IB 1998 answers

1998 Part IB Paper 1
Mechanics
Answers

1. (a) $\frac{-9I}{7ma}$ (b) $2aI$ $\frac{6I^2}{7m}$ (c) two

2. (a) $\frac{a}{2}$

   (b) $20\sqrt{2}$ mm s^{-1} $\leftarrow$ 0.4 mm s^{-2} $\downarrow$ + 0.2 mm s^{-2} $\leftarrow$

3. (a) (i) $\frac{I}{m}$ (ii) $\frac{\mu I}{m}$ in the opposite sense to (i)

   (b) both $\frac{3I}{5m}$ but in opposite directions

4. (a) $\boldsymbol{r}=\boldsymbol{r}_e + \boldsymbol{r}_p + \boldsymbol{r}_t$

   (i) $\dot{\boldsymbol{r}} = \dot{\boldsymbol{r}}_e + \dot{\boldsymbol{r}}_t + \omega\times(\boldsymbol{r}_p + \boldsymbol{r}_t)$

   (ii) $\ddot{\boldsymbol{r}} = \ddot{\boldsymbol{r}}_e + \ddot{\boldsymbol{r}}_t + 2\omega\times\dot{\boldsymbol{r}}_t + \omega\times(\omega\times(\boldsymbol{r}_p + \boldsymbol{r}_t))$

   (b) $\frac{|\omega|\ R\ \cos\lambda}{2}$ where R is the radius of the Earth

5. (a) ω, 2ω anticlockwise (b) $\sqrt{2}a\omega^2$, 0 (c) ω^2 anticlockwise

   (d) $\frac{\sqrt{61}ma\omega^2}{6}$ $\frac{ma\omega^2}{6}$

6. (a) $\frac{a\pi}{4}$ to the left (b) $\frac{5}{12}ma^2\pi$

   (c) π $\frac{a\pi}{4}$ to the right $\frac{5}{12}ma^2\pi$

   (d) $\pm\frac{a}{2}$

1998 Part IB Paper 2

Answers

1 b) (i) 22000 kNm, 1100 kN,

(ii) 70 N/mm^2, 26 N/mm^2

c) $125.10^{-6}W$ N/mm^2, $39.1.10^{-6}W$ N/mm^2 (W in kN), 2580 kN

2 a) 662 με, -329 με

b) 150 N/mm^2 at 18.4° anticlockwise from x, -50 N/mm^2

c) 538 με, -205 με

d) 3

3 b) $\alpha = \dfrac{(k+1) \pm \sqrt{(k+1)}}{k}$ where αL = distance of hinge from fixed end.

c) 113.2 kN, 86.8 kN, Yes

d) 305 × 102 × 33 kg/m

4 c) $24m/19L^2$

5 a) (i) $\pm WL/8$

(ii) 0, $WL/4$

b) (i) $-9WL/48$, $5WL/32$

(ii) $-WL/4$, $WL/2$

6 c) ≈ 417 kN, ≈ 4 mm

1998 Part IB Paper 3

Materials

Answers

1. (a) $M1 = \dfrac{E^{1/2}}{\rho}$

 (b) $M2 = \dfrac{\sigma^{2/3}}{\rho}$

 (c) CFRP. Minimum mass ≈ 1.4 kg per fork

 (d) Cost, toughness, formability, manufacturability.

2. (a) Carbon content, microstructure.

 (b) (i) ≈50%/50% ferrite and pearlite (0.4 wt % C),

 (ii) martensite (0.8 wt. % C),

 (iii) eutectoid pearlite (0.8 wt. % C),

 (iv) ferrite (< 0.01wt. % C).

 Order in decreasing hardness: (ii), (iii), (i), (iv).

 (d) Fine eutectoid pearlite or tempered martensite

3. (b) Compositions at the α and β ends of the phase field at 150°C.

