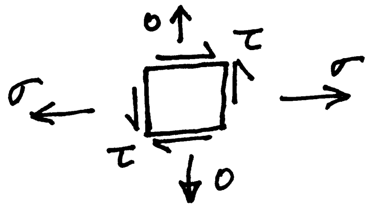
P2 – Structures. IB Examination 1998

– Solutions

1.(a) von Mises Yield Condition (from data book)

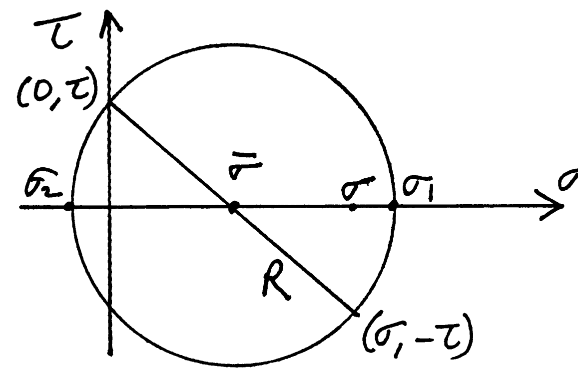
$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2 = 2Y^2$$

For a set of combined stresses



& $\sigma_3 = 0$

The Mohr's circle is:



$$\therefore \bar{\sigma} = \frac{\sigma}{2} \qquad R = \sqrt{\left(\frac{\sigma}{2}\right)^2+\tau^2}$$

$$\Rightarrow \sigma_1 = \frac{\sigma}{2}+\sqrt{\left(\frac{\sigma}{2}\right)^2+\tau^2} \qquad \sigma_2 = \frac{\sigma}{2}-\sqrt{\left(\frac{\sigma}{2}\right)^2+\tau^2}$$

$$\therefore (\sigma_1-\sigma_2)^2 = 4R^2$$

$$(\sigma_1-\sigma_3)^2 = \left(\frac{\sigma}{2}+R\right)^2 = \frac{\sigma^2}{4}+\sigma R+R^2$$

$$(\sigma_2-\sigma_3)^2 = \left(\frac{\sigma}{2}-R\right)^2 = \frac{\sigma^2}{4}-\sigma R+R^2$$

$\therefore$ von Mises can be written $6R^2+\frac{\sigma^2}{2} = 2Y^2$

Substitute for R and rearrange

$$\underline{\sigma^2+3\tau^2 = Y^2}$$ Q.E.D. (6 marks)

(answered correctly if Mohr's circle drawn correctly).

(b)(i) Beams cross-sectional area $= 0.357\ m^2$

Density of steel (data book) $= 7840\ kg/m^3$

$\therefore$ load/unit length $= 7840 \times 9.81 \times 0.357 = 27.5\ kN/m$

↑ (g - many candidates left this out)

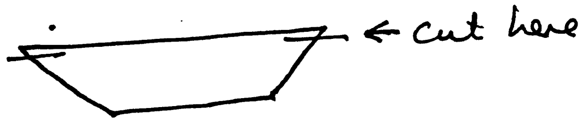
$\therefore$ Shear force at root of cantilever $= 40 \times 27.5 = \underline{\underline{1100\ kN}}$

& Bending moment $= \dfrac{27.5 \times 40^2}{2} = \underline{\underline{22{,}000\ kNm}}$

(4 marks)

(ii) Longitudinal stress $= \dfrac{My}{I} = \dfrac{22000 \times 10^6 \times 1000}{0.319 . 10^{12}} = \underline{\underline{70\ N/mm^2}}$

Shear stress $= \dfrac{FA\bar{y}}{I.t}$



$A\bar{y}$ = 1st moment of area of top flange about centroid (many errors here)

$= 10{,}000 \times 15 \times 1000 = 150 \times 10^6\ mm^3$

$\therefore \tau = \dfrac{1100 \times 10^3 . 150 \times 10^6}{0.319 . 10^{12} \times 2 \times 10} = \underline{\underline{26\ N/mm^2}}$

↑ two webs ↑ web thickness

(4 marks)

(c) Effect of W will be to

(i) add W to shear force, thus increasing shear stress by $\dfrac{W . 150 \times 10^6}{0.319 . 10^{12} \times 2 \times 10} = 23.5 \times 10^{-6}\ W$

(W in Newtons)

(c) cont

(ii) add 40W to moment, thus increasing axial stress by $\dfrac{40 \times 10^3 \times W \times 1000}{0.319 \times 10^{12}} = 125.4 . 10^{-6}\, W$

(iii) add torque of $5 . W . 10^3$, increasing shear stress by $\tau = \dfrac{T}{2 A_e t} = \dfrac{5000\, W}{2 . 8000 . 2000 . 10} = 15.6 . 10^{-6}\, W$

(enclosed area of section)

To find limiting value of W, substitute the <u>total</u> stresses into the von Mises limit.

