to obtain the relationship between r_1, r_2, and h. [4]

(d) The pressure on the outside of the thrust reverser is p_a. Find the pressure difference across the thrust reverser, and hence the bending moment (per unit distance into the page) at B. [6]

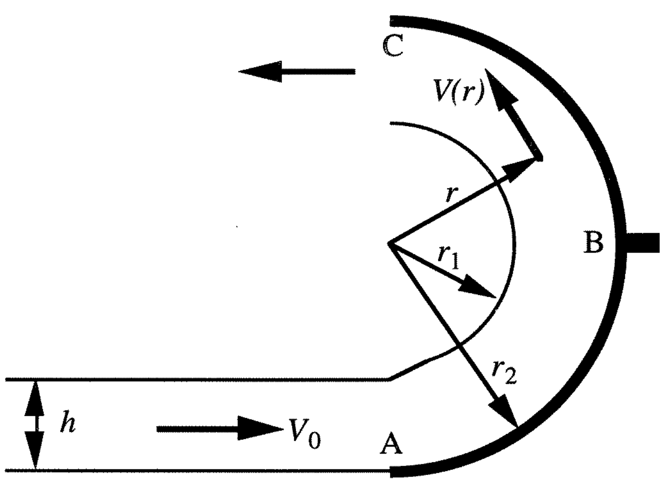

Fig. 4

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SECTION B

5 In a 'ramjet' propulsion unit, the flow first passes through an inlet section, where it is slowed before entering the combustion chamber. After combustion, the products leave the ramjet through a nozzle. The flow is steady throughout the ramjet.

(a) If the oncoming flow has temperature 216 K and velocity 718 m/s (relative to the ramjet), and the velocity in the combustion chamber is negligible, show that the temperature at inlet to the combustion chamber is 471.2 K. You may assume that the flow is adiabatic, and behaves as a perfect gas with the properties of air. [4]

(b) The combustion chamber is supplied with liquid n-octane at a temperature of 25 °C. The air/fuel ratio is stoichiometric, combustion is complete, and there is no heat loss. Show that the temperature of the products leaving the combustion chamber is 2500 K. You may assume that the products behave as semi-perfect gases, and that differences in the properties of 'atmospheric nitrogen' and nitrogen are negligible. [10]

(c) An analysis of the flow in the nozzle, assuming that the products behave as perfect gases, predicts that the ratio of the nozzle exit to nozzle inlet temperature is $2/(\gamma+1)$, where γ is the ratio of specific heats of the gas mixture. Calculate the nozzle exit temperature. The values of the specific heats of the mixture components at the temperatures of interest are given below: [6]

Component	N_2	CO_2	H_2O
c_p (kJ/kg)	1.30	1.40	2.95
c_v (kJ/kg)	1.01	1.21	2.49

6 (a) In the theory of radiative heat transfer, the view factor between body 1 and body 2 is denoted as F_{12}. Define F_{12}, and show from first principles that

$$A_1 F_{12} = A_2 F_{21},$$

where A_1 and A_2 are the radiating areas of bodies 1 and 2. [6]

(b) A large vessel, whose walls are maintained at 300 K, contains a workpiece and a heater (Fig. 5). Both heater and workpiece consist of planar surfaces, insulated on the sides not facing one another. The heater has area $0.1\ \mathrm{m}^2$ and emissivity 0.8. The view factor between the heater and the workpiece is 0.75. The workpiece area is $0.15\ \mathrm{m}^2$, and its emissivity is 0.4. If the heater is maintained at 600 K, find the temperature of the workpiece, and the overall heat flow rate from the heater. [8]

(c) If, due to a fault in the insulation, the workpiece loses heat from its rear face at a rate of 100 W, find the new heater temperature required to maintain the workpiece temperature at the value calculated in (b). [6]

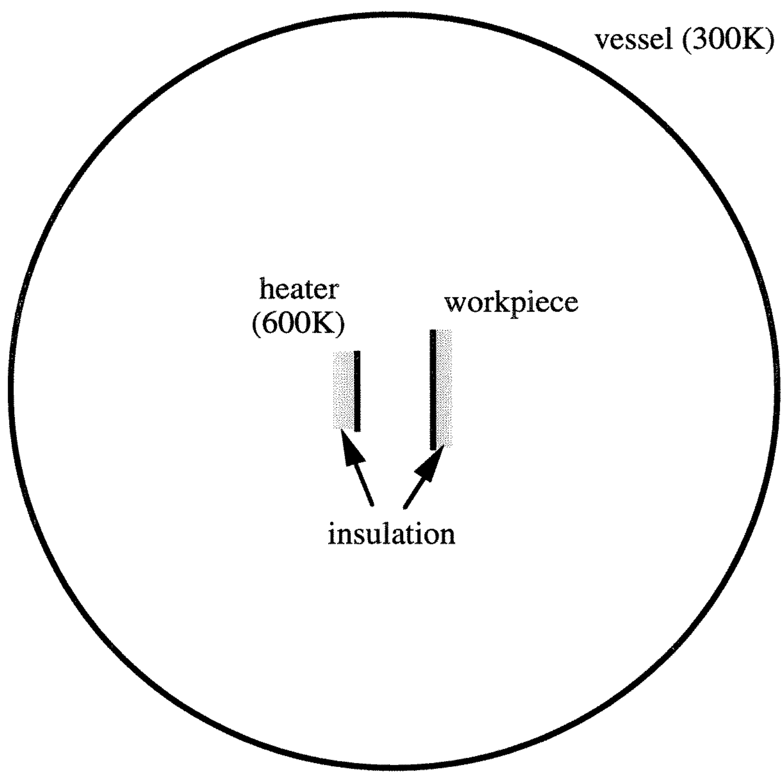

Fig. 5

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