 Lever rule; $W_\alpha = \dfrac{C_0 - C_\beta}{C_\alpha - C_\beta}$, $W_\beta = \dfrac{C_\alpha - C_0}{C_\alpha - C_\beta}$

 (c) $W_\alpha = 0.66$ wt % $W_\beta = 0.34$ wt %

 (d) 60 wt % Sn and 40 wt % Pb (easily melted, minimises heat damage).

5. (d) $P_s \approx 0.995$ (specimen rejection rate of 0.5%).

6. (c) $\varepsilon_m(t) = \dfrac{\sigma}{E_1} + \dfrac{\sigma t}{\eta_1}$

 (d) E = 1.25 GPa, η = 28 TPa.s

 Polyester.

1998 Part 1B Paper 4

Answers

Fluid Mechanics and Heat Transfer

1 (b) (i) $V_2 = V_1/0.8$

(ii) $8.034\, m/s$

2 $$p = \rho\frac{k_1^2 r^2}{2} + C_1$$

$$p = -\frac{\rho k_2^2}{2r^2} + C_2$$

$$k_1^2 = R^2(p_\infty - p_0)/\rho$$

$$k_2^2 = (p_\infty - p_0)/\rho R^2$$

3 -0.266

-0.162

4 $$u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - yh\right) + \frac{Uy}{h}$$

6

5 23.2°C
42.3°C

6 59.7%

1998 Part 1B - PAPER 5 - ANSWERS

1. a) $i_c = h_{fe}i_b + h_{oe}v_{ce}$ $\quad v_{be} = h_{ie}i_b + h_{re}v_{ce}$ $\quad$ b) $R_C = 400\ \Omega$; $R_B = 154\ \text{k}\Omega$

c) $R_L = 400\ \Omega$; Mid-band gain $\dfrac{v_o}{v_i} = -\dfrac{h_{fe}R_C R_L}{h_{ie} + R_C + R_L} = -50$

d) $\dfrac{v_o}{v_i} = -\dfrac{h_{fe}R_C R_L}{(h_{ie} + 1/j\omega C_1)(R_L + R_C + 1/j\omega C_2)}$ $\quad$ e) $f_{3dB} = 15.5$ Hz

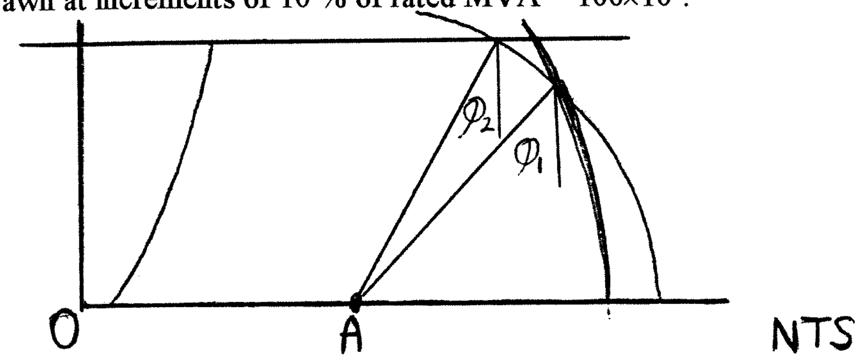
2. b) Common mode gain $= -\dfrac{h_{fe}R_C}{h_{ie} + 2R_T(h_{fe}+1)}$; Differential gain $= -\dfrac{h_{fe}R_C}{h_{ie}}$;

$R_C = 5\ \text{k}\Omega$; $R_T = 4.97\ \text{k}\Omega$ c) Differential gain $= -\dfrac{h_{fe}R_C}{h_{ie} + R_E(h_{fe}+1)}$;

Gain reduction factor $= 1 + \dfrac{R_E(h_{fe}+1)}{h_{ie}}$ d) $R_E = 44.8\ \Omega$

3. b) (i) $P_{delta} = 51.7$ kW; $P_{star} = 13.8$ kW (ii) $I_{line} = 110.4$ A
(iii) Power factor = 0.825 lagging c) (i) 828 μF; (ii) 430 μF; 17.5 % and 13.1 % reduction in line current respectively.