Thus

$$(70 + 125 . 10^{-6} W)^2 + 3(26 + 23.5 W . 10^{-6} + 15.6 . W . 10^{-6})^2 = 450^2$$

Becomes Quadratic in W

Solve $\Rightarrow$ $\underline{\underline{W = 2580\ \text{KN}}}$ (6 marks)

(Many errors - wrong area used in calculations - wrong formulae used - not all components of load considered).

2 (a) $\sigma_x = 130\ \text{N/mm}^2 \qquad \sigma_y = -30\ \text{N/mm}^2$

$$\therefore \varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$= \frac{1}{210.10^3}(130 + 0{\cdot}3 . 30) = \underline{\underline{662\,\mu\varepsilon}}$$

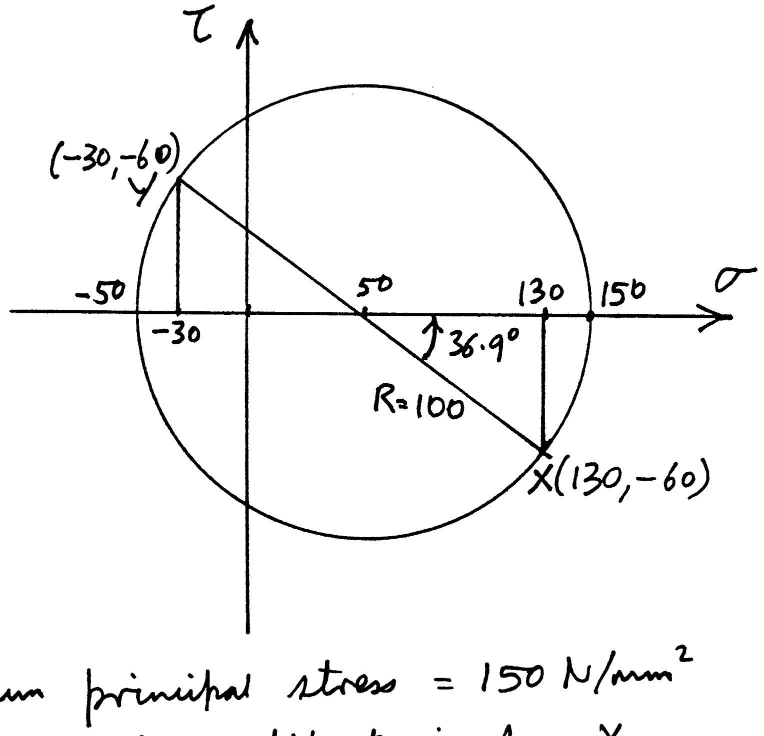
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

$$= \frac{1}{210.10^3}(-30 - 0{\cdot}3 . 130) = \underline{\underline{-329\,\mu\varepsilon}}$$

(6 marks)

(most common error was to get the sign wrong, or to get the wrong values for E & ν from the data book)

(b)



Maximum principal stress = 150 N/mm^2 at 18·4° anticlockwise from X

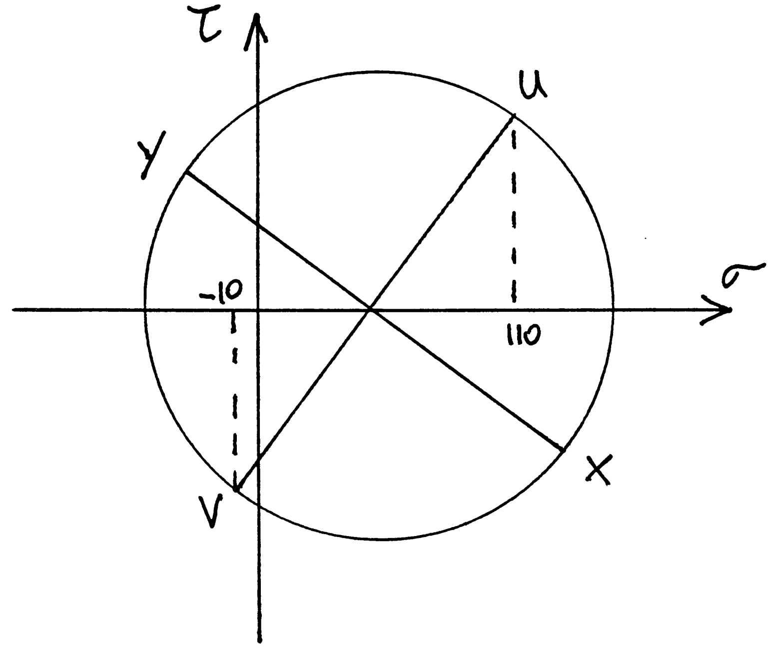
(6 marks)

Minimum principal stress = -50 N/mm^2 at 71·6° clockwise from X

(Common errors – incorrect orientation of Mohr's circle – it *is* important that the shear stress is drawn with the correct sign, otherwise the orientation of the principal axes will be wrong. It is also important that the scale on the τ & σ axes is the same, otherwise the figure will not be a circle.)

(Incorrect spelling of principal : $\neq$ principle !!)

(c)



ε_u (45° anticlockwise from x)

$$= \frac{1}{210.10^3}\left(110 + 0{\cdot}3 . 10\right) = 538\ \mu\varepsilon$$

ε_v (45° clockwise from x)

$$= \frac{1}{210.10^3}\left(-10 - 0{\cdot}3 . 110\right) = -205\ \mu\varepsilon$$

(6 marks)

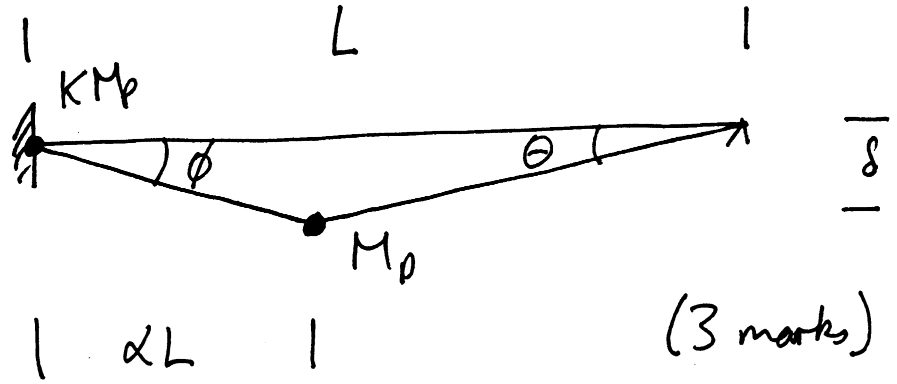
(d) A Mohr's circle requires 3 parameters to draw it (e.g centre, radius, orientation)

All equations relating stress to strain are linear

∴ 3 strain measurements are needed. (2 marks)

(It is not sufficient to say that three gauges are needed to measure the three principal strains, since the orientation of these is not known beforehand).

3. (a)



(3 marks)

(b) Compatability $\phi = \dfrac{\delta}{\alpha L}$

$$\theta = \frac{\delta}{(1-\alpha)L}$$

Work done by load $= \omega L \dfrac{\delta}{2}$

(Since average distance moved by load $= \dfrac{\delta}{2}$)

Work done in hinges

$$= \frac{KM_p \cdot \delta}{\alpha L} + M_p\,\delta\left(\frac{1}{\alpha L} + \frac{1}{(1-\alpha)L}\right)$$

Equate and rearrange

$$\frac{\omega L^2}{2M_p} = \frac{K}{\alpha} + \frac{1}{\alpha} + \frac{1}{(1-\alpha)} = J$$

minimum value of collapse load when $\dfrac{dJ}{d\alpha} = 0$

Thus $$-\frac{2(K+1)}{\alpha^2} + \frac{2}{(1-\alpha)^2} = 0$$

or $$\alpha^2 K - 2(K+1)\alpha + (K+1) = 0$$

quadratic in α

$$\alpha = \frac{(K+1) \pm \sqrt{(K+1)}}{K}$$

Solve and substitute to get value of $\frac{wL^2}{2M_p}$

(8 marks)

(This part done well by most candidates)

(c) $K = 0.7 \Rightarrow \alpha = \frac{1.7 \pm \sqrt{1.7}}{0.7}$

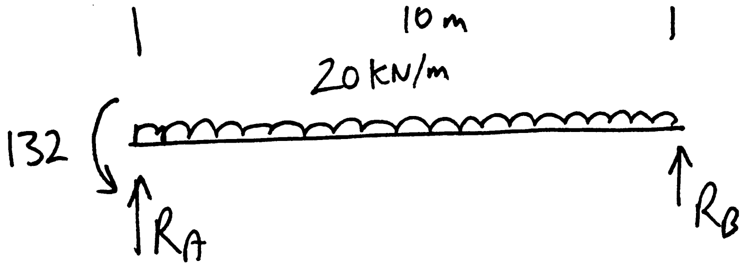
(take −ve root since $\alpha \leq 1$)

$\Rightarrow \alpha = 0.566$

$$\therefore \frac{wL^2}{2M_p} = \frac{0.7+1}{0.566} + \frac{1}{(1-0.566)} = 5.31$$

$$\Rightarrow M_p = \frac{20.100}{2.5.31} = \underline{\underline{188\ \text{kNm}}}$$