4. b) i) Prime-mover limit - horizontal line at 800×10^6; Stator heating limit - circle of radius 1000×10^6, centre A; Rotor heating limit - circle of radius 1620×10^6, centre O; Length OA = 720×10^6; Stability limit - intersection of circles and horizontal lines drawn at increments of 10 % of rated MVA = 100×10^6.



(iii) Power factor between 0.64 and 0.8 lagging.

5. d) $T_{max} = 99.5$ Nm; N = 1035 rpm e) Extra rotor resistance = 0.778 Ω

6. a) L = 0.2 μHm^{-1}; $Z_0 = 50\ \Omega$; Wavelength = 2.5 m d) $Z_{in} = Z_L = (30 + j40)\ \Omega$ - this is because the transmission line is exactly one wavelength long.

7. a) $\omega^2\varepsilon_0\mu_0 = \beta^2$ b) ***E***, ***H*** and direction of propagation mutually orthogonal, such that ***E***×***H*** gives direction of propagation; $\eta_0 = \dfrac{|\boldsymbol{E}|}{|\boldsymbol{H}|} = \sqrt{\dfrac{\mu_0}{\varepsilon_0}}$; $\overline{\mathbf{H}} = \mathbf{u}_y \dfrac{E_0}{\eta_0} \exp j(\omega t - \beta z)$

c) E = 980 Vm^{-1}; H = 2.6 Am^{-1} d) Power pu area = 1182 Wm^{-2}; E = 717 Vm^{-1};
H = 3.30 Am^{-1}

ENGINEERING TRIPOS

PART IB Paper 6 1998

ANSWERS

1. (a) $G(s) = \frac{e^{-s}}{1+s} \quad k(s) = \left(\frac{0.1}{s} + 0.2\right)$

   (b) $H(s) = \frac{se^{-s}}{s(s+1)+e^{-s}(0.1+0.2s)}$

   (c) 0

2. (c) A = 2/3, $\phi = \pi$

3. (b)
$$\begin{aligned} a &\approx 0.1 \\ b &\approx 10 \\ c &\approx 0.05 \\ \omega_n &\approx 2 \end{aligned}$$

   (c) GM = 1.8
   PM = 12°

4. (c) $PM = 45°$

5.

6. (b) 17 bits/sample
   SNR = 104 dB

   (c) 31,250 samples/frame (= 176 × 177
   = 153 × 204)

SJG/sjb

ENGINEERING TRIPOS PART IB 1998

PAPER 7 – Mathematical Methods

Answers

1. (b) $a^2b^2/8$

2. (i) $T_{max} = 1/27$ at $(2/3, 1/3)$
   $\underline{q} = -\lambda[y(1-2x+y)\underline{i} + (x-2y)(1-x)\underline{j}]$
   (ii) Heat loss $= \lambda/6 + \lambda/6 + \lambda/3 = 2\lambda/3$

3. Wave points convect with speed c along the characteristics such that $\phi = \phi_0 e^{-kt}$
   $\phi(x,t) = 5e^{-kt}\sin(2\pi x)\cos(2\pi ct)$

4. $C = \dfrac{f''(\alpha)}{2f'(\alpha)}$ 1.000

5. $a_0 = 0.459$ $a_1 = 1.169$ $a_2 = -0.336$

6. $R(\tau) = \dfrac{a+b}{\pi[(a+b)^2 + \tau^2]}$

7. $\{0, -2j, 0, 2j\}$

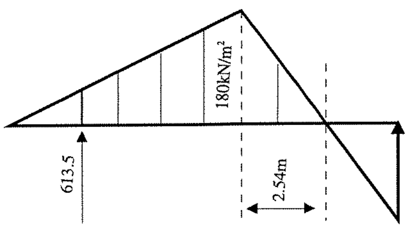
8. $(M-\mu_M)/\sigma_M = 1.6$ Not significant at 5% level
   $(S-\mu_S)/\sigma_S = 1.3$ Not significant at 5% level
   $[(M+S) - \mu_{M+S}]/\sigma_{M+S} = 2.06$ Significant at 5% (and 2%) level