$$KM_p = \underline{\underline{132\ \text{kNm}}}$$



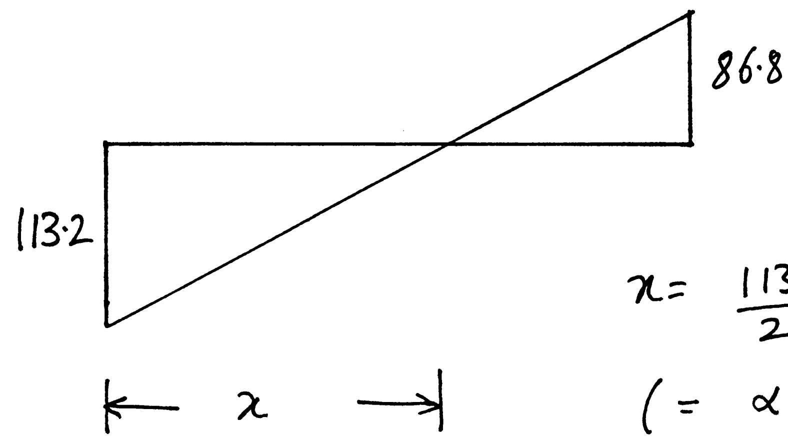
Take moments about left hand end

$$10 R_B + 132 = 20 . 10 . 5$$

$$\Rightarrow R_B = 86.8 \text{ kN}$$

$$R_A = 200 - 86.8 = 113.2 \text{ kN}$$

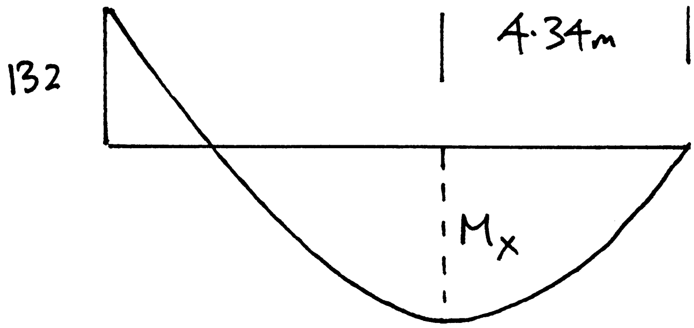
$\therefore$ Sheer force diagram



$$x = \frac{113.2}{200} \times 10 = 5.66\text{m}$$

$$(= \alpha \times 10\text{m})$$

Bending moment



$$M_x = 86.8 \times 4.34 - 20 \times \frac{4.34^2}{2} = 188 \text{ kNm}$$

$$= M_p$$

$\therefore$ It has been shown that the maximum bending moment occurs at the hinge position, and that the bending moment no where exceeds M_p. $\therefore$ The correct solution has been found.

(6 marks)

N.B - It is not sufficient to show that the moment at the hinge position $= M_p$. It must also be shown to be a maximum.

(This section was very badly done. Very few candidates used the results of part (b) to get the answer - instead, they performed a completely irrelevant elastic analysis).

(d) M_p required $= 188$ kNm

σ_p allowed $= 400\ N/mm^2$

$\therefore Z_p$ required $= \dfrac{188.10^6}{400} = 470{,}000\ mm^3$

$= 470\ cm^3$

A suitable section (data book, p10) would be $\underline{305 \times 102 \times 33\ kg/m}$

Z_p (about major axis) $= \underline{\underline{479.9\ cm^3}}$

4 (a) Yield lines will generally be straight

Each region is rigid and will have an axis of rotation

The axes of rotation of the two regions, and the yield line between them, must meet at a point, (projected if necessary).

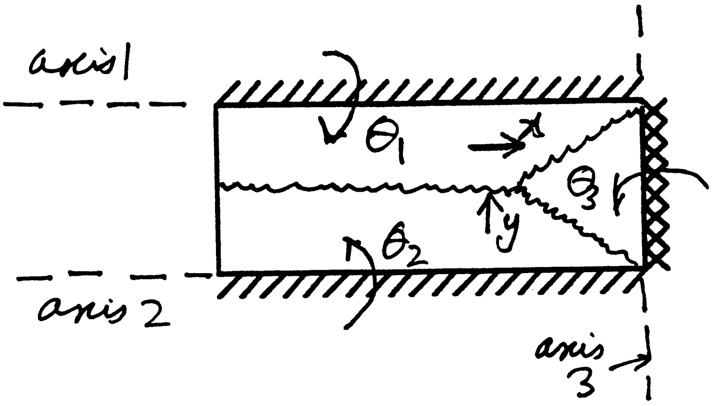
Yield lines generally pass along supports.

(N.B. Yield lines often pass under point loads, and can preserve symmetry, but neither of these is a rigid rule.) (5 marks)

(b) See next sheet.

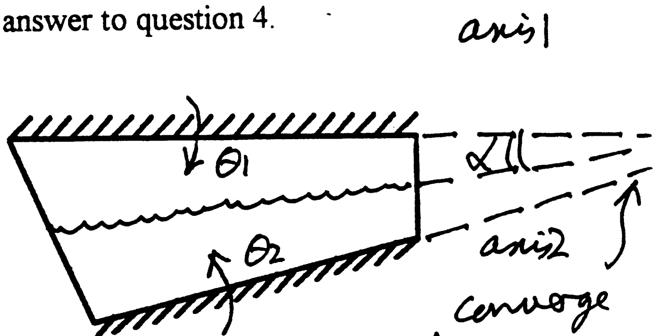
(i), (ii) and (iii) were well done, as was (v) however, in (iv) many candidates omitted the axis of rotation of the region at the column and got an incompatible mechanism. For (vi), many candidates drew many yield lines with no attempt to define the axes of rotation, or any thought about the way the slab would deform – the result was nonsense).