1998 PART IB PAPER 8

ANSWERS

1. (a) 613.5 kN/m

(b)



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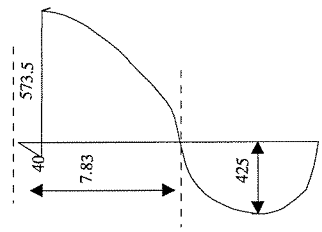
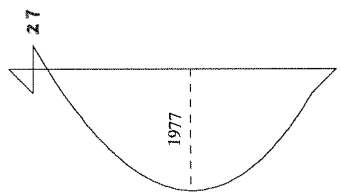


Shear kN/m



Moments kNm/m

(c) 661mm

3. (a) (i) σ_v = 16.2 kPa

(ii)

	σ_v	u	σ_v'	σ_h'	σ_h	
wall top (1m)	16.2	0	16.2	4.8	4.8	) Trapezoidal
wall bottom (7.1m)	137.6	59.8	77.8	23.0	82.8	)

(iii) σ_v base ≈ 59.4 kPa

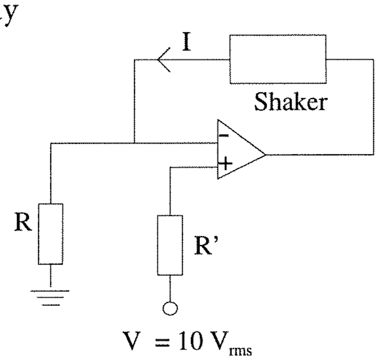
(b) Unfortunately the uplift water pressure on the base is u = 6.1 x 9.8 = 59.8 kPa.
So the tank is on the point of floating, therefore it is unsatisfactory to rely on friction, so raise embankment by 1m (say) or anchor tank to underlying ground by ground anchors.

4. (a) (i) f_u = 22.5 Hz

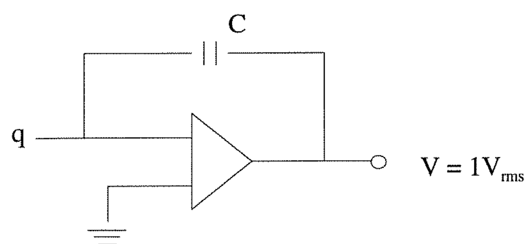
(ii) ± 4.5 mm

(iii) 7.0 mms^{-2}

(b) $I_{rms} = 2.5A$
$R = 4\Omega$
$R' \approx 1k\Omega$ say



(c)



6. (a)

	buried	uncovered	spanning
ω_n (rads $^{-1}$)	120	90	52
k(MN/m)	240	80	13.2

(b) 8Hz (but anything from 2 - 10 Hz would work)

7 (a) 12

(b) .2636m and 10143 r.p.m.

(c) 19164 N/m

(d) 2 stages

9 (b) $494.33\ m^2$

(c) 109 kN/engine and 362.5 m/s

(d) 13311m

(e) 24.5 kN/engine

10 (c) 0.066 eV

11 (a) $2 \times 10^{21}\ m^{-3}$

(b) $E = 10^6$ V/m
d = 2 microns

(c) 0.64 mA

(d) 1.6V

12 (b) To make a good ohmic contact
Arsenic-n-type and Boron (most common), Gallium or Aluminium for p-type.

(c) (i) ~ 1.5 - 2.5 microns dependent upon reading from graphs and a slightly shallower value if Arsenic is used, because of lower diffusion coefficient.