This sheet should be handed in with the answer to question 4.



(2 parameters x, y)

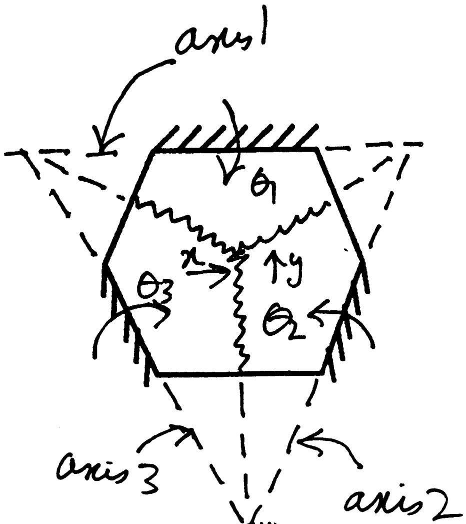
(i)



one parameter - angle α

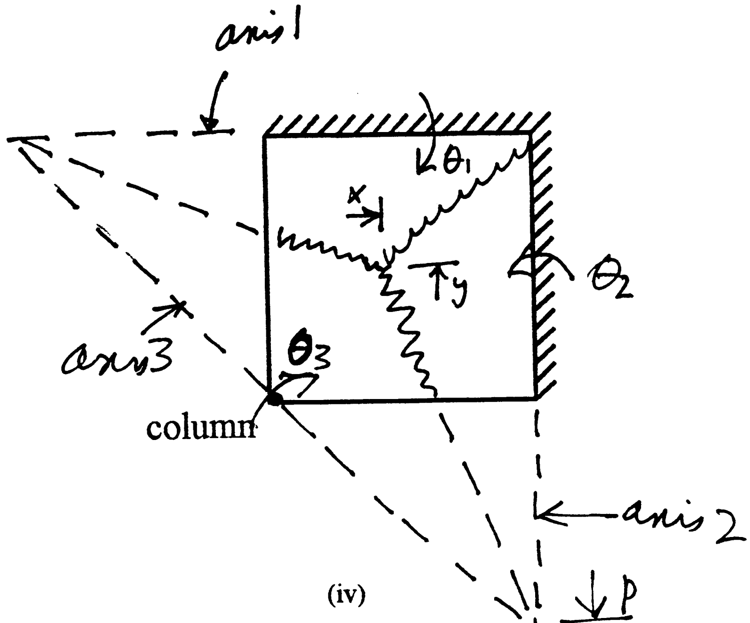
converge to point (very important)

(ii)



(iii)

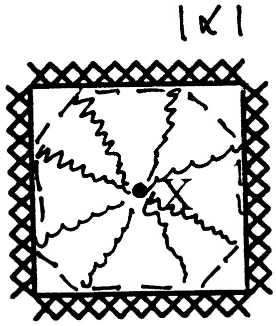
(2 parameters, x, y)



(iv)

3 parameters (x, y, P)

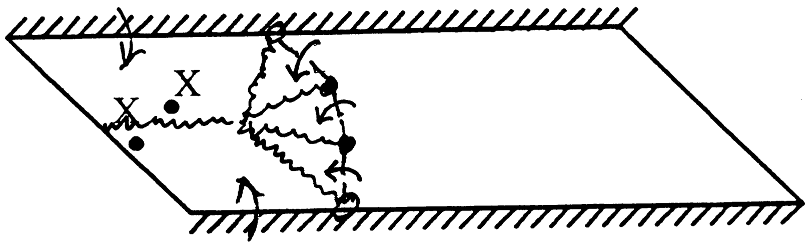
N.B. orientation of axis 3 <u>not</u> fixed



If symmetry preserved, one parameter α

(v)

If no symmetry, 8 parameters



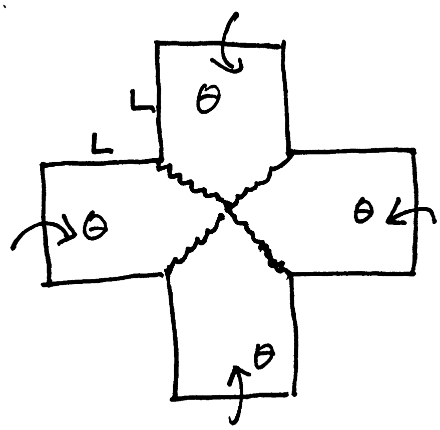
Two parameters for each point •
One parameter for each point ○

(vi)

(Many alternatives)

(10 marks)

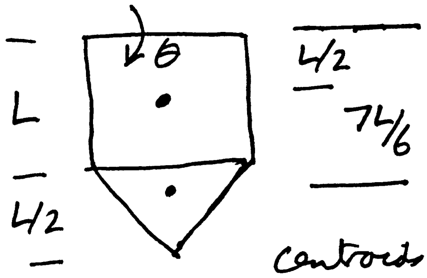
(C)



Work done in yield line
= (projection of yield line onto axis of rotation) × (angle of rotation)

$$= 4 \times \left(\frac{L}{2} \times \theta \times 2\right) \times m \quad = \quad 4mL\theta$$

Work done by load



$$= 4w\left(L^2 \cdot \frac{L}{2}\theta + \frac{L^2}{4} \cdot \frac{7L}{6} \cdot \theta\right)$$

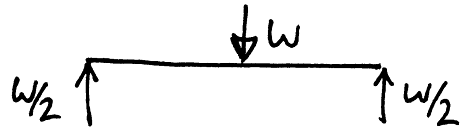
$$= \frac{19wL^3\theta}{6}$$

Equate $\quad 4mL\theta = \dfrac{19wL^3\theta}{6}$

$$\Rightarrow \quad w = \frac{24}{19}\frac{m}{L^2}$$ (5 marks)

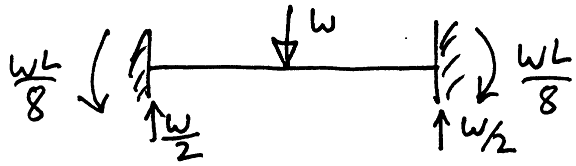
(done well. biggest error was failure to locate centroid of triangle correctly).

5 (a) (i)



Max moment $= -\frac{WL}{4}$ (sagging) (1 mark)

(ii)

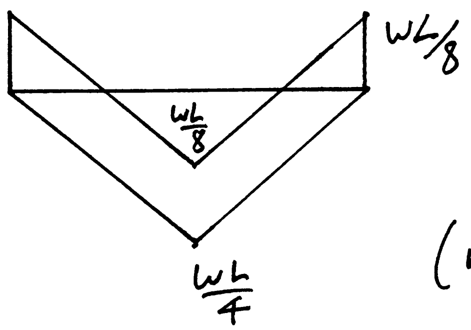


(from data book)

∴ Moment under load

$= \frac{WL}{8} - \frac{WL}{4} = -\frac{WL}{8}$ (2 marks)

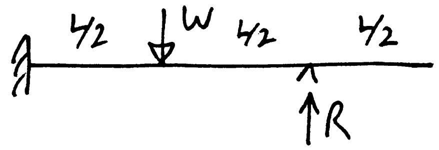
(iii)



(1 mark)

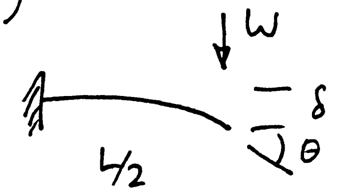
(many silly errors here!)

(b) (i)



(one degree indeterminate)

Need to find R.



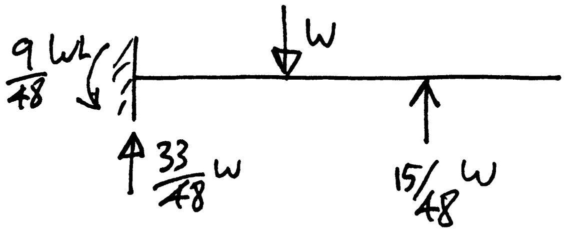
$\delta = \frac{WL^3}{24EI}$ $\theta = \frac{WL^2}{8EI}$

∴ Deflection at reaction $= \frac{WL^3}{24EI} + \frac{WL^2}{8EI}\cdot\frac{L}{2} = \frac{5}{48}\frac{WL^3}{EI}$

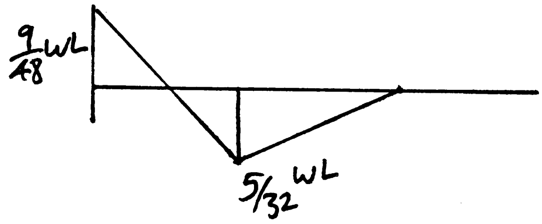
Deflection due to R alone $= \dfrac{RL^3}{3EI}$

Equate $\Rightarrow R = \dfrac{15}{48} W$

$\therefore$ loading is

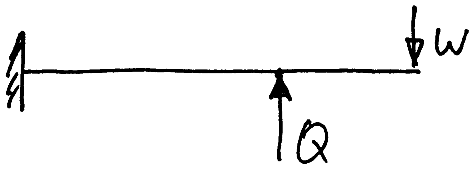


$\therefore$ Bending moment



(4 marks)

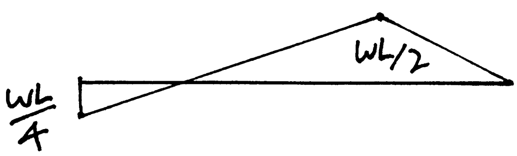
(ii)



Could repeat (i) or

(directly from data book, p 6)

$\therefore$ Bending moment



(4 marks)

(c) Maximum moment over sleeper goes up from $\frac{WL}{8}$ to $\frac{WL}{2}$ (4 fold increase)

If rail acts as clamped at sleeper normally, moment at mid span goes from $\frac{WL}{8}$ to $\frac{5}{32}WL$ (i.e. only 25% increase).

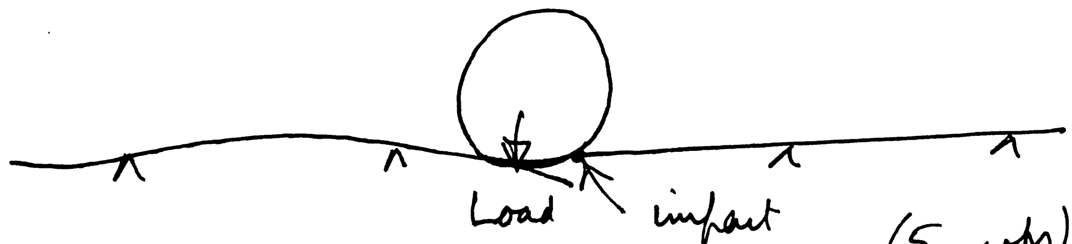
Therefore, expect subsequent failure over sleeper.

(3 marks)

(d) Sleepers will not act as clamped supports. Thus, the mid-span moment will be higher than $W/8$ with unbroken rail. Moment over sleeper in unbroken rail not affected by this assumption.

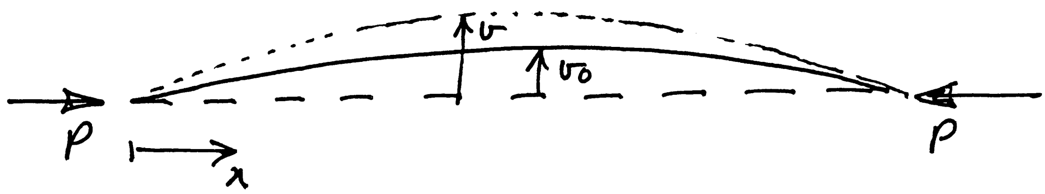
Ballast is not a rigid support. Thus, even in unbroken rail, with wheel over sleeper, there will still be bending of rail.

Wheel is round, so assumption of point load valid, but there will be a significant impact load when the rail is broken



(5 marks)

6 (a)



$$M = Pv = -EI\left(\frac{d^2v}{dx^2} - \frac{d^2v_0}{dx^2}\right)$$

$$\frac{d^2v_0}{dx^2} = -\frac{\pi^2}{L^2} v_0 \sin\frac{\pi x}{L}$$

$$\therefore \frac{P}{EI}\cdot v + \frac{d^2v}{dx^2} = -\frac{\pi^2}{L^2} v_0 \sin\frac{\pi x}{L}$$

Assume v = sinusoidal

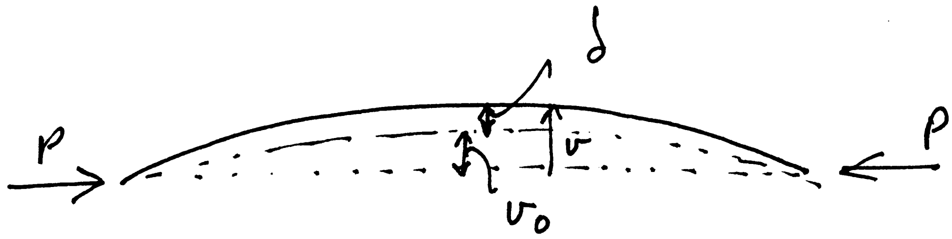
$$\therefore v = a \sin\frac{\pi x}{L} \qquad \frac{d^2v}{dx^2} = -a\frac{\pi^2}{L^2}\sin\frac{\pi x}{L}$$

Substitute and rearrange

$$a = \frac{v_0}{\left(1 - \frac{PL^2}{EI\pi^2}\right)} = \frac{v_0}{(1 - P/P_{cr})}$$

(7 marks)

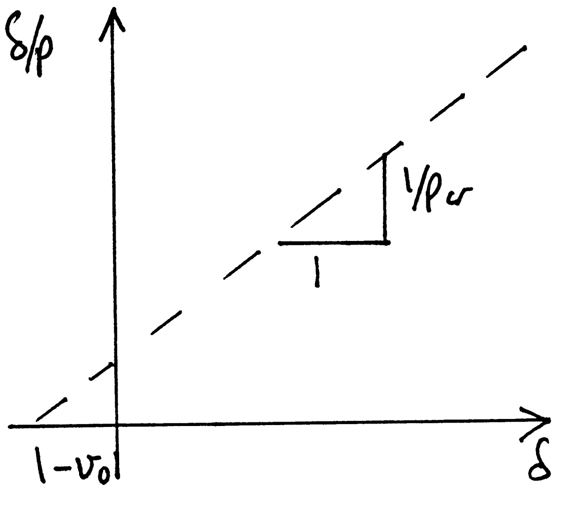
(b)



$$v = v_0 + \delta = \frac{v_0}{(1 - P/P_{cr})}$$

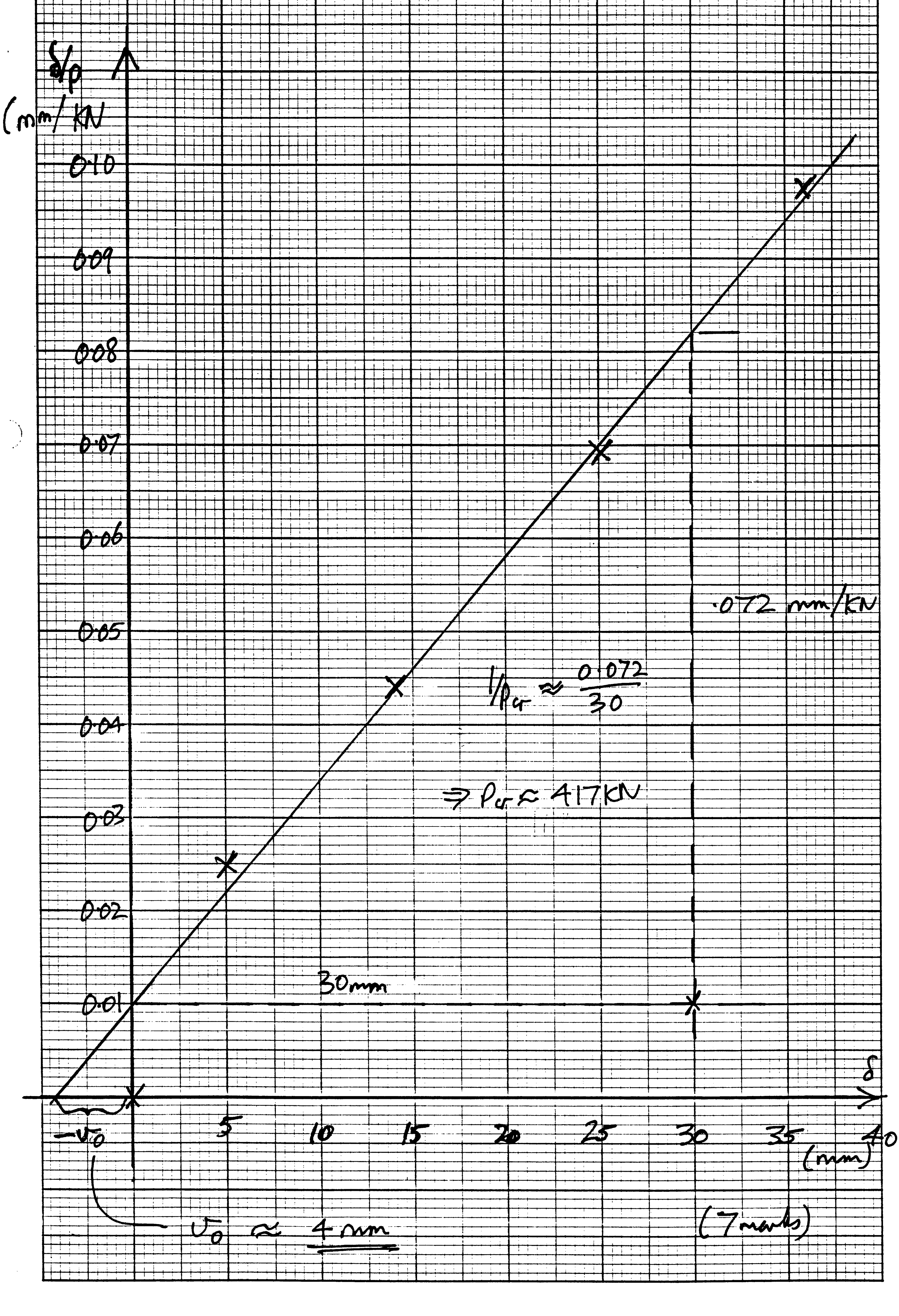
$$\Rightarrow v_0 - \frac{v_0 P}{P_{cr}} + \delta - \frac{\delta P}{P_{cr}} = v_0$$

$$\Rightarrow \frac{\delta}{P} = \frac{\delta}{P_{cr}} + \frac{v_0}{P_{cr}}$$ (3 marks)



(3 marks)

P2/6/3



δ/p
(mm/KN
0·10
0·09
0·08
0·07
0·06
0·05
0·04
0·03
0·02
0·01
·072 mm/KN
1/Pcr ≈ 0·072 / 30
⇒ Pcr ≈ 417KN
30mm
δ
−v0
5
10
15
20
25
30
35
40
(mm)
v0 ≈ 4 mm
(7 marks)